Exact Algorithms and Fixed-Parameter Tractability

Summary of Dagstuhl Seminar 05301

24. July–29. July 2005

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1 Motivation

It seems that by now almost everybody in our community has accepted that P $\neq$ NP holds true, although we do not have the slightest idea how to prove it. Regardless, P $\neq$ NP means that there are no polynomial time algorithms for NP-hard problems, and that super-polynomial time algorithms are the best we can hope for when dealing with exact algorithms for NP-hard problems.

Recently, there have been some fascinating activities on exact algorithms. For example, there has been a long sequence of papers on exact algorithms for 3-satisfiability. The current best algorithm for this problem is due to Iwama and Tamaki (2003) and needs roughly 1.324^n time for instances with n variables. Fomin and Kratsch (2003) have just developed a 1.993^n exact algorithm for determining the tree-width of an n-vertex graph - this is clearly not the last result on this problem. Fedin and Kulikov (2002) gave an exact $2^{m/4}$ algorithm for the Max-Cut problem on graphs with m edges. Williams (2004) gave an exact 1.732^n algorithm for Max-Cut in n-vertex graphs. There also has been a number of ever improving algorithms for the Vertex Cover problem; the current best is from Chandran and Grandoni (2004) and it finds a Vertex Cover of size k in a graph of size n in time $1.2745^k \cdot k^4 + kn$.

However, many very interesting questions are still wide open and have received little if any attention. We mention just two examples here:

- Held and Karp (1962) designed a dynamic programming algorithm for the n-city traveling salesman problem with a running time of roughly 2^n. This running time has not been improved in more than 40 years.
- Nesetril and Poljak (1985) observed that a simple algorithm based on matrix multiplication finds a clique of size k in an n-vertex graph in

Dagstuhl Seminar Proceedings 05301
Exact Algorithms and Fixed-Parameter Tractability
http://drops.dagstuhl.de/opus/volltexte/2006/439

time $n^{0.79k}$. No improvement has been made since then.

More qualitatively, we ask if there are algorithms solving the satisfiability problem or the traveling salesman problem in time $2^{o(n)}$ or the clique problem in time $n^{o(k)}$. Very little is known about such lower bound questions.

The idea of fixed-parameter tractability is to approach hard algorithmic problems by isolating problem parameters that can be expected to be small in certain applications and then develop algorithms that are polynomial in the instance size n, except for an arbitrary dependence on the parameter. More precisely, a problem is fixed-parameter tractable if its running time is $f(k)p(n)$, where f is an arbitrary function and p a polynomial. Since the choice of suitable parameters allows for a great flexibility, fixed-parameter algorithms have found their way into practical applications such diverse as computational biology, database systems, computational linguistics, and automated verification. The algorithmic methods developed in this area are not far from those used in the exact algorithms mentioned above. As a matter of fact, the fast algorithms for Vertex Cover have been developed in the context of fixed-parameter tractability. But beyond these algorithmic results, parameterized complexity also offers a well developed theory of intractability, and this theory may provide us with the right tools to systematically approach a theory of lower bounds for exact algorithms. For example, parameterized complexity allows us to establish exponential lower bounds on hard problems, modulo complexity assumptions. Indeed, recent work has substantiated the fact that this approach has deep links with the existence of feasible PTAS's, and limited nondeterminism; so we see the exciting emerging interplay between a number of hitherto unlinked groups of researchers.

To summarize, the area of Exact Algorithms is still in a rudimentary stage, but it is full of fascinating and difficult open problems. Fixed-parameter tractability is a branch of algorithms and complexity theory that seems very well suited to approach some of these questions. Connections between the two areas have recently evolved, ranging from very practical questions in algorithms design to the fundamental complexity theoretic problems.

2 The Workshop

The workshop brought together leading researchers from exact algorithms and parameterized complexity theory. If not before, it became very clear during the workshop that the areas are quickly growing together. Topics of the workshop ranged from very practical aspects of algorithm engineering for applications in computational biology to theoretical and mathematical topics such structural parameterized complexity and matroid theory. The technical program consisted of 7 invited one hour talks and 25 half hour talks. There will be a special issue of the journal *Theory of Computing Systems*, edited by Rod Downey, devoted to work arizing from this seminar.

The main topics of the seminar can be grouped under the following headings. We briefly mention some of the specific subjects and talks, for a more comprehensive picture we refer the reader to the list of abstracts of all talks given at the seminar.

Better Exact and Parameterized Algorithms

With new algorithms for a wide range of problems, this was the core topic of the workshop. U. Schöning gave an introductory survey talk on various techniques for designing exact algorithms. The Boolean satisfiability problem, probably the best studied problem in the field, was considerd in talks by A. Kulikov, M. Robson, and R. Schuler. P. Rossmanith presented a new algorithm for the Steiner tree problem, which marked the first real progress on this problem for more than 30 years. D. Marx presented algorithms for closest substring problems that employed seemingly unrelated results on hypergraphs in an intriguing way.

Applications and Algorithm Engineering

M. Langston reported on his collaboration with biologists, and in particular on how techniques from parameterized complexity for vertex cover and clique problems went into the design of algorithms for large scale biological problems. T. Tantau presented new algorithms for inferring haplotype information from genotype data. I. van Rooij, a Psychologist, explained how she uses ideas from parameterized complexity for modeling cognitive processes.

Decompositions of Graphs and Matroids and their Algorithmic Applications

After a general introduction to matroids by G. Whittle, P. Hlineny talked about branch decompositions of binary matroids and on his algorithm for efficiently computing such decompositions. S.I. Oum presented his work on rank width, which marked a breakthrough on several open questions related to clique width. This work has close connections to the matroid theory mentioned before. F. Fomin showed how fairly simple bounds on path width can be used to get improved exact algorithms for various problems.

New Developments in Parameterized Complexity

J. Flum presented the new theory of bounded fixed-parameter tractability, in which a singly exponential restriction is put on the parameter dependence of fpt-algorithms and thus a better correspondence between theory and practice can be achieved. J. Buss reported on new ways placing parameterized problems on the fitting levels of the W-hierarchy. Y. Chen presented new results establishing a close correpondence between parameterized complexity theory and subexponential complexity theory.