07261 Abstracts Collection
Fair Division
— Dagstuhl Seminar —

Steven J. Brams[1], Kirk Pruhs[2] and Gerhard Woeginger[3]

[1] New York Univ., US
steven.brams@nyu.edu
[2] Univ. of Pittsburgh, US
kirk@cs.pitt.edu
[3] TU Eindhoven, NL

Abstract. From 24.06. to 29.06.2007, the Dagstuhl Seminar 07261 "Fair Division" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Economics, Fairness, Allocation, Political Science

07261 Summary – Fair Division

The problem of fair division-dividing goods or "bads" (e.g., costs) among entities in an impartial and equitable way-is one of the most important problems that society faces. A Google search on the phrase "fair allocation" returns over 100K links, referring to the division of sports tickets, health resources, computer networking resources, voting power, intellectual property licenses, costs of environmental improvements, etc.

Keywords: Economics, Fairness, Allocation, Political Science

Joint work of: Brams, Steven J.; Pruhs, Kirk; Woeginger, Gerhard

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2007/1243

The Geometry of Efficient Fair Division

Julius B. Barbanel (Union College - Schenectady, USA)

We consider the problem of dividing a "cake" C among n players. Our cake C is a measure space on which each player uses a countably additive non-atomic probability measure to evaluate pieces of cake, i.e., measurable subsets of the measure space.

We consider two natural geometric structures that one can associate with this situation. One we call the Individual Pieces Set, or IPS, and the other we call the Radon-Nikodym Set, or RNS.

Let us suppose that the measures used by the players are $m_1, m_2, \ldots, m_n$. The IPS is the set

$\{\langle m_1(P_1), m_2(P_2), \ldots, m_n(P_n)\rangle \colon \langle P_1, P_2, \ldots, P_n\rangle$ is a partition of C into measurable subsets$\}$

Define a new measure $\mu = m_1 + m_2 + \ldots + m_n$. For each $i \in \{1, 2, \ldots, n\}$, let $f_i \colon C \to \mathbf{R}$ be the Radon-Nikodym derivative of m_i with respect to μ. Then, for each such i and any measurable $A \subseteq C$, $m_i = \int_A f_i d\mu$ and, for every $a \in C$, $f_1(a) + f_2(a) + \ldots + f_n(a) = 1$ (except possibly on a measure zero set). Let S denote the standard simplex in $\mathbf{R}^n$ (i.e., $S = \{\langle x_1, x_2, \ldots, x_n\rangle \colon$ each $x_i \geq 0$ and $x_1 + x_2 + \ldots + x_n = 1\}$). For each $a \in C$, we define $f(a) = \langle f_1(a), f_2(a), \ldots, f_n(a)\rangle$. Then (after redefining on a set of measure zero if necessary), for every such a, $f(a) \in S$, and so f is a function from C to S. Intuitively, $f(a)$ describes the relative worth that each player attaches to a, as compared to the other players. The RNS is the set $\{f(a) \colon a \in C\}$.

We will discuss how envy-freeness, Pareto maximality, and other fairness and efficiency properties are reflected in the IPS, in the RNS, and in the interaction between these two structures.

Keywords: Fair division, envy-free, Pareto maximal

Divide-and-Conquer: A Proportional, Minimal-Envy Cake-Cutting Procedure

Steven Brams (New York University, USA)

Properties of discrete cake-cutting procedures that use a minimal number of cuts (n - 1 if there are n players) are analyzed. None is always envy-free or efficient, but divide-and-conquer (D& C) minimizes the maximum number of players that any single player may envy. It works by asking n ? 2 players successively to place marks on a cake that divide it into equal or approximately equal halves, then halves of these halves, and so on. Among other properties, D& C (i) ensures players of more than 1/n shares if their marks are different and (ii) is strategyproof for risk-averse players. However, D& C may not allow players to obtain proportional, connected pieces if they have unequal entitlements. Possible applications of D& C to land division are briefly discussed.

