Line Planning on Paths and Tree Networks with Applications to the Quito Trolebús System

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Abstract. Line planning is an important step in the strategic planning process of a public transportation system. In this paper, we discuss an optimization model for this problem in order to minimize operation costs while guaranteeing a certain level of quality of service, in terms of available transport capacity. We analyze the problem for path and tree network topologies as well as several categories of line operation that are important for the Quito Trolebús system. It turns out that, from a computational complexity worst case point of view, the problem is hard in all but the most simple variants. In practice, however, instances based on real data from the Trolebús System in Quito can be solved quite well, and significant optimization potentials can be demonstrated.

1 Introduction

The major cities of South America are facing an enormous and constantly increasing demand for transportation and, unfortunately, also increase vehicular congestion, with all its negative effects. In Quito, the elongated topography of the city with 1.8 millions inhabitants (the urban area being 60 km long and 8 km wide) aggravates vehicular congestion even more, such that traffic almost completely breaks down during rush hours. As a consequence, the local government faces the necessity of improving the public mass transit system.

A low-cost option that has produced satisfactory results in recent years has been the implementation of major corridors of transportation. These corridors consist of street tracks that are reserved exclusively for high-capacity bus units, which, in this way, can operate independently of the rest of the traffic. Even though the topology of a corridor is extremely simple (just a path), bus operation on it is non-trivial. In fact, it is usually organized in a complex system of several dozen lines, which cover, in an overlapping way, different parts of the corridor, and which can operate in different ways, e.g., as "normal lines" or as "express lines" (stopping only at distinguished express stations), as "open lines" (unidirectional) or "closed lines" (bidirectional lines), and in any combination of these categories. The corridor lines are often complemented by feeding lines that transport passengers between special transshipment terminals of the corridor and the nearby neighborhoods.

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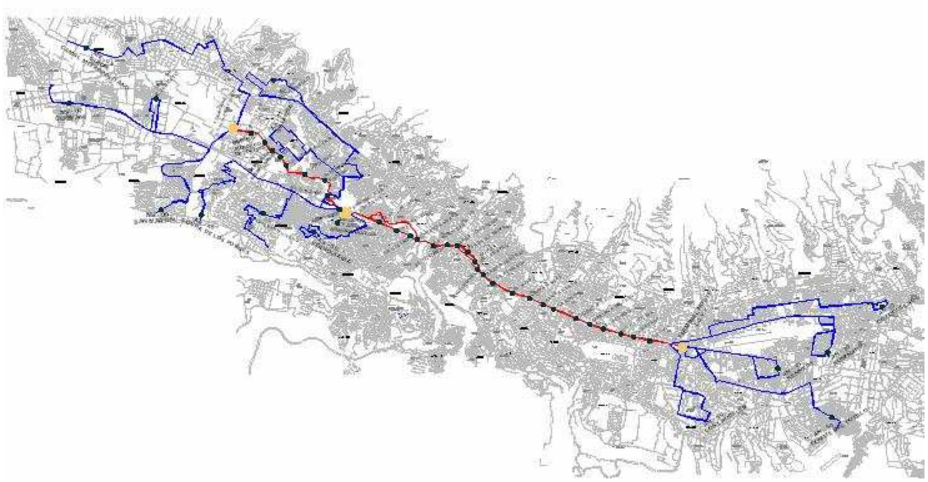

Fig. 1: Trolebús system and feeder line system in Quito.

In Quito, the most important of such corridors is the so-called *Trolebús System* (TS), see Figure 1. TS is currently the largest public transportation system in Quito, carrying around $250,000$ passengers daily. However, the dramatic increase in transportation demand has had a negative impact on the quality of service, with overcrowded buses and long waiting times being commonly experienced by passengers. At the same time, operation costs have been continuously increasing. With the aim of contributing to the improvement of this situation, we have been working on optimization models that can be applied to improve the operation of the TS and similar corridor transportation systems. The question that we investigate is whether the design of the corridor line system can be optimized using mathematical methods in order to improve the quality of service and/or lower operation costs by a better vehicle utilization.

