Analysis of the Parameters of Transfers in Rapid Transit Network Design

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Abstract. The rapid transit network design problem consists of the location of train alignments and stations in an urban traffic context. The originality of our study is to incorporate into the location model the decisions about the transportation mode and the route, to be chosen for urban trips. This paper proposes a new design model which includes transfers between train lines. The objective of the model is to maximize the number of expected users in the transit network taking limited budgets into consideration, in addition to location and allocation constraints. Furthermore, the transfer costs are considered in the generalized public costs when the users change lines. Waiting time to take the metro and walking time to transfer is included in the formulation of the costs. The analysis of transfer parameters is carried out using a test network. Some computational experience is included in the paper.

1 Introduction

Increasing mobility caused by the growth of cities is the reason why new lines of rail transit have been constructed. A crucial part of the network design process consists of the location of stations and alignments between them. In the paper [1] an approach to the network design problem, based on the previous selection of the key nodes (those providing a high number of trips) was described. Therefore, the transit network is defined on the edges which connect the key node set.

The transit system involves the node and edge locations at upper level and considers the user traffic behavior at lower level. At upper level the main factor

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to consider is maximum coverage of the demand using public mode, taking constraints of our model and the budget constraints into account. Traffic demand leads to alternative configurations of networks, comparing private trip cost with public trip cost, the latter depending on previous location decisions.

Customers choose the most convenient routes and modes in order to carry out their trips. A decisive factor for attracting passengers to the public mode is to offer direct trips without transfers. Transferring is annoying and it is undesirable for customers. In our approach, transfers are explicitly considered in accordance with the central role played by user mentality.

The previous references to the rapid transit network design (RTND), [2] , [3], [4], [5], [6], [7] consider travel cost as the time spent in travelling without taking into account any transfer cost.

The layout of the paper is as follows. In Section 2 the RTND is formulated including transfers between lines. In Section 3 the transfer costs are consistently introduced into the previous model. In Section 4 a transfer parametric analysis is implemented. The paper finalizes with conclusions and further research.

2 Rapid Transit Network Design Model with transfers

We assume that a set $N = \{i : i = 1, \dots, I\}$ of potential locations for the stations is given. Let E be the set of feasible edges linking the potential stations. Thus, we have an undirected graph $\mathcal{G} = (N, E)$ from which the transit network is going to be designed. For each node $i \in N$, let $N(i)$ denote the set of nodes adjacent to it. A matrix of distances $D = (d_{ij})$ between pairs of points of N is also known. The travel patterns are given by the origin-destination matrix $G = (g_p)$, where g_p is the demand of the pair $p = (q, r) \in P$ and P is the set of pairs of demand.

The cost structure is as follows. Let c_{ij} and c_i denote the cost of constructing a section of an alignment on edge ij and that of constructing a station at node i, respectively. According to the available budget the length of the public lines will be bounded; for this purpose, there are bounds $length^l_{\min}$, $length^l_{\max}$, $l = 1, \dots, L$, on the length of line l and bounds $Tlength_{\min}$ and $Tlength_{\max}$ on the total length of the lines of the network.

In regard to the demand, let uc_p^{PUB} be the user's generalized cost of travelling within the constructed transit network and let uc_p^{PRIV} be the user's generalized cost using the private mode. Observe that this cost does not depend on the final topology of the transit network.

The problem we are dealing with consists of choosing a number of lines $\mathcal{L} = \{l : l = 1, \dots, L\}$ covering as much as possible travel demand between the points of N, subject to the line length constraints and other constraints.

The decision variables are defined as follows:

• The station selection variables: $y^l_i = 1$, if station i of line l is constructed; and $y^l_i = 0$ otherwise.

• The edge selection variables: $x^l_{ij} = 1$, if edge ij of the line l is constructed; and $x^l_{ij} = 0$ otherwise.

• The following variables are defined for indicating whether or not the demand would use the transit network in case edge ij is selected. Specifically, $u_{ij}^p = 1$, if the demand of pair p would use edge ij in the public network, $u_{ij}^p = 0$ otherwise.