Keywords: Cake-cutting, proportionality, envy-freeness, efficiency, strategyproofness

Joint work of: Brams, Steven J.; Jones, Michael A.; Klamler, Christian

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1221

Cake Cutting Really is Not a Piece of Cake and Balanced Allocations of Cake

Jeff Edmonds (York University - Toronto, CA)

We consider the well-known cake cutting problem in which a protocol wants to divide a cake among $n \geq 2$ players in such a way that each player believes that they got a fair share. The standard Robertson-Webb model allows the protocol to make two types of queries, Evaluation and Cut, to the players. A deterministic divide-and-conquer protocol with complexity $O(n \log n)$ is known.

We provide the first an $\Omega(n \log n)$ lower bound on the complexity of any deterministic protocol in the standard model. This improves previous lower bounds, in that the protocol is allowed to assign to a player a piece that is a union of intervals and only guarantee approximate fairness. We accomplish this by lower bounding the complexity to find, for a single player, a piece of cake that is both rich in value, and thin in width. We then introduce a version of cake cutting in which the players are able to cut with only finite precision. In this case, we can extend the $\Omega(n \log n)$ lower bound to include randomized protocols.

We give a randomized algorithm for the well known caking cutting problem that achieves approximate fairness, and has complexity $O(n)$.

The heart of this this result involves extending the standard offline multiple-choice balls and bins analysis to the case where the underlying resources/bins/machines have different utilities to different players/balls/jobs.

Keywords: Cake Cutting nlogn lower bound and O(n) upper bound

Joint work of: Edmonds, Jeff; Pruhs, Kirk

Equilibria for two parallel links: The strong price of anarchy versus the price of anarchy

Leah Epstein (University of Haifa, IL)

Following recent interest in the "strong price of anarchy" SPOA), we consider this measure, as well as the well known "price of anarchy" (POA) for the job scheduling problem on two uniformly related parallel machines (or links). The atomic players are the jobs, and the delay of a job is the completion time of the machine running it. The social goal is to minimize the maximum delay of any job. Thus the cost (or social cost) in this case is the makespan of the schedule. The selfish goal of each job is to minimize its delay, i.e., the delay of the machine that it chooses to run on.

A pure Nash equilibrium is a schedule where no job can obtain a smaller delay by selfishly moving to a different configuration (machine), while other jobs remain in their original positions. A strong equilibrium is a schedule where no (non-empty) subset of jobs exists, where all jobs in this subset can benefit from

changing their configuration. We say that all jobs in a subset benefit from moving to a different machine if all of them have a strictly smaller delay as a result of moving (while the other jobs remain in their positions, and may possibly have a larger delay as a result).

The SPOA is the worst case ratio between the social cost of a (pure) strong equilibrium and the cost of an optimal assignment, that is, the minimum achievable social cost. The POA is a standard measure which takes into account not only strong equilibria but any (pure) equilibrium. These two measures consolidate and give the same results for some problems, whereas for other problems, the SPOA gives much more meaningful results than the POA.

We study the behavior of the SPOA versus the behavior of the POA for this scheduling problem and give tight results for both these measures. We find the exact SPOA for any possible speed ratio $s \geq 1$ of the machines, and compare it to the exact POA which we also find. We show that for a wide range of speeds ratios these two measures are very different ($1.618 < s < 2.247$), whereas for other values of s, these two measures give the exact same bound. We extend all our results for cases where a machine may have an initial load resulting from jobs that can only be assigned to this machine, and show tight bounds on the SPOA and the POA for three such variants as well.

Keywords: Nash equilibrium, strong equilibrium, uniformly related machyines

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1222

Competitive Queue Management for Latency Sensitive Packets

Amos Fiat (Tel Aviv University, IL)

We consider the online problem of non-preemptive queue management. An online sequence of packets arrive, each of which has an associated intrinsic value. Packets can be accepted to a FIFO queue, or discarded. The profit gained by transmitting a packet diminishes over time and is equal to its value minus the delay.

This corresponds to the well known and strongly motivated Naor's model in operations research.