Mathematical optimization approaches to line planning have received growing attention in the operations research and the mathematical programming community in the last two decades, see Odoni, Rousseau, and Wilson [1] and Bussieck, Winter, and Zimmermann [2] for an overview. In particular, integer programming approaches to line planning have been considered since the late nineties. Bussieck, Kreuzer, and Zimmermann [3] (see also Bussieck [4]) and Claessens, van Dijk, and Zwaneveld [5] both propose cut-and-branch approaches to select lines from a previously generated pool of potential lines. Both articles are based on a "system-split" of the demand, i.e., an a priori distribution of the passenger flow on the arcs of the transportation network; these "aggregated demands" are then covered by lines of sufficient capacity. Bussieck, Lindner, and Lübbecke [6] extend this work by incorporating nonlinear components. Goossens, van Hoesel, and Kroon [7,8] improve the models and algorithms and show that real-world railway problems can be solved within reasonable time and quality. Approaches that integrate line planning and passenger routing have recently been proposed by Borndörfer, Grötschel, and Pfetsch [9,10], and by Schöbel and Scholl [11,12]. The latter authors consider an expanded line-network that allows to minimize the number of transfers or the transfer time.

All of these articles consider general network topologies, but do not analyze line operation categories such as express lines, or open lines, probably because the line planning problem on general graphs is already hard without them. The corridor topology, however, opens up a chance to investigate complex line operation categories in a practically relevant setting. It also brings up the question whether perhaps some cases associated with different line operation categories can be solved in polynomial time. It will turn out in Section 3 that this is indeed the case if only closed lines and a homogeneous vehicle fleet are used; in all other cases, however, the problem is hard (there is one open case left). From a practical point of view, however, TS instances can be solved quite well. Indeed, our results show significant optimization potentials with respect to the currently operated solution, see Section 4.

2 A Flow-Based Model for Line Planning

We consider a bus transportation network as a digraph $D = (V, A)$, where each bus station is represented by a node $v \in V$ and arcs represent direct links between stations, i.e., $(i, j) \in A$ if and only if some bus may visit station j directly after station i. The fleet of buses is often heterogeneous; for instance, in Quito it contains trolley-buses and several other types of buses used for the feeding lines. We call a specific type of bus a transportation *mode* and define $\mathcal{M}$ to be the set of all transportation modes in the system, where each transportation mode $m \in \mathcal{M}$ has a specific capacity $\kappa_m \in \mathbb{Z}^+$. For each $m \in \mathcal{M}$, certain stations referred to as *terminals* are identified, where buses of mode m may start or end a service route. An *open line* for a mode m is a directed path whose first and last nodes are different terminals. Similarly, a *closed line* for m is a circuit containing at least one terminal. We consider for each $m \in \mathcal{M}$ a *line pool* $\mathcal{L}^m$, i.e., a set of a priori selected (open or closed) lines that can potentially be established. We denote by $\mathcal{L} := \cup_{m \in M} \mathcal{L}^m$ the set of all possible lines and by $\mathcal{L}^m_a$ the set of lines of mode m using arc a. For a line $\ell \in \mathcal{L}$, $c_\ell \in \mathbb{R}_+$ is the cost of a single trip via ℓ. Transportation demand is usually expressed in terms of an origin-destination matrix $(d_{uv}) \in \mathbb{Z}_+^{V \times V}$, where each entry d_{uv} indicates the number of passengers traveling from station u to station v within a certain time horizon T. In the following we assume that each passenger has been routed along some specific directed (u, v)-path in a preprocessing step, such that an *aggregated transportation demand* g_a on each arc a of the network has been computed.

We will consider three network topologies that are related to the TS structure. On the main corridor, trolley-buses move on a single path and are usually not allowed to overtake. This suggests to define a transportation network consisting of two directed paths (one for each transportation direction). Any line moving from a station u to a station v must stop at all intermediate stations. We call such a network topology a *Quito-Graph* (QG). However, transport authorities are considering the possibility of allowing trolley-buses to overtake at certain segments of the main corridor in the future. This would make it possible to introduce *express lines* that stop only at certain stations. The trips between two express stations can be modeled using respectively longer arcs. We call a network of this type a *Quito-Hopping-Graph* (QHG). Finally, when considering both feeding lines and the main corridor together, we observe that the TS network can be modeled as a *tree*, since feeding lines are simple paths that start at transshipment stations along the main corridor.