• Mode choice variables: $z_p = 1$ if the generalized cost for the demand of pair p within the public network $\mathcal{A}$ is less than that of the private mode; $z_p = 0$ otherwise.

• Flow routing variables: $w_{ij}^{pl} = 1$ if the demand p traverses the edge ij using line l, 0 otherwise.

• Transfer variables: $v_i^{pl} = 1$ if demand p transfers to line l in station i.

The RTND model with transfers is stated in the following terms:

• Objective function: Trip coverage

$$\max \sum_{p\in P} g_p z_p$$

• Length constraints

$$\sum_{ij\in E} d_{ij} x_{ij}^l \in [\text{length}_{\text{min}}^l, \text{length}_{\text{max}}^l], \quad l \in \mathcal{L} \tag{1}$$

$$\sum_{l\in\mathcal{L}} \sum_{ij\in E} d_{ij} x_{ij}^l \in [\text{Tlength}_{\text{min}}, \text{Tlength}_{\text{max}}] \tag{2}$$

• Alignment location constraints

$$\sum_{j\in N(o_l)} x_{o_l j}^l = 1, \quad l \in \mathcal{L} \tag{3}$$

$$\sum_{i\in N(d_l)} x_{id_l}^l = 1, \quad l \in \mathcal{L} \tag{4}$$

$$y_{o_l}^l = y_{d_l}^l = 1, \ l \in \mathcal{L} \tag{5}$$

$$\sum_{j\in N(i)} x_{ij}^l = 2y_i^l, \quad i \in N \setminus \{o_l, d_l\}, \ l \in \mathcal{L} \tag{6}$$

$$x_{ij}^l = x_{ji}^l \, ij \in E, \, l \in \mathcal{L} \tag{7}$$

• Routing demand constraints

$$\sum_{j\in N(q)} u_{qj}^p = 1, \quad p = (q,r) \in P \tag{8}$$

$$\sum_{i\in N(q)} u_{iq}^p = 0, \quad p = (q,r) \in P \tag{9}$$

$$\sum_{i\in N(r)} u_{ir}^p = 1, \quad p = (q,r) \in P \tag{10}$$

$$\sum_{j\in N(r)} u_{rj}^p = 0, \quad p = (q,r) \in P \tag{11}$$

$$\sum_{i\in N(j)} u_{ij}^p - \sum_{k\in N(j)} u_{jk}^p = 0, \ j \in N \setminus \{q,r\}, p = (q,r) \in P \tag{12}$$

• Location-Allocation constraints

$$u_{ij}^p + z_p - 1 \le \sum_{l=1}^{L} x_{ij}^l, \ , \ ij \in, \ p = (q,r) \in P \tag{13}$$

• Splitting demand constraints

$$uc_p^{PUB} - uc_p^{PRIV} - M(1 - z_p) \le 0 \, p = (q,r) \in P \tag{14}$$

where uc_p^{PRI} is a data and uc_p^{PUB} will be defined in the next subsection. M is an enough big number.

- Transfer constraints

$$w_{ij}^{pl} \leq x_{ij}^{l}, \quad ij \in E, p = (q,r) \in P, l = 1, \ldots, L \tag{15}$$
$$u_{ij}^{p} + z_p - 1 \leq \sum_{l=1}^{L} w_{ij}^{pl}, \quad p = (q,r) \in P, \quad ij \in E \tag{16}$$
$$u_{ij}^{p} - z_p + 1 \geq \sum_{l=1}^{L} w_{ij}^{pl}, \quad p = (q,r) \in P, \quad ij \in E \tag{17}$$
$$\sum_{ij \in E, l \in \mathcal{L}} w_{ij}^{pl} \leq M\, z_p, \quad p = (q,r) \in P \tag{18}$$
$$\sum_{j \in N(i)} w_{ij}^{pl} - \sum_{j \in N(i)} w_{ji}^{pl} \geq 2\, v_i^{pl} - 1, \quad i \in N \setminus \{r\}, \quad p = (q,r) \in P, \quad l \in \mathcal{L} \tag{19}$$
$$\sum_{j \in N(i)} w_{ij}^{pl} - \sum_{j \in N(i)} w_{ji}^{pl} \leq 2\, v_i^{pl}, \; i \in N \setminus \{r\}, \quad p = (q,r) \in P, \quad l \in \mathcal{L} \tag{20}$$
$$x_{ij}^{l}, y_i^{l}, u_{ij}^{p}, z_p, w_{ij}^{pl}, v_i^{pl} \in \{0,1\}.$$

Constraints (1) and (2) impose lower and upper bounds on the individual and total line lengths.