We give a queue management algorithm with a competitive ratio equal to the golden ratio ($\phi \approx 1.618$) in the case that all packets have the same value, along with a matching lower bound. We also derive $\Theta(1)$ upper and lower bounds on the competitive ratio when packets have different intrinsic values (in the case of differentiated services). We can extend our results to deal with more general models for loss of value over time.

Finally, we re-interpret our online algorithms in the context of selfish agents, producing an online mechanism that approximates the optimal social welfare to within a constant factor.

Keywords: Online Buffer Mangement, Competitive Analysis of Algorithms

Joint work of: Fiat, Amos; Mansour, Yishay; Nadav, Uri

Object Division by Market Procedures

Claus-Jochen Haake (Universität Bielefeld, D)

We discuss a model, in which two agents may distribute finitely many objects among themselves. The conflict is resolved by means of a market procedure. Roughly, in a market procedure, we specify price(s) for the objects to be divided and the final allocation is achieved by individual demand. Therefore, negotiations over the final allocation can be replaced by letting agents "buy" their favorite bundles.

Depending on the specifications, the procedure serves to implement bargaining solutions such as the discrete Raiffa solution, the Kalai-Smorodinsky solution and the Perles-Maschler solution. The latter is axiomatized using the superadditivity axiom, which in the present context is readily interpreted as resolving a specific source of conflict potential.

In the second paper, we use the idea of attaching prices to objects to reach the Kalai-Smorodinsky solution, or, put differently, the Adjusted Winner point, through strategic interaction. We construct two games for two persons, in which the AW allocation prevails in Nash and subgame perfect equilibrium, respectively. The games again rely on an underlying market for objects, in which prices are specified in a an appropriate way.

Envy in Experiments

Dorothea Herreiner (Loyola Marymount University, USA)

This talk discusses different notions of envy and the empirical evidence for their relevance for distributive justice.
In particular, it links the concept of envy freeness to the recent experimental and survey-based literature on fair division.

Keywords: Fairness, Envy, Experiments

Some Recent Results on Pie Cutting

Michael A. Jones (Montclair State University, USA)

In 1993, David Gale suggested that there is a difference between the fair division of a cake and a pie. The difference between a cake and a pie is topological:

A cake can be viewed as a rectangle valued along its horizontal axis - a line segment, while a pie is viewed as a disk valued along its circumference - a circle. A cake can be turned into a pie if the endpoints of the line segment are identified. In this talk, I will provide a survey of some recent results on pie-cutting.

Vertical, parallel cuts divide a cake into pieces, and radial cuts from the center divide a pie into wedge-shaped pieces. We restrict our attention to allocations that use the minimal number of cuts necessary to divide cakes or pies. Gale also asked a simple question: Does there always exist an envy-free and efficient allocation of a pie? An allocation of pie into n wedge-shaped sectors among n players is

- envy-free if no player prefers another sector to her own, and
- efficient if, based on each player's measure, every other possible allocation of sectors either gives the same values or else decreases the value assigned to at least one player (in which "efficient" is among other allocations into n sectors).

For two players, Barbanel and Brams (2007) answer Gale's question and provide the condition on the players' preferences for an envy-free, efficient, and equitable allocation to exist, in which equitable means that each player assigns the same value to the sector that she receives. Thomson (2007) also answers Gale's question in the affirmative for two players. Brams, Jones, and Klamler (2007) extend the definition of envy-freeness to unequal entitlements, in which one player is entitled to p of the pie while the other is entitled to $1 - p$, and use the Universal Chord Theorem to prove that an efficient, envy-free, proportional allocation exists for two players.

The existence of an allocation with specific properties does not imply a procedure whereby the players would truthfully reveal their preferences to arrive at the allocation. Barbanel and Brams (2007) provide a two-player, moving-knife procedure that yields an envy-free and efficient allocation when the players' measures are absolutely continuous with respect to one another. They provide a three-player, moving-knife procedure that produces an allocation that is envy-free, but need not be efficient nor equitable. Brams, Jones, and Klamler (2007) provide a two-player, moving-knife procedure that yields a rational, proportional outcome that will almost assuredly leave a surplus, thereby guaranteeing that the allocation is not efficient. A more information-intensive procedure requiring a referee yields a proportional, envy-free, and efficient allocation.