The *line planning problem* is to choose a set of lines $L \subseteq \mathcal{L}$ and frequencies for the lines in L in such a way that there is enough transportation capacity to cover the aggregated demand on each arc of the network. It can be formulated as an integer programming problem, that we denote by *Demand Covering Model with Fixed Costs* (DCM-FC):

$$\min \quad \sum_{m\in\mathcal{M}} \sum_{\ell\in\mathcal{L}^m} (c_\ell\, f_\ell + K_\ell\, y_\ell) \tag{1}$$

subject to

$$\sum_{m\in\mathcal{M}} \sum_{\ell\in\mathcal{L}_a^m} \kappa_m\, f_\ell \geq g_a, \qquad \forall\, a \in A \tag{2}$$

$$0 \leq f_\ell \leq f_\ell^{\max}\, y_\ell \qquad \forall\, \ell \in \mathcal{L} \tag{3}$$

$$f_\ell \in \mathbb{Z}_+ \qquad \forall\, \ell \in \mathcal{L} \tag{4}$$

$$y_\ell \in \{0,1\} \qquad \forall\, \ell \in \mathcal{L}. \tag{5}$$

Here, f_ℓ is an integer variable representing the frequency assigned to line $\ell \in \mathcal{L}$, and y_ℓ is a 0/1-variable that indicates whether a line is chosen in the solution ($y_\ell = 1$) or not ($y_\ell = 0$). The cost of line $\ell \in \mathcal{L}$ involves a fixed component K_ℓ as well as an operating cost $c_\ell\, f_\ell$ that depends on the frequency. The objective function (1) aims at minimizing the total operation costs. Constraints (2) ensure that the aggregated transportation demand is covered. Constraints (3) couple the line selection variables y_ℓ and the frequency variables f_ℓ and they impose upper bounds $f_\ell^{\max}$, for all $\ell \in \mathcal{L}$ on line frequencies. Finally, (4) and (5) are integrality constraints for the frequencies.

When fixed costs are zero ($K_\ell = 0$, $\forall \ell \in \mathcal{L}$), the model simplifies to the following form, that we denote by *Demand Covering Model* (DCM):

$$\min \quad \sum_{m\in\mathcal{M}} \sum_{\ell\in\mathcal{L}^m} c_\ell\, f_\ell \tag{6}$$

subject to

$$\sum_{m\in\mathcal{M}} \sum_{\ell\in\mathcal{L}_a^m} \kappa_m\, f_\ell \geq g_a, \qquad \forall\, a \in A \tag{7}$$

$$0 \leq f_\ell \leq f_\ell^{\max} \qquad \forall\, \ell \in \mathcal{L} \tag{8}$$

$$f_\ell \in \mathbb{Z}_+ \qquad \forall\, \ell \in \mathcal{L}. \tag{9}$$

DCM is a simplified version of the models appearing in Claessens, van Dijk, and Zwaneveld [5] and Bussieck, Kreuzer, and Zimmermann [3].

3 Computational Complexity

Solving DCM is NP-hard for general graphs, as the problem includes the *Set Covering Problem* as a special case ($\kappa \equiv 1, g \equiv 1, f^{\max} \equiv 1$), see also Schöbel and Scholl [11]. We now investigate how the network topology and several other factors affect the computational complexity of the model.

3.1 Fixed Costs are Hard

We first observe that fixed costs make the problem difficult. A reduction from the 0/1 Knapsack Problem can be used to prove:

Proposition 1 *DCM-FC is NP-hard, even if the underlying transportation network is a Quito graph consisting of two nodes joined by an arc, only closed lines are allowed, and there is only one transportation mode.*

3.2 Multiple Modes are Hard

It will turn out in Section 3.5 that the homogenous fleet case ($|\mathcal{M}| = 1$) allows a further simplification of the model DCM that leads to special complexity results. We therefore first discuss the case of *multiple modes* ($|\mathcal{M}| \geq 2$). Before doing this, however, let us consider an undirected version of the problem for Quito graphs.