Constraints (3) and (4) guarantee that each line starts and ends at its specified origin and destination. Constraints (5) ensure that all the origins and destinations belong to $\mathcal{A}$. Constraints (6) impose that each line must be a path between the corresponding origin and destination.

Constraints (7), (8) and (9) guarantee demand conservation. Constraints (10) and (11) were introduced to ensure that identity $z_p = 1$ implies that the demand of the pair p goes through the public network and $z_p = 0$ if it uses the private network. Constraints (12) guarantee that the demand is routed on an edge only if this edge belongs to the public system.

Constraints (13) ensure that the demand is routed on an edge if it has been previously constructed. Constraints (14) force the demand to be assigned to public mode if public cost is less than private cost.

Note that this formulation does not include the common sub-tour elimination constraints. Therefore, when a solution contains a cycle, additional constraints can be imposed in order to avoid the presence of cycles in the solution network. Note that well developed networks (e.g. Paris, London, Moscow, Tokyo and Madrid) often contain circular lines. It has also been proved by Laporte, Mesa and Ortega (1997, [**?**]) that the inclusion of a circle line increases the effectiveness of the network and thus the inclusion of cycles can be interesting.

Transfer constraints (15) permit the flow to use edge ij of line l only if edge ij have already been established for public mode. Constraints (16) guarantee that if the flow of edge ij is carried through any line, the public mode and its flow at edge ij must already be chosen. Constraints (17) establish that any flow for the pair p can use any edge of the public network. Constraints (18) impose that if a transfer is made at node i then the flow leaving from this node is bigger than the flow coming in. Constraints (19) and (20) impose that if the flow leaving out is less than the flow coming in at a node i, then a transfer is made at this node.

3 Transit costs in transfers

Time spent in transference between lines is characterized by certain cost parameters. We have studied different values of the parameters which define transfer cost in order to conclude how they influence the RTND solution.

The paper is focused on the study of the increase in the cost using the public mode, assuming that users may transfer from one line to another. The transfer process has been modeled at RTND through constraints (15) to (20), but now we will give details about how the public mode cost is influenced by transfers.

The public cost for each demand uc_p^{PUB} is the sum of two terms: one term relative to *travel time* spent moving in the transit vehicle on the rapid transit network and another term related to the *transfer time* spent in transferring from one line to another.

The first term has been considered in all the references, and it is computed by the traveling time, which is calculated by the sum of the travel distance divided by the average velocity of the lines $\hat{\lambda}$. Hence, non-transfer public costs (NTPUB) are defined as follows:

$$uc_p^{NTPUB} = \frac{1}{\hat{\lambda}} \sum_{ij \in E} d_{ij} u_{ij}^p, \; p = (q, r) \in P \; (16)$$

In the concept of vehicle velocity, we include vehicle moving time and the time spent at the station required to permit boarding and alighting of the passengers. These values are average values for standard lines. The line velocity may be considered by taking an average value of 20 kilometers per hour. In that case, the average velocity, $\hat{\lambda}$, is 1/3 kilometers per minute.

As was pointed out below, the demand is very sensible to transfer time. Thus, the transfer cost of each demand and station is considered as the sum of two terms: 1) one value fixed for each station i, uc_i, that represents the average walking time between line platforms and 2) another value on the waiting time for taking the next train of a different line.

