Abstract. From Jan 18 to Jan 22, 2010, the Dagstuhl Seminar 10031 “Quantitative Models: Expressiveness and Analysis” was held in Schloss Dagstuhl – Leibniz Center for Informatics. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Quantitative models, quantitative analysis, timed and hybrid systems, probabilistic systems, weighted automata

10031 Executive Summary – Quantitative Models: Expressiveness and Analysis

Quantitative models and quantitative analysis in Computer Science are currently intensively studied, resulting in a revision of the foundation of Computer Science where classical yes/no answers are replaced by quantitative analyses. The potential application areas are huge, e.g., performance analysis, operational research or embedded systems.

The aim of the seminar was to address three fundamental topics which are closely related: quantitative analysis of real-time and hybrid systems; probabilistic analysis and stochastic automata; weighted automata. These three areas of research have mainly evolved independently so far and the relationship between them has emerged only recently. The seminar brought together leading researchers of the three areas, with the goal of future highly productive cross-fertilizations.

Christel Baier, Manfred Droste, Paul Gastin and Kim Guldstrand Larsen
Automata-based Axiom Pinpointing

Franz Baader (TU Dresden, DE)

Axiom pinpointing has been introduced in description logics (DL) to help the user understand the reasons why consequences hold by computing minimal subsets of the knowledge base that have the consequence in question (MinA). Most of the pinpointing algorithms described in the DL literature are obtained as extensions of tableau-based reasoning algorithms for computing consequences from DL knowledge bases. In this paper, we show that automata-based algorithms for reasoning in DLs and other logics can also be extended to pinpointing algorithms. The idea is that the tree automaton constructed by the automata-based approach can be transformed into a weighted tree automaton whose so-called behaviour (on the unique unlabeled infinite tree) yields a pinpointing formula, i.e., a monotone Boolean formula whose minimal valuations correspond to the MinAs. We also develop an approach for computing the behaviour of a given weighted tree automaton.

Keywords: Knowledge Representation, Description Logics, Automata and Logic, Weighted Tree Automata

Full Paper:
http://lat.inf.tu-dresden.de/research/papers/2010/BaPe-JAR.pdf


Probabilistic semantics for timed automata

Nathalie Bertrand (INRIA - Rennes, FR)

We will present a probabilistic semantics for timed automata meant to measure the likelihood of properties in timed systems. First we will explain how qualitative verification of LTL can be performed under this semantics, using a finite-state abstraction. Then, we will tackle quantitative questions, and give an approximation algorithm to compute the probability for a property to be satisfied in a timed automaton.

This talk is based on joint work with Christel Baier, Patricia Bouyer, Thomas Brihaye, Marcus Grosser and Nicolas Markey.
Managing resources in timed systems

Patricia Bouyer-Decitre (ENS - Cachan, FR)

We study the problems of existence and construction of infinite schedules for finite weighted automata and one-clock weighted timed automata, subject to boundary constraints on the accumulated weight. More specifically, we consider automata equipped with positive and negative weights on transitions and locations, corresponding to the production and consumption of some resource (e.g. energy). We ask the question whether there exists an infinite path for which the accumulated weight for any finite prefix satisfies certain constraints (e.g. remains between 0 and some given upper-bound). We also consider a game version of the above, where certain transitions may be uncontrollable.

This talk should be after Kim Larsen's talk and before Nicolas Markey's talk.

Keywords: Weighted timed automata, energy constraints

Full Paper: http://www.lsv.ens-cachan.fr/Publis/PAPERS/PDF/BFLMS-formats08.pdf


CTL Model Checking of Weighted Automata

Peter Buchholz (TU Dortmund, DE)

The talk introduces an extension of CTL denoted as CTL\$ which allows one to formulate requirements on paths of a weighted automaton (WA) with transitions weights from some semiring. For a WA defined over the Boolean semiring CTL\$ corresponds to CTL, for the probabilistic semiring CTL\$ is able to mimic RTCTL an extension of CTL for probabilistic systems. For other semiring like the idempotent semirings max/+, min/+ or max/min CTL\$ defines a new logic and allows the model checking of these types of automata.

