Integrated Vehicle Routing and Crew Scheduling (IVRCS) in Waste Management
Part II

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Abstract. Waste management involves an efficient planning of collection-vehicles and corresponding crews. We present an optimization model and algorithm for the crew scheduling problem which is the second part of an integrative solution approach. We differentiate between a network based duty generation phase and a duty optimization phase where an optimal set of annual duties for all employees is selected.

Keywords. Crew Scheduling, Waste Management, Integer Programming, Column Generation, Lagrangean Relaxation

1 Introduction and Problem Description

Planning waste management for a given waste disposal area involves the two major resources collection-vehicles and crews. The overall goal of our joint project, which includes the Institute of Mathematical Optimization (Technische Universität Braunschweig) and the Chair of Transportation Systems and Logistics (Technische Universität Dortmund) as well as two waste management companies, is an integrative approach for planning the routes and the crews of the vehicles. In the first phase of our three-phase approach we generate so called daily crew tasks which contain routes operated by a single crew at a particular day within a given disposal horizon, for example, two weeks, considering various practical requirements (see Figure 1). The goal is to minimize the number of crews/vehicles required for the entire disposal process. With the obtained number of crews, in phase two we re-optimize the daily crew tasks to increase the robustness of the routes. In the third phase we assign employees to the generated daily crew tasks for all working days of a planning horizon of one year such that the constraints concerning crew scheduling are satisfied and the benefits for the employees and the company are maximal. In this paper we focus on the above mentioned third phase and present a network based modeling and solution approach for the given crew scheduling problem in waste management.

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2 Mathematical Modeling

We differentiate the crew scheduling problem in waste management between two main parts: the generation of feasible annual duties for all employees and an optimal selection of duties which we will deal with first [1], [2]. Therefore, let $P$ be the set of all employees, $I(p)$ the set of all annual duties for an employee $p \in P$, $T$ the set of all working days and $J$ the set of all possible jobs. Each job $j \in J$ represents a daily crew task and a specific activity an employee can perform as part of a crew, where possible activities are working as a driver or as one of two loaders. Then the set partitioning formulation with additional constraints becomes

$$
\text{min} \sum_p \sum_i c_i^p x_i^p
$$

subject to

$$
\sum_p \sum_i a_{ij}^p x_i^p = 1 \quad \forall t \in T \ \forall j \in J
$$

$$
\sum_i x_i^p = 1 \quad \forall p \in P
$$

$$
\sum_p \sum_i b_i^p x_i^p \geq S_t \quad \forall t \in T
$$

$$
x_i^p \in \{0, 1\} \quad \forall p \in P \ \forall i \in I
$$

where $c_i^p$ is the cost of duty $i$ of employee $p$ and parameter $a_{ij}^p \in \{0, 1\}$ denotes if duty $i$ of employee $p$ covers job $j$ on day $t$. The binary variables $x_i^p$ show whether a certain duty $i$ of employee $p$ is selected or not in order to minimize the costs in the objective function (1). Constraints (2) and (3) ensure that for each day

**Fig. 1.** daily crew tasks for a waste disposal cycle of 2 weeks
of the planning horizon and each daily crew task all required jobs are covered and for each employee $p$ only one duty is selected, respectively. If the equality constraint in (3) is replaced by an inequality constraint a reduced number of employees is possible, too. The additional constraints (4) guarantee a variable safety stock $S_t$ of so called jumpers which are a subset of employees also allowed to work on standby in order to replace absent colleagues. Here $b^p_t \in \{0, 1\}$ shows if employee $p$ is on standby in duty $i$ on day $t$.

For the generation of (all) feasible annual duties we construct a network/directed graph for each employee in which each node represents a combination of an activity and a specific day of the planning horizon. Possible activities are working as a driver or loader, being on standby, holiday wishes or a priori known absences. Feasible connections of activities between different working days are handled via directed arcs. Figure (2) shows possible switch-overs between jobs in the same or different daily crew tasks and actual district knowledge of an employee. Additional aspects concerning the generation of duties, like different holiday wishes, can be incorporated, too (see Figure 3).

![Fig. 2. network: switching between jobs and daily crew tasks](image)

Then each feasible flow through a given network represents an annual duty for the corresponding employee. Thereby, all aspects that cannot be integrated directly into the network, like working time models or legal rules, have to be checked. In order to price the value of more or less suitable duties in a column generation phase we use arc costs with different weights. For example, complicating but not avoiding unnecessary switching between different jobs or preferred holiday wishes can be handled in this way.
3 Solution Approach

As the number of possible annual duties for all employees can become too huge to be solved in a reasonable time we developed a problem specific algorithm which is based on a lagrangean relaxation of our set partitioning model. We relax constraints (2) and (4) and punish their violation in the objective function via the corresponding lagrangean multipliers $\lambda_{ij}^1$ and $\lambda_{t}^2$, respectively [3], [4].

\[
Z(LR_\lambda) = \min \sum_p \sum_i x_i^p x_i^p + \sum_t \sum_{ij} \lambda_{ij}^1 (1 - \sum_p \sum_i a_{ij}^p x_i^p) + \sum_t \lambda_{t}^2 (S_t - \sum_p \sum_i b_{t}^p x_i^p)
\]

s.t. \[ \sum_i x_i^p = 1 \quad \forall p \in P \]
\[ x_i^p \in \{0, 1\} \quad \forall p \in P \quad \forall i \in I \]

The above shown lagrangean relaxation of our problem, which can be solved easily for given lagrangean multipliers, yields a lower bound for the original problem as relaxed constraints may be violated in an optimal solution. Thus, we solve the lagrangean dual problem $\max_\lambda Z(LR_\lambda)$ to find an optimal solution for the original set partitioning problem. Based on a former work by Caprara, Fischetti and Toth [5] we developed an algorithm which uses subgradient optimization to gain optimal lagrangean multipliers and apply column generation techniques to operate on a smaller subset of promising duties [6], [1], [2].
4 Conclusions and Outlook

In our ongoing research project we are currently in the phase of implementing final aspects of the algorithm and computational testing with real life data of the companies. First test results are promising but especially the subgradient method and its various parameters still require a lot of fine tuning. In addition, some new aspects like more workload balanced crews or duties have just arisen and have to be checked for a possible integration in our models and algorithms.

References