In our approach we assume uc_i as a parameter which depends on the travel time spent for any demand that transfers at station i from the previous board platform line up to the board platform of the next line. This value is given in minutes and it can model the cumulative sum of walking times between platforms and the annoyance associated to the transfers, which depends on the station characteristics. This value would be in average about 3 to 5 minutes.

The transfer waiting cost depends on the frequency of the line of the train to take in order to continue the trip towards the destination. The line frequency is considered fixed and represents a parameter of the model. Considering that the planning period is of 1 hour, and that the time cost will be given in minutes, the wait time is assumed equal to one divided by twice the frequency of the line, $2 * f_l$. So if the average frequency is of 6 vehicle per hour then the average waiting time is of 5 minutes. For a line with double frequency, 12 vehicles per hour, the waiting time will be of 2.5 minutes.

With these considerations, the public cost expression with transfer of a pair p is:

• UserPublic cost
$uc_p^{PUB} = \frac{1}{\lambda} \sum_{ij \in E} d_{ij} u_{ij}^p + \sum_{i \in N \setminus \{r\}, l \in L_i} \left(uc_i + \frac{1}{2f_l} \right) v_i^{pl}$
$$p = (q, r) \in P \qquad (17)$$

where L_i is the set of lines that use the node i.

In this context, for a distance of 3 kilometers and for an average velocity of 20 km/h, the travel time is 9 minutes. If the fixed transfer time is of 3 minutes more, and the wait time of 5 minutes, the total travel time is of 17 minutes.

4 Transfer parametric analysis

The previous model has been tested on the 6-node network shown in Figure 1, where each node i has an associated cost (variable c_i in the model) and each edge has been weighted by means of a pair (c_{ij}, d_{ij}) its components respectively representing the cost of constructing edge ij and the generalized public cost of using edge ij to connect both nodes.

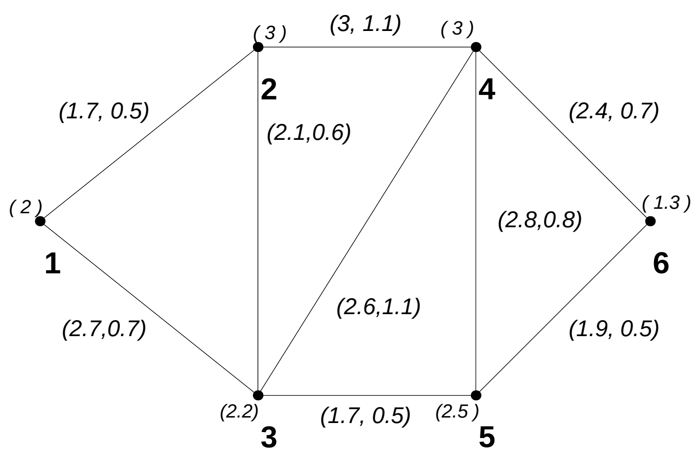

Fig. 1. Network considered.

The origin-destination demands $g_p, p = (q, r) \in P$ and the private cost uc_p^{PRIV} for each demand pair $p \in P$ are given in the following matrices:

$$\text{G} = \begin{pmatrix} - & 9 & 26 & 19 & 13 & 12 \\ 11 & - & 14 & 26 & 7 & 18 \\ 30 & 19 & - & 30 & 24 & 8 \\ 21 & 9 & 11 & - & 22 & 16 \\ 14 & 14 & 8 & 9 & - & 20 \\ 26 & 1 & 22 & 24 & 13 & - \end{pmatrix};$$

$$
\mathrm{uc}^{\mathrm{PRIV}} = \begin{pmatrix} - & 1.6 & 0.8 & 2 & 2.6 & 2.5 \\ 2 & - & 0.9 & 1.2 & 1.5 & 2.5 \\ 1.5 & 1.4 & - & 1.3 & 0.9 & 2 \\ 1.9 & 2 & 1.9 & - & 1.8 & 2 \\ 3 & 1.5 & 2 & 2 & - & 1.5 \\ 2.1 & 2.7 & 2.2 & 1 & 1.5 & - \end{pmatrix}
$$

Bounds on maximum and minimum lengths of the total public network have been established at 4 and 2, respectively. The solutions presented in the following scenarios have been obtained by using CPLEX 8.0 on a Pentium IV laptop computer at 2.56 Mghz, provided by 1 Gbyte of RAM.

