Grammar-Based Integer Programing Models for Multi-Activity Shift Scheduling

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Abstract. We present a new implicit formulation for shift scheduling problems, using context-free grammars to model regulation in the composition of shifts. From the grammar, we generate an integer programming (IP) model allowing the same set of shifts as Dantzig's set covering model. When solved by a state-of-the- art IP solver on problems allowing a small number of shifts, our model, the set covering formulation and a typical implicit model from the literature yield comparable solving times. Moreover, on instances where many shifts are allowed, our model is superior and can encode a wider variety of constraints. Among others, multi-activity cases, which cannot be modeled by existing implicit formulations, can easily be captured with grammars.

Keywords. Shift Scheduling, Implicit models, Integer Programming, Context-free grammars

1 Introduction

Shift scheduling problems deal with the assignment of employees to shifts over a given interval of time called the *planning horizon*. The planning horizon is divided into *periods* of equal length, typically from 15 minutes to one hour, for which the number of employees required to perform different tasks is given. Our goal is to find a set of shifts satisfying these requirements.

To define a *shift*, we need to determine its starting time, its length and specify which activity is to be performed at each period. An *activity* can be a specific work task as well as a lunch break or a relief break. Usually, the assignment of activities to a shift is constrained by different rules mainly arising from work regulation agreements and ergonomic matters. *Multi-activity shift scheduling problems* arise from contexts where more then one work-activity must

Dagstuhl Seminar Proceedings 09261
Models and Algorithms for Optimization in Logistics
http://drops.dagstuhl.de/opus/volltexte/2009/2177

be performed, may call for special regulation on sequences of activities. Given an optimization criteria, the objective is to find the best set of shifts statisfying the organization regulation and meeting the labor requirements.

See [1] and [2] for an exhaustive overview of models and methods for problems related to staff scheduling and rostering.

2 Using Context-Free Grammars for Shift Scheduling

We approach the shift scheduling problem using formal languages theory. In [3] and [4], the authors suggest ways to use the expressiveness of automata and context-free grammars to formulate constraints over shifts and generate integer programming (IP) models that encapsulate all feasible shifts for any employee. However, when the number of employees in the problems get larger, their ideas were shown to be inadequate in practice.

Our contribution is an implicit IP model for shift scheduling problems based on context-free grammars, that can address problems involving many complex constraints such as the one arising from multi-activity contexts with as many employees as needed.

The idea is to formulate the constraints over the shifts of the problem using a context-free grammar in a way that the words of the language defined by the grammar are the allowed shifts for this problem. From the grammar, we build a directed acyclic graph (DAG, see [5]) and, from this DAG, derive an IP model. A solution to this IP model, when put together with demand constraints and a given optimization criteria, gives the best set of allowed shifts for the problem.

We proved that the linear relaxation of a model arising from a context-free grammar forms a totally unimodular matrix.

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