For $n > 2$ players, the results are less positive. Brams, Jones, and Klamler (2007) provide examples for three players that demonstrate that proportional allocations may fail to exist when entitlements are not equal. Barbanel and Brams (2007) demonstrate measures for four players (with equal entitlements) in which no envy-free, efficient allocation of pie exists; their solution requires that the measures are not absolutely continuous with respect to one another. The aforementioned examples for three or four players can be naturally extended to more players. I will conclude with a list of open questions.

I will discuss results from the following papers:

– Barbanel and Brams, "Cutting a pie is not a piece of cake." (Preprint, 2007)

– Brams, Jones, and Klamler, "Proportional pie-cutting." (Preprint, 2007) http://www.csam.montclair.edu/jonesma/pie-cutting.pdf

– Thomson, "Children crying at birthday parties. Why?" Economic Theory 31 (2007) 501-521.

Keywords: Pie cutting, procedures, Universal Chord Theorem

See also: Thomson, "Children crying at birthday parties. Why?" Economic Theory 31 (2007) 501-521.

Some Recent Results on Pie Cutting

Michael A. Jones (Montclair State University, USA)

For cake cutting, cuts are parallel to an axis and yield rectangular pieces. As such, cutting a cake is viewed as dividing a line segment. For pie cutting, cuts are radial from the center of a disc to the circumference and yield sectors or wedge-shaped pieces. As such, cutting a pie is viewed as dividing a circle. There is clearly a relationship between cutting a cake and cutting a pie. Once a circular pie has a single cut, then it can be straightened out into a segment, looking like a cake. Isn't a cake just a pie that has been cut? Gale (1993) suggested that this topology was a significant difference. This note is to summarize and compare some of the recent results on pie cutting that appear in Barbanel and Brams (2007) and Brams, Jones, and Klamler (2007). The geometric framework presented in Barbanel and Brams (2007) is used to prove and to explain results in Brams, Jones, and Klamler (2007).

Keywords: Pie cutting, envy-free, proportional, undominated

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1224

Strong Price of Anarchy for Machine Load Balancing

Haim Kaplan (Tel Aviv University, IL)

As defined by Aumann in 1959, a strong equilibrium is a Nash equilibrium that is resilient to deviations by coalitions. We give tight bounds on the strong price of anarchy for load balancing on related machines. We also give tight bounds for k-strong equilibria, where the size of a deviating coalition is at most k, for unrelated machines.

Keywords: Game theory, Strong Nash equilibria, Load balancing, Price of Anarchy

Joint work of: Amos Fiat; Meital Levy; Kaplan, Haim; Olonetsky, Svetlana

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1225

Optimal Allocation without Transfer Payments

Todd R. Kaplan (University of Exeter, GB)

Often an organization or government must allocate goods without collecting payment in return. This may pose a difficult problem when the allocator does not know the values or needs of the agents receiving those goods. We first discuss a mechanism from Brams and Kaplan (2004) for allocating several heterogeneous goods among two parties. This mechanism combines sequential choices with a structured form of trading. We then talk about the case (Chakravarty and Kaplan, 2007) where the agents have private information (do not know each others' preferences), objects are homogeneous, and the agents can send a costly but wasteful signal to the designer. We show that for a large class of distributions of valuations, ignoring these costly signals by giving agents equal share (or using lotteries if the goods are indivisible) maximizes the social surplus. In other cases, those that send the highest signal should receive the goods; however, we then show that there exist cases where more complicated mechanisms are superior.

Multicast cost sharing and Nash equilibria

Claire Kenyon-Mathieu (Brown Univ. - Providence, USA)

We consider a multicast game played by a set of selfish noncooperative players (i.e., nodes) on a rooted undirected graph. Players arrive one by one and each connects to the root by greedily choosing a path minimizing its cost; the cost of using an edge is split equally among all users of the edge. How large can the sum of the players' costs be, compared to the cost of a "socially optimal" solution, defined to be a minimum Steiner tree connecting the n players to the root? We show that the ratio is at least logarithmic in n and at most squared logarithmic. One can view this multicast game as a variant of online Steiner tree with a different cost sharing mechanism.