Observe that if the line pool contains only closed lines, then each line using an arc $a = (u, v)$ must also use the arc $\overline{a} = (v, u)$, on which the bus is traveling in the opposite direction. Hence, both the arc set of the network and the arc set of each line can be partitioned into pairs of antiparallel arcs. Substituting these pairs by undirected edges, any instance of DCM with closed lines can be reduced to an equivalent *undirected* instance on an undirected graph $G = (V, E)$, where new aggregated demands on the edges are computed as follows:

$$g'_{uv} := \max\{g_{(u,v)}, g_{(v,u)}\}, \qquad \text{for all } (u, v) \in A.$$

In this version of the problem, the lines correspond to simple undirected paths in G, having the same costs. The task is to assign frequencies to these paths to cover the edge demands at minimum cost. Figure 2 gives an example of this problem transformation.
Using a reduction from the 3-Dimensional Matching Problem, one can prove:

Proposition 2 *If $|\mathcal{M}| \geq 2$, then DCM is NP-Hard even for undirected Quito graphs and if fixed costs are zero.*

3.3 Trees are Hard

Feeding line systems transport passengers from the main corridor to the neighborhoods. Each feeding line starts at a transshipment terminal, visits a set of

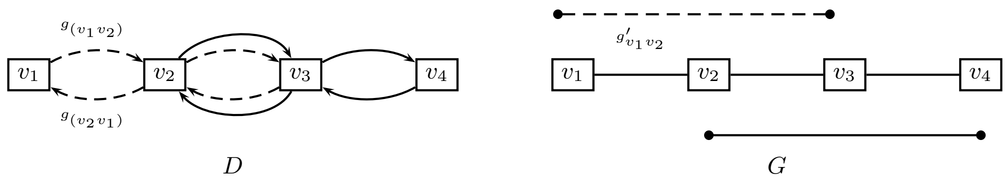

Fig. 2: Constructing the undirected version of DCM on a Quito graph. The closed lines $(v_1, v_2, v_3, v_2, v_1)$ and $(v_2, v_3, v_4, v_3, v_2)$ in D are substituted by simple undirected paths in G.

consecutive stations up to certain *turn-over station*, and returns back to the transshipment terminal stopping at the same stations on the way. Since only closed lines are admissible, there is again an undirected version of the DCM involving feeder lines. The underlying graph for this problem is a tree, with several terminals as initial nodes, and simple paths starting from it. Thus, each line is represented by an undirected path linking one terminal with a certain node where the turn-over takes place. The following result can be proved using a reduction from the 3-Dimensional Matching Problem.

Proposition 3 *DCM on trees is NP-hard, even if only closed lines and a homogeneous transportation fleet* ($|\mathcal{M}| = 1$) *is used and fixed costs are zero.*

3.4 Hopping is Hard

In this section we consider the Quito Hopping Graph topology. To this end let $D = (V, A)$ be defined by the set $V = \{v_1, v_2, \dots, v_n\}$ of nodes representing all bus stations in the sequence along the path, and let $V_X \subseteq V$ be a subset of *express stations*. Similarly, there are express terminals, where express buses are allowed to start or end their routes.

Express lines are allowed to stop only at nodes from V_X, while *normal* (i.e., non-express) lines visit any node. Two nodes are joined by an arc if the corresponding stations can be visited consecutively by some line. Hence, the set of arcs is partitioned into three classes: a subset A_N containing arcs that may only be used by normal lines, a set A_X of arcs that may only be used by express lines, and a set A_S of "shared arcs". We assume that a transportation demand has been previously assigned to each arc of the network using some system split method. Using a reduction from 3-Dimensional Matching similar as for Proposition 2, one can prove:

Proposition 4 *DCM on Quito Hopping Graphs is NP-hard, even if only closed lines are considered and fixed costs are zero.*

3.5 Easy and Open Cases

We investigate now the Demand Covering Model on Quito graphs for a homogeneous transportation fleet ($|\mathcal{M}| = 1$) and fixed costs of zero. This model, that we denote by *Demand Covering Model with Homogeneous Fleet* (DCM-HF), can be further simplified and formulated in the following matrix form:

$$\min \quad c^T f \tag{10}$$

subject to

$$A_H f \geq \tilde{g} \tag{11}$$

$$f \leq f^{max} \tag{12}$$

$$f \in \mathbb{Z}_+^{|\mathcal{L}|}. \tag{13}$$

Here, $\tilde{g}_a := \lceil \frac{g_a}{\kappa} \rceil$ for all $a \in A$, are the *transformed aggregated demands*, $c \in \mathbb{R}^{|\mathcal{L}|}$ is the vector of line (operating) costs, $f^{max} \in \mathbb{Z}_+^{|\mathcal{L}|}$ denotes the vector of upper bounds on the frequencies, and $A_H \in \{0,1\}^{|A| \times |\mathcal{L}|}$ is the arc-line incidence matrix.