Although different parametric analyses can be carried out for the model, we have emphasized the sensitivity of the solutions with respect to the dispersion of values associated to transfer costs.

Applying a calibration process to the public costs of the edges, the centralized value $uc_{ij}^{PUB} = 0.685$ was obtained. Taking this result into account, the value 0.75 was considered a central value. Different decrements and increments of size 0.25 have been used to grade the dispersion with respect to the average 0.75. Therefore, a variation of modulus 0.25 will be identified as low dispersion and if the difference with respect to the central value is 0.50, then we will say that the dispersion is high.

4.1 The effect of varying the transfer cost between lines on the configuration of the optimum network

An average speed of 20 km/h has been assumed for transit on all lines. The cost for transferring was established at 3 min. Private cost was six times higher than public cost; this proportion stimulates a desirable competition between transportation modes. The congestion level has been assumed equal to 1.5 remaining outside of this first parametric analysis.

The following tables show optimum configurations of the rapid transit networks for the constraints indicated; namely, the number of lines that will compose the final network and the range for the maximum length of each line. Each table deals with a different scenario:

Table 1 : All transfer costs coincide in a central value (0.75).
Table 2 : Low dispersion for the distribution of transfer costs in a 2-line network (0.5 and 1).
Table 3 : Low dispersion for the distribution of transfer costs in a 3-line network (0.5, 0.75 and 1).
Table 4 : High dispersion for the distribution of transfer costs in a 2-line network (0.25 and 1.25).
Table 5 : High dispersion for the distribution of transfer costs in a 3-line network (0.25, 0.75 and 1.25).

The analysis of the results lead us to the following conclusions:

Line Num.	Length Range	Optimum Lines	Obj. Func.	Line Lengths	Exec. Time(s)
2	[0.5, 2.5] [0.5, 2.5]	n_1-n_2-n_3-n_4 n_3-n_5-n_6-n_4	444	2.2 1.7	57.7
3	[0.5, 1.5] [0.5, 1.5] [0.5, 1.5]	n_3-n_5-n_6 n_1-n_3-n_2 n_3-n_4	404	1 1.3 1.1	1666.72

Table 1. All transfer coefficients are equal to 0.75

Line Num.	Length Range	Optimum Lines	Obj. Func.	Line Lengths	Exec. Time(s)
2	[0.5, 2.5] [0.5, 2.5]	n_2-n_1-n_3-n_5-n_4 n_5-n_6-n_4	456	2.5 1.2	95.23
2	[0.5, 3] [0.5, 2.5]	n_1-n_3-n_5-n_4-n_6 n_2-n_3	470	2.7 0.6	50.5
2	[0.5, 3.5] [0.5, 2.5]	n_2-n_1-n_3-n_5-n_4-n_6 n_2-n_3	470	3.2 0.6	61.2

Table 2. Low dispersion of transfer coefficients (2 lines)

1. A wide range for the line lengths produces better values of the objective function.
2. When the dispersion of transfer cost increases (i.e., waiting time is very heterogeneous for all lines), the flow shift is higher. Subsequently, the best results for a 2-line network are obtained when the dispersion is low and, on the other hand, the objective function reaches higher values for the 3-line network when the dispersion is high. This fact does not alter although the range of the line lengths varies.
3. The required execution time descends when the dispersion of transfer costs and the range for line lengths increase, which behaves even better on the 3-line network, as the tables show.

4.2 The effect of varying the train frequency of the lines on the network configuration

The following tables show optimum configurations of the rapid transit networks for the train frequency indicated.

Table 6 : Low frequency of the train flow on the 2-line network.
Table 7 : High frequency of the train flow on the 2-line network.