Furthermore, we consider what happens if the players, in a second phase, are allowed to change their paths in order to decrease their costs. Thus, in the second phase players play best response dynamics until eventually a Nash equilibrium is reached. We show that the price of anarchy is at least logarithmic and at most the cube of logarithmic.

Joint work of: Charikar, Moses; Karloff, Howard; Kenyon-Mathieu, Claire; Naor, Seffi; Saks, Michael

The Housemates' Problem

Marc Kilgour (Wilfrid Laurier University, CA)

A house with n rooms is rented by n housemates. The Housemates' Problem asks whether it is possible to assign rents to the individual rooms, totaling the rent for the house, so that each housemate prefers a different room. The Housemates' Problem was proposed and elaborated, and several solutions developed, in six papers published between 1999 and 2004. These papers are reviewed in this presentation, and several directions for further development are suggested.

Keywords: Fair division, preference, bid matrix, feasible assignment, maxsum assignment

Better Ways to Cut a Cake - Revisited

Christian Klamler (Universität Graz, A)

Procedures to divide a cake among n people with n-1 cuts (the minimum number) are analyzed and compared. For 2 persons, cut-and-choose, while envy-free and efficient, limits the cutter to exactly 50% if he or she is ignorant of the chooser's preferences, whereas the chooser can generally obtain more. By comparison, a new 2-person surplus procedure (SP'), which induces the players to be truthful in order to maximize their minimum allocations, leads to a proportionally equitable division of the surplus - the part that remains after each player receives 50% - by giving each person a certain proportion of the surplus as he or she values it.

For $n \geq 3$ persons, a new equitable procedure (EP) yields a maximally equitable division of a cake. This division gives all players the highest common value that they can achieve and induces truthfulness, but it may not be envy-free. The applicability of SP' and EP to the fair division of a heterogeneous, divisible good, like land, is briefly discussed.

Keywords: Fair division, cake-cutting, envy-freeness, strategy-proofness

Joint work of: Brams, Steven; Jones, Michael; Klamler, Christian

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1227

Approximating min-max k-clustering

Asaf Levin (The Hebrew University of Jerusalem, IL)

We consider theproblems of set partitioning into k clusters with minimum of the maximum cost of a cluster. The cost function is given by an oracle, and we assume that it satisfies some natural structural constraints.

That is, we assume that the cost function is monotone, the cost of a singleton is zero, and we assume that for all $S \cap S' \neq \emptyset$ the following holds $c(S) + c(S') \geq c(S \cup S')$. For this problem we present a $(2k-1)$-approximation algorithm for $k \geq 3$, a 2-approximation algorithm for $k = 2$, and we also show a lower bound of k on the performance guarantee of any polynomial-time algorithm.

We then consider special cases of this problem arising in vehicle routing problems, and present improved results.

Keywords: Approximation algorithms

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2007/1228

Self-interested players in repeated games: Minimizing regret

Katrina Ligett (CMU - Pittsburgh, USA)

While in some situations, games are mediated by a central authority who can attempt to maximize fairness or other notions of social welfare, in many systems, players interact in a game, and the social welfare achieved is determined solely by the strategies they employ.

For this reason, it is interesting to study the outcomes achieved by players operating in a self-interested manner in various settings.

Recent work in computer science has attempted to study self-interested behavior by quantifying the ratio between the social value achieved by the worst Nash equilibrium and that of the social optimum. In this talk, we present an alternative definition of self-interested behavior in games, that of minimizing regret. We present some new results quantifying the global consequences of regret-minimizing strategies in a variety of classes of games, focusing in particular on a market game where the social utility function is a measure of fairness.