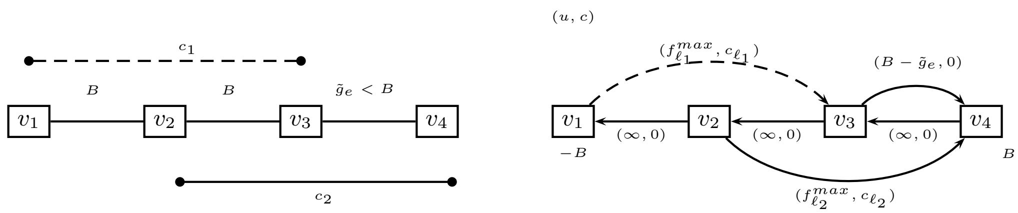

Fig. 3: Transforming undirected DCM-HF on Quito graphs to a minimum cost flow problem.

Closed Lines The undirected version of DCM-HF on Quito Graphs can be reduced to a minimum cost flow problem as follows. Let $G = (V, E)$ be an undirected Quito Graph with n nodes $v_1, \ldots, v_n$. We define $B := \max_{e \in E}\{\tilde{g}_e\}$ and define $\hat{D} = (V, \hat{A})$ to be a directed network on the node set of G, whose arc set is the disjoint union of three subsets: a set $\hat{A}_1$ containing all "backward arcs" of the form (v_i, v_{i-1}), for all $i \in \{2, 3, \ldots, n\}$; a set $\hat{A}_2$ that contains one "line arc" (v_i, v_j), with $i < j$ for every line having its ends points at v_i and v_j; and a set $\hat{A}_3$ containing one "slack arc" (v_i, v_{i+1}) for each edge $\{v_i, v_{i+1}\}$ in G with $B - \tilde{g}_e > 0$. Flow demands are defined as follows (negative demands meaning that the node is a source of flow):

$$b_{v_i} = \begin{cases} -B, & \text{if } i = 1, \\ B, & \text{if } i = n, \\ 0, & \text{otherwise.} \end{cases}$$

Arc costs are equal to zero and capacities are set to infinity on the arcs belonging to $\hat{A}_1$. For each arc in $\hat{A}_2$ representing a line $\ell \in \mathcal{L}$, the cost is equal to c_ℓ and the capacity is set to f_ℓ^{max}. Finally, each slack arc in $\hat{A}_3$ associated to an edge e from G has capacity equal to $B - \tilde{g}_e$ and cost equal to zero. Figure 3 shows an example. Interpreting the values of a feasible flow on the line arcs as transportation capacities of the respective lines is the key to proving:

Proposition 5 *DCM-HF can be solved in polynomial time on undirected Quito Graphs.*

Open and Closed lines If both open and closed lines are present in the line pool, the symmetry of the problem is broken and the reduction of the last section does no longer work. We have not yet been able to determine the complexity of this case, but we show next that this problem is at least as difficult as the Exact Perfect Matching Problem, whose complexity is open.

The *Exact Perfect Matching Problem* (EPMP, see e.g. [13]) is a *perfect matching problem* defined on a bipartite graph with red and blue edges; there is also an integer k given. The task is to determine whether there exists a perfect matching containing exactly k blue edges. The complexity of this problem is unknown. We have proven the following proposition.

Proposition 6 *Every instance of EPMP can be transformed to an instance of DCM-HF in polynomial time.*

4 Optimizing the Trolebús System

We have carried out a computational study with various DCM models for the three network topologies considered in the previous section, based on data provided by the Trolebús System operator. The models were solved using the IP-solver SCIP [14] in its standard configuration, which was sufficient to obtain optimal solutions within a few seconds. All experiments were performed on a 3.0 GHz Pentium 4 PC with 512 MB RAM running Suse Linux 10.0.

The total fleet of the TS consists of 113 trolley-buses for the corridor and 89 normal buses for two different types of the feeding lines. The transportation network has 528 nodes, 52 of them located along the main corridor.