Line Num.	Length Range	Optimum Lines	Obj. Func.	Line Lengths	Exec. Time(s)
3	[0.5, 1.5] [0.5, 1.5] [0.5, 1.5]	n_1-n_3-n_2 n_3-n_5-n_4 n_3-n_5-n_6	425	1.3 1.3 1	1252.69
3	[0.5, 2] [0.5, 2] [0.5, 1.5]	n_1-n_3-n_5-n_4 n_2-n_3 n_5-n_6-n_4	470	2 1.2 0.6	248.61
3	[0.5, 2.5] [0.5, 2.5] [0.5, 1.5]	n_1-n_3-n_5-n_4 n_5-n_6-n_4 n_2-n_3	470	2 1.2 0.6	248.61
3	[0.5, 3] [0.5, 2.5] [0.5, 1.5]	n_1-n_3-n_5-n_4-n_6 n_2-n_3 n_1-n_2	470	2.7 0.6 0.5	173.64

Table 3. Low dispersion of transfer coefficients (3 lines)

Line Num.	Length Range	Optimum Lines	Obj. Func.	Line Lengths	Exec. Time(s)
2	[0.5, 2.5] [0.5, 2.5]	n_1-n_3-n_5-n_6-n_4 n_2-n_3	447	2.4 0.6	64.63
2	[0.5, 3] [0.5, 2.5]	n_2-n_1-n_3-n_5-n_6-n_4 n_2-n_4	456	2.9 1.1	46.81
2	[0.5, 3.5] [0.5, 2.5]	n_1-n_3-n_5-n_4-n_6 n_2-n_3	456	2.7 0.6	38.81

Table 4. High dispersion of transfer coefficients (2 lines)

4.3 The effect of varying the train frequency on the remainder parameter set

Table 8 contains the results obtained for a 3-line network whose congestion level is 5, range for line lengths is $[0, 2.5]$ and total network length is less than 5. Moreover, the maximum value for the objective function is 496 (the total demand) and $\lambda = 0.33$.

The main aim of this subsection consists of showing how the solution varies when the line frequency increases. For this purpose, the first column of Table 8 collects an increasing sequence of values for the frequency while the remainder columns show the associated values in relation to the objective function, the total length and the execution time.

As can be noted, when frequency increases, the waiting time for riderships decreases and, subsequently, the use of public network increases (as shows the sequence of values corresponding to the objective function).

In relation to the total length of the lines, Table 8 shows, from a frequency greater than 6, how the system does not require a minimization of the location

Line Num.	Length Range	Optimum Lines	Obj. Func.	Line Lengths	Exec. Time(s)
3	[0.5, 1.5] [0.5, 1.5] [0.5, 1.5]	n_1-n_3-n_2 n_3-n_5-n_4 n_3-n_5-n_6	446	1.3 1.3 1	243.13
3	[0.5, 2] [0.5, 2] [0.5, 1.5]	n_2-n_1-n_3-n_5 n_3-n_4-n_6 n_5-n_6	456	1.7 1.8 0.5	432.17
3	[0.5, 2.5] [0.5, 2.5] [0.5, 1.5]	n_1-n_3-n_5-n_6-n_4 n_2-n_3 n_3-n_5	470	2.4 0.6 0.5	194.06
3	[0.5, 3] [0.5, 2.5] [0.5, 1.5]	n_5-n_3-n_2-n_4-n_6 n_1-n_2 n_5-n_6	470	2.9 0.5 0.5	142.63

Table 5. High dispersion of transfer coefficients (3 lines)

Congestion Level	Optimum Lines	Obj. Func.	Line Costs	Exec. Time(s)
1.2	n_1-n_2-n_3-n_5-n_6-n_4 n_5-n_6	331	23.8 5.7	494.47
1.5	n_1-n_2-n_3-n_5-n_6-n_4 n_5-n_6	444	23.8 5.7	97.76
2.2	n_2-n_3-n_5-n_6-n_4 n_1-n_3	496	20.1 6.9	22.99

Table 6. Frequency interval = (12 trains per hour, 6 trains per hour).

cost and then, takes advantage of all available resources although the improvement in terms of the objective function is minimum.