In this talk, we consider a repeated game played on a fixed graph, where on each timestep, each of the players must select a node in the graph to play. A player's reward is the number of nodes to which she is the closest player (in case of ties, they split the credit). For example, the nodes might be tourist destinations in Paris and the players might be souvenir sellers deciding where to set up their stands. Their income is based on the number of tourists (nodes) who buy from them (in this example, tourists always buy from the nearest stand). The social utility is the minimum player utility, so social welfare is maximized when all players achieve the same utility. One way to maximize social welfare is for all k players to place their stands on a single location and thus split the n tourists evenly, giving social welfare n/k. This clearly need not be an equilibrium. However, we are able to show that the worst Nash equilibrium can be no worse than n/2(k-1), and regret-minimizing algorithms achieve the same guarantee, even though they need not converge.

Keywords: Regret-minimization, game theory, repeated games, online games, price of anarchy

Joint work of: Blum, Avrim; Hajiaghayi, Mohammad Taghi; Ligett, Katrina; Roth, Aaron

On Approximately Fair Allocations of Indivisible Goods

Evangelos Markakis (CWI - Amsterdam, NL)

We study the problem of fairly allocating a set of indivisible goods to a set of people from an algorithmic perspective.

Fair division has been a central topic in the economic literature and several concepts of fairness have been suggested. The criterion that we focus on is envy-freeness. In our model, a monotone utility function is associated with every player specifying the value of each subset of the goods for the player. An allocation is envy-free if every player prefers her own share than the share of any other player.

When the goods are divisible, envy-free allocations always exist. In the presence of indivisibilities, we show that there exist allocations in which the envy is bounded by the maximum marginal utility, and present a simple algorithm for computing such allocations. We then look at the optimization problem of finding an allocation with minimum possible envy. In the general case the problem is not solvable or approximable in polynomial time unless P = NP. We consider natural special cases (e.g. additive utilities) which are closely related to a class of job scheduling problems.

Approximation algorithms as well as inapproximability results are obtained. Finally we investigate the problem of designing truthful mechanisms for producing allocations with bounded envy.

Keywords: Envy-free, Fair division, approximation algorithms

Joint work of: Lipton, Richard; Markakis, Evangelos; Mossel, Elchanan; Saberi, Amin

Full Paper:
http://homepages.cwi.nl/∼vangelis/research/ec04-fair.ps

See also: ACM Conference on Electronic Commerce, New York, NY, May 2004

Efficient cost sharing with a cheap residual claimant

Hervé Moulin (Rice University - Houston, USA)

For the cooperative production problem where the commons is a one dimensional convex cost function, I propose the residual mechanism to implement the efficient production level . In contrast to the familiar cost sharing methods such as serial, average and incremental, the residual mechanism may subsidize an agent with a null demand. IFor a large class of smooth cost functions, the residual mechanism generates a budget surplus that is, even in the worst case, vanishes

as 1/logn where n is the number of participants. Compare with the serial, average and incremental mechanisms, of which the budget surplus, in the worst case, converges to the efficient surplus as n grows.

The second problem is the assignment among n agents of p identical objects and cash transfers to compensate the losers. We assume p<n, and compute the optimal competitive performance among all VCG mechanisms generating no budget deficit. It goes to zero exponentially fast in n if the number of objects is fixed; and as $(n)^(1/2)$ uniformly in p. The mechanism generates envy, and net utilities are not co-monotonic to valuations. When $p > n/2$, it may even fail to achieve voluntary participation.

Keywords: Assignment, cost sharing, Vickrey-Clarke-Groves mechanisms, competitive analysis

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2007/1231

Adjusted Winner: Extensions to 3 or More People

Allison M. Pacelli (Williams College - Williamstown, USA)

Adjusted Winner is a procedure developed by Steven Brams and Alan Taylor that guarantees an efficient, equitable, and envy-free division of goods for two people. For three or more people, it is not always possible to find such an allocation. Using mixed integer programming however, we are able to find such a solution if it exists. This is work in progress.

Keywords: Adjusted Winner

Joint work of: Pacelli, Allison M.; Craft, David L.