Table 1 reports some operational parameters for the line plan currently implemented by the TS operator in the main corridor (QG) and in the feeder line system (FLS): cost, average number of transfers per passenger, average travel times, and the accumulated frequency. We refer to this line plan as the *reference plan*. The statistics are given for time slices of one hour during the day. For the time interval 06:00–07:00, the reference plan does not provide enough capacity to cover the transportation demand with the nominal maximum capacity of a trolley bus ($\kappa = 180$); in fact, the solution requires 210 passengers to be transported by each bus unit on average, i.e., the buses are overcrowded. Passenger transfers were computed using the method described in Bouma and Oltrogge [15] (the frequency variables were fixed to the values given by the reference plan). Traveling times between stations were taken from historical data for QG and

Table 1: The current operation of the Quito Trolebús System (main corridor and feeding lines).

	Quito Graph				Feeding Lines			
T	Cost	# Tr.	Travel Time	$\sum_{\ell \in \mathcal{L}} f_\ell$	Cost	# Tr.	Travel Time	$\sum_{\ell \in \mathcal{L}} f_\ell$
06:00-07:00*	5379	–	–	57	3806.8	0.478	49.66	59
07:00-08:00	7271	0	30.7	79	4144.6	0.457	46.32	65
08:00-09:00	7246	0	28.1	83	3330.4	0.456	44.94	53
09:00-10:00	5991	0	24.3	75	3251.0	0.506	44.74	52
12:00-13:00	4858	0.0140	21.1	62	2873.6	0.452	41.16	46
13:00-14:00	4941	0.0322	21.8	63	3323.6	0.504	45.18	52
16:00-17:00	4945	0.0150	28.3	62	3473.6	0.500	46.77	54
17:00-18:00	7188	0	30.9	81	3455.8	0.415	42.89	53
18:00-19:00	7457	0	30.1	85	3050.0	0.394	43.29	48
19:00-20:00	6044	0	28.3	79	3050.2	0.548	52.47	49
20:00-21:00	5343	0	30.6	72	2597.6	0.661	56.09	41

Table 2: Optimizing the Quito Trolebús System using model DCM-HF on QG.

	Closed Lines					Closed+Open Lines				
T	Cost	# Tr.	Travel Time	$\sum_{\ell \in \mathcal{L}} f_\ell$	$\lvert L \rvert$	Cost	# Tr	Travel Time	$\sum_{\ell \in \mathcal{L}} f_\ell$	$\lvert L \rvert$
06:00-07:00	6275	0	30.02	79	19	4560.3	0	29.30	79	25
07:00-08:00	6911	0.00226	31.19	88	20	5232.7	0.00226	30.09	88	28
08:00-09:00	4792	0.00023	25.68	65	18	3785.8	0.00023	25.99	65	28
09:00-10:00	2992	0.00119	24.39	38	16	2522.2	0.00113	23.14	38	20
12:00-13:00	2230	0	20.05	26	10	2195.7	0	20.51	26	11
13:00-14:00	2342	0	21.54	28	11	2289.1	0	21.44	30	14
16:00-17:00	3234	0	26.33	39	13	2942.8	0	26.24	39	19
17:00-18:00	4847	0	29.02	58	16	4108.6	0	28.64	58	18
18:00-19:00	4625	0	27.08	58	17	3922.7	0.0116	26.79	60	20
19:00-20:00	3062	0	26.46	40	16	2667.2	0	26.50	41	17
20:00-21:00	1843	0	25.70	23	9	1711.4	0	26.10	24	10

FLS and estimated for express arcs in QHG. The transfer time for a change from line ℓ_1 to line ℓ_2 was estimated as $\frac{T}{2f_{\ell_2}}$.

As a first experiment, we carried out line planning for the main corridor based on the DCM-HF model on QG. We considered each one-hour time slice as an independent instance and ran two tests on it. In the first the line pool $\mathcal{L}$ consists of 66 closed lines and in the second one $\mathcal{L}$ contains 66 closed lines and 132 open lines. Table 2 reports the results obtained for this setting. Significant cost savings were obtained even in the case when only closed lines are allowed. The cost of our solution is smaller than that of the reference plan, with an average decrease of \$ 2,119.31 per hour and a global decrease of \$ 40,267. The total number of transfers increased in the morning time intervals, but decreased dramatically during midday and in the afternoon. The total number of transfers is 125, the average travel time is 25.56 minutes, compared to 26.4 minutes in the reference plan. If both open and closed lines are considered, solution costs are reduced even more. This can be explained by an asymmetry in the demand data. In fact, most passengers move in the S-N direction in the morning and return to their homes traveling in the N-S direction in the afternoon. The number of transfers is about the same as for the closed line scenario, except for time slices 15:00–16:00 and 18:00-19:00, where substantial increases are registered; the total number of transfers is 453. Nevertheless, average travel time is only 25.38 minutes.