Calculation times considerably increase from frequencies greater than 10, due to the options of finding efficient routes inside the public network increase. Subsequently, since the size of possible solutions is greater, the time for exploring will become higher.

5 Comparative tests

This section is devoted to the comparison between results obtained in presence/absence of transfers. The general context for parameter values (network with 3 lines at most, congestion factor equals to 5 and range for total length equals to $[0, 5]$) remains.

Congestion Level	Optimum Lines	Obj. Func.	Line Costs	Exec. Time(s)
1.2	n_1-n_2-n_3-n_5-n_6-n_4 n_5-n_6	414	23.8 5.7	253.14
1.5	n_2-n_3-n_5-n_4-n_6 n_1-n_3	470	21 6.9	297.95
2.2	n_1-n_2-n_3-n_5 n_2-n_4-n_6	496	15.2 12.7	0.63

Table 7. Frequency interval = (20 trains per hour, 10 trains per hour)

Frequency	Obj. Function	Total length	Exec. Time
4	14	4.7	3.51
5	27	3.3	4.87
6	47	4.9	11.63
7	115	4.5	9.95
8	132	4.8	3.75
9	172	4.8	9.4
10	174	4.5	36.52
12	207	4.8	262.07
14	282	4.8	152.81
16	282	4.8	282.51
18	301	4.8	311.2
20	301	4.6	297.96
30	341	4.9	336.03

Table 8. Influence of line frequency on the other parameters

5.1 The effect of varying the maximum lengths of the lines

Assuming a model without transfers where the speed in the public mode of transportation is 20 km/h, frequency is 10 trains per hour for all lines and an access time for boarding equals to 3 minutes, Tables 9 and 10 respectively show the optimum configurations, the objective function values, the line lengths and the execution times for the model with and without transfers.

As can be observed, when the line capacity in the model with transfers increases, the portion of demand which uses the public network suffers an increment but limited by a maximum value (174).

For the case without transfers, this upper boundary does not appear due to all the existing demand chooses the public mode of transportation, once the capacity of the lines can take values greater.

Computation time is higher in the model with transfers as consequence of its inherent complexity.

Max. Line Length	Lines	Obj. Function	Line Length	Exec. Time
[0.5,1]	n_3-n_5-n_6 n_4-n_5 n_1-n_2	89	1 0.8 0.5	9.1
[0.5,1.5]	n_1-n_3-n_5 n_3-n_5-n_6 n_4-n_5-n_6	112	1.2 1.0 1.5	276.2
[0.5,2]	n_1-n_3-n_5-n_6 n_5-n_4-n_6 n_1-n_2-n_4	153	1.7 1.5 1.6	271.69
[0.5,3]	n_1-n_2-n_3-n_5-n_6 n_4-n_5 n_3-n_4	174	2.1 0.8 1.1	36.59
[0.5,4]	n_1-n_2-n_3-n_5-n_6 n_4-n_5-n_6 n_3-n_4	174	2.1 1.3 1.1	22.23

Table 9. Model with Transfers

5.2 The effect of varying the maximum length of the network

The constraint of maximum length for the total network gives rise to different configurations (see Tables 11 and 12) in relation to the line number whose maximum was assumed 3. Results obtained suggest similar conclusions to the obtained in the previous test.

5.3 The effect of varying the congestion factor

In this section, models with and without transfers are compared from the point of view of the congestion influence. Both settings consider networks composed of three lines at maximum.

The values of the model parameters are the same: an average speed of 20 km/h assumed for the transit on all lines, cost for transferring 3 minutes and frequency of 10 trains per hour. Line lengths belong to range [0.5, 2.5] and total length of the network is in interval the [0, 4].

Table 13 states that when the congestion in the private mode with transfers increases then a higher portion of the demand uses the public network. Specifically, the maximum possible demand (496) is reached for a congestion factor equals to 15 and a total length of the network close to 4.

The model without transfers (Table 14) has a similar behaviour but the increasing in the preference of using the public mode is more significative.

