Fair Division

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The problem of fair division—dividing goods or "bads" (e.g., costs) among entities in an impartial and equitable way—is one of the most important problems that society faces. A Google search on the phrase "fair allocation" returns over 100K links, referring to the division of sports tickets, health resources, computer networking resources, voting power, intellectual property licenses, costs of environmental improvements, etc.

There is an enormous but scattered literature on fair division in the fields of economics, political science, mathematics, operations research, and computer science, among others. In the recent years, there have been several academic books, and one popular book, on the subject.

Predictably, researchers in different disciplines study different aspects of fair division. They publish in different journals, attend different conferences, and even use different terminology. Thus, the impact of a development in one field may take years to be felt in another field.

Many problems that arise in fair division demand formal protocols, in part because of the many actors or the numerous activities they undertake that must be processed, and in part because of the need for consistency and transparency. For example, the 1982 Convention of the Law of the Sea, which was signed by 159 countries, specifies a simple cut-and-choose protocol for dividing seabed mining tracts. As more business and society interactions migrate to the web, it will become even more critical to have formal, well-studied protocols for fair division.

The general setting for most academic research is simple: There is a collection of goods or bads that need to be divided among a set of entities, but there are conditions on feasible allocations. For example, if the goods to be divided are divisible, like money or land, the situation is very different from that in which the goods are indivisible, such as most marital property in a divorce.

There are many ways to formalize "fairness," including max-min fairness, proportional fairness, envy-free fairness, etc. These variations may or may not lead to stable allocations, resulting in so-called Nash equilibria in a game.

Recognizing the problem created by different definitions of, and approaches to, fair division, we invited top researchers and promising young scientists—including a few advanced graduate students—to the seminar. We encouraged the top researchers, several

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of whom had authored books or done pioneering work in their fields, to outline major research approaches and discuss important open problems. Most of the young scientists reported on their research, which tended to reflect the latest trends and innovative tools that have been applied in a variety of areas. All speakers were asked to avoid highly technical or specialized vocabulary so that people outside their disciplines could better understand the questions and issues they were raising.

To give some flavor of the contributions, we briefly mention some of the main themes of the different presentations.

1. Economists and political scientists analyzed and compared various axioms of fairness, showing which were compatible and which were not. They also discussed problems that arise in implementing various procedures, some of which become manifest in experiments and some of which arise in real-world applications.

2. Economists and game theorists analyzed protocols that induce actors to be truthful in reporting their interests or valuations of items. In the absence of such protocols, problems of "incentive compatibility," which can lead to attempts to manipulate procedures, occur.

3. Computer scientists and mathematicians analyzed the complexity of achieving various goals. For example, a proof that a certain property of fairness requires a complex protocol may make an economist reconsider whether this property is sufficiently important to justify the increased complexity it requires. Approximate algorithms may be used in such circumstances.

4. Computer scientists addressed the "price of anarchy," which occurs when selfish choices on a network lead to inferior outcomes. Such issues often come up in scheduling tasks efficiently and equitably.

5. Computer scientists and game theorists analyzed rules that guarantee players minimum payoffs, whatever the choices of other players. Most of the literature n cake cutting uses such rules, which are especially useful if the players are "risk-averse. But the resulting outcomes may lead to inferior payoffs for the players, which they can sometimes improve on without such guarantees.

6. Mathematicians analyzed the consequences of measurability assumptions, such as whether players' measures are atomic or nonatomic, countably additive or finitely additive, etc. Different assumptions about measurability allow for different divisions. Thus, in dividing land, noncontiguous pieces may have less value than contiguous pieces, illustrating measures that are not additive.

To conclude, we believe the seminar opened up the eyes of many participants to aspects of fair division not normally studied within their own disciplines. The lively intellectual interchange may well spawn cross-disciplinary research collaborations. In fact, we know of three participants from different disciplines who met at the seminar and are now collaborating on a joint paper.