Table 3 shows the results for the QHG instances, i.e., if express lines are considered. To this purpose, we identified 17 express stations along the main corridor. We considered a line pool with 84 closed lines and 168 open lines, of which 18 closed and 36 open lines were express lines.

In both scenarios (closed lines and closed+open lines) the cost increased compared with the results obtained for QG. The global cost for the transportation plan with only closed lines was \$ 60,825, which still represents savings of 36%, when compared to the current plan. The total number of transfers increased in comparison to QG, mainly for time slices 11:00-12:00 (from 7 to 458 transfers) and 21:00-22:00 (from 0 to 288 transfers) in the scenario with open+closed lines. The increases in cost and number of transfers are, however, compensated by better service for passengers, in terms that average travel time was reduced to 23.66 minutes if only closed lines are considered and 23.35 if closed and open lines are included in $\mathcal{L}$.

Our last experiment consisted in computing a line plan for the feeder line system. The TS has three independent systems of feeder lines that intersect the main corridor at three different transshipment terminals and contain 12, 17, and 13 turn-over stations, respectively. Currently, the vehicle fleet used for serving the feeder lines is heterogeneous and contains two types of buses with transportation capacities $\kappa_1 = 90$ and $\kappa_2 = 110$. Two planning scenarios were considered, depending on the number of "branches" that a feeder line is permitted to visit. In the first scenario, feeder lines are required to visit only one branch, i.e., they are paths having the transshipment terminal as one end node. In the second scenario, up to two branches may be visited by the same line, i.e., feeder lines are paths that contain the terminal in any position. In the first scenario,

Table 3: Optimizing the Quito Trolebús System using express lines.

T	Closed Lines					Closed+Open Lines				
	Cost	# Tr.	Travel T.	$\sum_{l\in\mathcal{L}} f_l$	$\lvert L\rvert$	Cost	# Tr.	Travel T	$\sum_{l\in\mathcal{L}} f_l$	$\lvert L\rvert$
06:00-07:00	6284	0	27.42	79	24	4892.2	0.0028	25.09	80	30
07:00-08:00	7092	0	27.66	87	21	5924.0	0	26.07	94	27
08:00-09:00	5167	0.00176	22.91	65	18	4556.6	0	22.91	74	25
09:00-10:00	3207	0.00251	21.82	39	19	2898.5	0.0102	21.58	42	21
12:00-13:00	2431	0	18.75	29	12	2407.6	0	18.60	29	13
13:00-14:00	2462	0.00365	20.10	28	12	2433.2	0	20.16	29	15
16:00-17:00	3772	0	23.48	44	16	3297.9	0.0017	23.44	44	23
17:00-18:00	5255	0.00214	25.75	61	16	4429.5	0.0067	25.70	61	22
18:00-19:00	5125	0	24.25	62	20	4257.9	0.0187	24.18	62	26
19:00-20:00	3446	0	24.22	43	18	2939.5	0.0092	24.49	44	24
20:00-21:00	2083	0.00702	24.45	26	14	1899.7	0.0136	24.29	26	15

Table 4: Optimizing the Quito Trolebús System including the feeder line systems.