Max. Line Length	Lines	Obj. Function	Line Length	Exec. Time
[0.5,1]	n_3-n_5-n_6 n_1-n_3 n_4-n_6	368	1.0 0.7 0.7	3.73
[0.5,1.5]	n_4-n_5-n_6 n_1-n_3-n_5 n_2-n_3	496	1.3 1.2 0.6	2.084
[0.5,2]	n_2-n_1-n_3-n_5 n_4-n_6-n_5 n_2-n_3-n_4	496	1.7 1.2 1.7	0.24
[0.5,3]	n_1-n_3-n_4-n_6 n_2-n_3-n_5-n_6	496	2.5 1.6	0.17
[0.5,4]	n_1-n_3-n_4-n_6 n_2-n_3-n_5-n_6	496	2.5 1.6	0.19

Table 10. Model without Transfers

Max Length	Lines	Obj. Function	Line Length	Exec. Time
[0,1]	n_3-n_5-n_6	38	1	3.78
[1,2]	n_1-n_3-n_5-n_6	77	1.7	64.05
[1,3]	n_1-n_2-n_3-n_5-n_6 n_4-n_5	147	2.1 0.8	271.69
[1,4]	n_1-n_2-n_3-n_5-n_6 n_4-n_5-n_6 n_1-n_2	163	2.1 1.3 0.5	75
[1,5]	n_1-n_2-n_3-n_5-n_6 n_3-n_4-n_5-n_6 n_3-n_5	174	2.1 2.4 0.5	29.36

Table 11. Model with Transfers

6 Conclusions and further research

The influence of transfer cost parameters have been studied for a test network in the context of the Rapid Transit Network Design problem. It has been established that an adequate penalization of the transfers has a big influence on travel behavior demands. Therefore, the public cost variable is sensitive to the transfer cost, and this variable is basic for network design decisions.

Another parameter analysis carried out in the paper deals with the influence of the line frequency, which has been assumed as a given fixed parameter which depends on design variables. Specifically, the line frequency is obtained as a function of the number of stations and the length of the lines. Again, it has been shown that these design variables have a direct influence on the time spent by a user moving along the lines. A more profound parametric analysis is our next

Max Length	Lines	Obj. Function	Line Length	Exec. Time
[0,1]	n_3-n_5-n_6	95	1	3.86
[1,2]	n_1-n_3-n_5-n_4	227	2	17.52
[1,3]	n_4-n_5-n_6 n_2-n_3-n_5 n_1-n_2	496	1.3 1.1 0.5	0.14
[1,4]	n_5-n_3-n_4-n_6 n_1-n_3-n_2	496	1.3 2.3	0.06
[1,5]	n_4-n_3-n_5-n_6-n_4 n_1-n_3-n_2	496	2.8 1.3	0.06

Table 12. Model without Transfers

Congestion Factor	Lines	Obj. Function	Line Length	Exec. Time
5	n_1-n_2-n_3-n_5-n_6 n_1-n_2 n_4-n_5-n_6	163	2.1 0.5 1.3	46.99
7	n_1-n_3-n_5-n_6-n_4 n_1-n_2 n_1-n_2-n_3	322	2.4 0.5 1.1	183.18
10	n_1-n_2-n_3-n_5-n_4 n_4-n_6-n_5	470	2.4 1.2	36.68
15	n_4-n_3-n_5-n_6 n_1-n_2-n_3 n_1-n_2	496	2.1 1.1 0.5	19.91

Table 13. Model with Transfers

objective in order to broaden the research thus gaining more insight into the rapid transit network design problem.

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Congestion Factor	Lines	Obj. Function	Line Length	Exec. Time
5	n_1-n_3-n_4-n_6 n_3-n_5 n_2-n_3	496	2.5 0.5 0.6	0.11
7	n_1-n_3-n_4-n_6 n_3-n_5 n_2-n_3	496	2.4 0.5 1.1	0.24
10	n_1-n_3-n_5-n_4-n_6 n_1-n_2-n_4	496	2.4 1.6	1.24
15	n_1-n_2-n_4-n_6 n_4-n_3-n_5	496	2.3 1.6	0.66

Table 14. Model without Transfers

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