Fair Division and Uncertainties within Joint International Trading Activities-Intelligent Resource Allocation and Process Optimization within International Resource Conflicts

Stefan Pickl (Univ. der Bundeswehr - Neubiberg, D)

The important conferences of Rio de Janeiro 1992 and Kyoto 1997 demand for new economic instruments which focus on environmental protection in both macro and micro economy. A sustainable development can only be guaranteed if the instrument is embedded in an optimal strategic energy management and in intelligent (fair) resource allocation principles. Game theoretic monitoring and verification instruments should be developed. Companies, operating within national registry requirements, must have accurate and up-to-date inventories and tools for tracking, analyzing, and reporting the data. Such financial markets

provide economic incentives for better management of GHG emissions. Tradable credits are derived from certified reductions recorded over a period of time and must be tracked and properly documented.

Thus, a successful trading system calls for fair division principles. The talk gives an overview about this topic and present a few solution principles which were developed by the author:

Game-theoretic solution concepts were theoretically defined and practically computed. Emission Trading Markets as general market situation can then be examined. Preferences in the context of the Kyoto protocol were analysed in real-world experiments. The results of those international experiments and fair division techniques might lead to an optimal market design. In the next phase the implementation of auctions will be treated and the computation of utility functions in this important environment should be characterized.

Keywords: Fair Allocation, Process Optimization, Business Intelligence

An Approximation Algorithm for Max-Min Fair Allocation of Indivisible Goods

Amin Saberi (Stanford University, USA)

I talk about the the first approximation algorithm for the problem of max-min fair allocation of indivisible goods. The approximation ratio of our algorithm is $1/\log^3 k\sqrt{k}$ where k is the number of agents. As a part of our algorithm, we design an iterative method for rounding a fractional matching on a tree which might be of independent interest.

Joint work of: Saberi, Amin; Asadpour, Arash

Dynamic Pricing for Impatient Bidders

Baruch Schieber (IBM TJ Watson Research Center, USA)

We study the following problem related to pricing over time. Assume there is a collection of bidders, each of whom is interested in buying a copy of an item of which there is an unlimited supply. Every bidder is associated with a time interval over which the bidder will consider buying a copy of the item, and a maximum value the bidder is willing to pay for the item. On every time unit the seller sets a price for the item. The seller's goal is to set the prices so as to maximize revenue from the sale of copies of items over the time period.

In the first model considered we assume that all bidders are impatient, that is, bidders buy the item at the first time unit within their bid interval that they can afford the price. To the best of our knowledge, this is the first work that considers this model. In the offline setting we assume that the seller knows the bids of all the bidders in advance. In the online setting we assume that at each

time unit the seller only knows the values of the bids that have arrived before or at that time unit. We give a polynomial time offline algorithm and prove upper and lower bounds on the competitiveness of deterministic and randomized online algorithms, compared with the optimal offline solution. The gap between the upper and lower bounds is quadratic.

We also consider the envy free model in which bidders are sold the item at the minimum price during their bid interval, as long as it is not over their limit value. We prove tight bounds on the competitiveness of deterministic online algorithms for this model, and upper and lower bounds on the competitiveness of randomized algorithms with quadratic gap. The lower bounds for the randomized case in both models use a novel general technique.

Keywords: Pricing, bid, envy free, online algorithm, competitive ratio

Preemptive Online Scheduling: Optimal Algorithms for All Speeds

Jiri Sgall (Academy of Sciences - Prague, CZ)

We study an online version of the classical problem of preemptive scheduling on uniformly related machines. We are given m machines with speeds and a sequence of jobs, each described by its processing time (length). The time needed to process a job with length p on a machine with speed s is p/s. In the preemptive version, each job may be divided into several pieces, which can be assigned to different machines in disjoint time slots. The objective is to find a schedule of all jobs in which the maximal completion time (makespan) is minimized. In the online problem, jobs arrive one-by-one and we need to assign each incoming job to some time slots on some machines, without any knowledge of the jobs that arrive later.

To compare with fair division problems, that also deal with infinitely divisible object and seek an even (or fair) allocation, here we also need to assign the pieces to non-overlapping times. On the other hand, minimizing the number of preemptions/cuts is not our objective.