T	One Branch						One+Two Branches						
	Cost	# Tr.	$\sum_{l\in\mathcal{L}} f_l$	$\lvert L\rvert$	T. Time	CPU	Cost	# Tr.	$\sum_{l\in\mathcal{L}} f_l$	$\lvert L\rvert$	T. Time	CPU	Gap
06:00-07:00	3142.4	0.501	59	44	53.08	0.01	2562.4	0.496	30	28	56.03	10000	6.96
07:00-08:00	3434.0	0.454	65	43	49.23	0.04	2794.0	0.454	33	32	54.31	10000	7.03
08:00-09:00	2740.8	0.481	53	42	48.60	0.02	2220.8	0.449	27	26	51.24	10000	6.21
09:00-10:00	2698.8	0.501	52	39	49.04	0.01	2198.8	0.499	27	24	51.76	0.23	3.25
12:00-13:00	2341.2	0.444	46	37	44.78	0.03	1881.2	0.425	23	22	47.80	0.66	4.68
13:00-14:00	2707.6	0.496	52	35	46.81	0.01	2207.6	0.494	27	24	49.80	10000	8.29
16:00-17:00	2804.6	0.496	53	37	48.88	0.01	2289.0	0.473	27	24	51.40	1.54	4.75
17:00-18:00	2837.8	0.409	54	41	46.20	0.01	2309.0	0.405	28	28	49.29	10000	7.42
18:00-19:00	2464.6	0.386	47	39	45.83	0.01	2002.4	0.383	24	24	48.37	1.38	4.33
19:00-20:00	2579.4	0.531	49	38	55.79	0.02	2110.6	0.521	26	24	58.02	1.38	4.33
20:00-21:00	2279.0	0.631	43	35	63.84	0.04	1872.2	0.622	22	22	68.34	0.23	3.01
Average	2443.6	0.549	46.2	36.1	55.42	0.020	1997.5	0.532	23.8	22.8	58.43	3692.0	4.99

a total of 84 lines were considered in the line pool (containing all three feeder systems), while in the second scenario 470 new lines were added. In our runs, we allowed an optimality gap of 5% and set a time limit of 10000 for each instance.

Table 4 reports the results (aggregated for all three feeder systems). As expected, the average number of transfers is much larger than in the previous experiments, since trips of the form "feeding line-main corridor-feeding line", which involve at least two transfers, are common in the solution. In both the "one branch" and "two branches" scenarios, the cost was reduced in comparison to the currently implemented solution by about 18% (one branch) and 32% (two branches). On the other hand, these savings are related to larger travel times for the passengers, which are slightly increased in all instances.

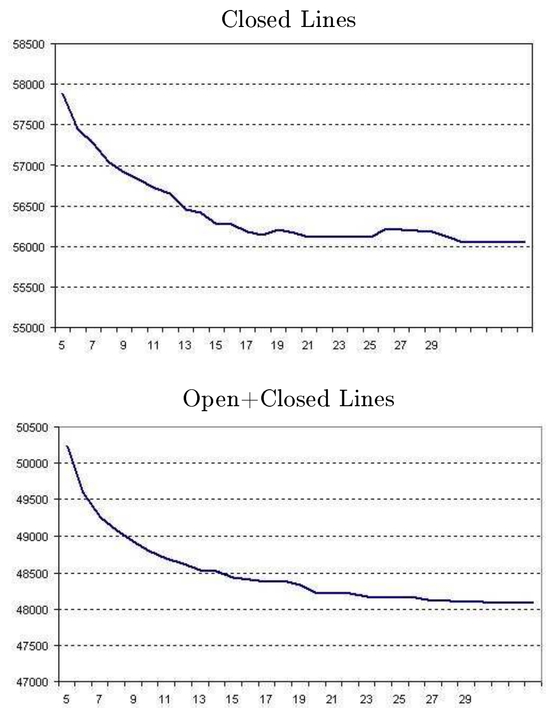

Fig. 4: Tradeoff cost vs. maximum number of lines.

The dramatic cost decrease in our solutions over the reference solution can be explained by two factors. First, our DCM model does not impose a limit on the number of lines in a solution. In practice, however, it is not desirable to have too many lines, as the whole system becomes too complicated for the user and the operator. Adding new binary variables to DCM that indicate whether a line is chosen in the solution or not, we carried out new experiments for the QG network topology limiting the allowed numbers of lines to a maximum between five (the number of lines currently used by the TS operator) and 30. Figure 4 summarizes the results for the whole day. As expected, the optimum solution cost increases as the number of allowed lines decrease, but the increase is less than 10% from 30 to 5 lines. A second reason can be found in the planning policies that the TS operator is currently using. Up to now, line planning has been carried out in a single step together with duty scheduling for the bus drivers by pre-assigning bus drivers to buses. It might be that this scheme is just too inflexible, since hard laboral constraints might discard some good solutions for the line planning problem. It would certainly be worthwhile to compute a vehicle and a duty schedule based on our line plans, in order to get a better assessment of the operational consequences of such an optimization.

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