Our main result is an optimal online algorithm for preemptive scheduling on uniformly related machines with the objective to minimize makespan. The algorithm is deterministic, yet it is optimal even among all randomized algorithms. In addition, it is optimal for any fixed combination of speeds of the machines, and thus our results subsume all the previous work on various special cases. Together with a new lower bound it follows that the overall competitive ratio of this optimal algorithm is between 2.054 and $e \approx 2.718$.

Keywords: Scheduling, preemption, makespan, online algorithms

Joint work of: Ebenlendr, Tomas; Jawor, Wojciech; Sgall, Jiri

Full Paper:
http://www.math.cas.cz/~sgall/ps/optrel.ps

See also: T. Ebenlendr, W. Jawor, J. Sgall: Preemptive Online Scheduling: Optimal Algorithms for All Speeds. To appear in Algorithmica. A preliminary version appeared in Proc. of the 14th European Symp. on Algorithms (ESA), Lecture Notes in Comput. Sci. 4168, pages 327-339, Springer, 2006.

A Pie That Can't Be Cut Fairly

Walter Stromquist (Swarthmore College, USA)

David Gale asked in 1993 whether, when a pie is to be divided among n claimants, it is always possible to find a division that is both envy free and undominated (Pareto optimal). The pie is cut along n radii, and the claimantsŠ preferences are described by different measures on the cake.

We answer Gale's question in the negative for n = 3 by exhibiting three measures for which no division of the pie can be both envy free and undominated. The measures are absolutely continuous with respect to each other and with respect to area.

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1219

Envy-free cake divisions cannot be found by finite protocols

Walter Stromquist (Swarthmore College, USA)

No finite protocol (even if unbounded) can guarantee an envy-free division of a cake among three or more players, if each player is to receive a single connected piece.

Keywords: Cake cutting, envy free, finite protocol

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1220

Cutting several cakes with linked preferences

Francis Edward Su (Harvey Mudd College, USA)

We present a method for addressing envy-free, multi-object division where the items to be divided may have linked preferences. This method is based on viewing the space of possible divisions as a triangulated space, and using a polytopal version of Sperner's lemma.

Keywords: Topology, combinatorics, Sperner's lemma, fair division, envy-free

Fair Division With Indivisible Items: Heuristic and Exact Algorithms

William A. Webb (Washington State Univ. - Pullman, USA)

Given n indivisible items, how can they be partitioned among k people, in specified ratios, in an optimal way. We give a survey of known results and indicate some new method which are both more general and potentially lead to better solutions.

Fictitious Play beats Simplex for fractional packing and covering

Neal E. Young (Univ. California - Riverside, USA)

Linear programs with non-negative coefficients are called fractional packing and covering linear programs. In this talk I'll describe a fast (1+epsilon)-approximation algorithm for such linear programs. For large instances, the algorithm is much faster than the simplex method, even for small epsilon.

Keywords: Linear program, packing, covering, fictitious play, Lagrangian relaxation, approximation algorithm

Joint work of: Koufogiannakis, Christos; Young, Neal E.

Maximizing the Minimum Load for Selfisch Agents

Rob van Stee (Universität Karlsruhe, D)

We consider the problem of maximizing the minimum load for machines that are controlled by selfish agents, who are only interested in maximizing their own profit. Unlike the classical load balancing problem, this problem has not been considered for selfish agents until now.

For a constant number of machines, m, we show a monotone polynomial time approximation scheme (PTAS) with running time that is linear in the number of jobs. It uses a new technique for reducing the number of jobs while remaining close to the optimal solution. We also present an FPTAS for the classical machine covering problem, i.e., where no selfish agents are involved (the previous best result for this case was a PTAS) and use this to give a monotone FPTAS.

Additionally, we give a monotone approximation algorithm with approximation ratio $\min(m, (2+\varepsilon)s_1/s_m)$ where $\varepsilon > 0$ can be chosen arbitrarily small and s_i is the (real) speed of machine i. Finally we give improved results for two machines.

Keywords: Scheduling, algorithmic mechanism design, maximizing minimum load

Joint work of: Epstein, Leah; van Stee, Rob

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1242