After defining the syntax and semantics of the logic, algorithms for model checking the logic on finite state WA are introduced and their complexity is analyzed for different semirings. Finally, we consider bisimulation of WAs and show that bisimilar WAs are indistinguishable under CTL\$ formulas.

Lattice-valued logics and weighted automata

Manfred Droste (Universität Leipzig, DE)

We show that $\mathcal{L}$-weighted automata, rational series, and $\mathcal{L}$-valued monadic second order logic have the same expressive power, for any bounded lattice $\mathcal{L}$ and for finite and infinite words.
We also prove that aperiodicity, star-freeness, and \( L \)-valued first order and LTL-definability coincide. This extends classical results of Kleene, Büchi, and others to arbitrary bounded lattices, without any distributivity assumption fundamental in the theory of weighted automata over semirings.

**Keywords:** Weighted automata, multi-valued logic, formal power series, MSO-logic

**Joint work of:** Droste, Manfred; Vogler; Heiko

*See also:* Manfred Droste and Heiko Vogler: Kleene and Büchi theorems for weighted automata and multi-valued logics over arbitrary bounded lattices, in 14th Int. Conf. on Developments in Language Theory (DLT), Lecture Notes in Computer Science, Springer, 2010, to appear.

**Continuous semirings and finite automata**

*Zoltan Esik (University of Szeged, HU)*

We use weighted finite automata to give complete descriptions of the equational theory of continuous semirings enriched with a Kleene star operation.

**Weighted Tree Automata over Multioperator Monoids**

*Zoltan Füloep (University of Szeged, HU)*

A multioperator monoid \( A \) is a commutative monoid with additional operations on its carrier set.

In general, these operations are not required to distribute over the monoid operation. A weighted tree automaton over \( A \) is a finite state tree automaton of which each transition is equipped with an operation of \( A \).

We show several examples and give a sketch of the Nivat-Engelfriet-like decomposition and the Kleene-result for this automaton model.

**Keywords:** Weighted tree automata and tree transducers, semirings, multioperator monoids

**Joint work of:** Fülöp, Zoltan; Maletti, Andreas; Stüber, Torsten; Vogler, Heiko

**Reconciling Weighted MSO and Probabilistic CTL**

*Paul Gastin (ENS - Cachan, FR)*

While a mature theory around logics such as MSO, LTL, and CTL has been developed in the pure boolean setting of finite automata, weighted automata lack such a natural connection with (temporal) logic and related verification algorithms.
We will identify weighted versions of MSO and CTL that generalize the classical logics and even other quantitative extensions such as probabilistic CTL. We establish expressiveness results on our logics giving translations from weighted and probabilistic CTL into weighted MSO.

*Joint work of:* Bollig, Benedikt; Gastin, Paul

**Quantitative Analysis under Fairness Constraints**

*Marcus Grosser (TU Dresden, DE)*

Fairness assumptions can be crucial for verifying progress, reactivity or other liveness properties for interleaving models. This also applies to Markov decision processes as an operational model for concurrent probabilistic systems and the task to establish tight lower or upper probability bounds for events that are specified by liveness properties.

In this talk, we study general notions of strong and weak fairness constraints for Markov decision processes, formalized in an action- or state-based setting. We present a polynomially time-bounded algorithm for the quantitative analysis of an MDP against omega-automata specifications under fair worst- or best-case scenarios. Furthermore, we discuss the treatment of strong/weak fairness and process fairness constraints in the context of partial order reduction techniques for Markov decision processes that have been realized in the model checker LiQuor and that rely on a variant of Peled’s ample set method.

**Stochastic Model Checking**

*Holger Hermanns (Universität des Saarlandes, DE)*

This is an invited tutorial on stochastic model checking.

We start off with a review of modelling principles for concurrent systems based on labelled transition systems (LTS). We then discuss how to add simple probabilistic experiments to this model, and learn about two possible options: (Discrete-time) Markov chains change the nature of the LTS model, while probabilistic automata provide a conservative extension of LTS and the underlying compositional theory. We then turn our attention to probabilistic experiments in continuous time, as they appear in continuous-time Markov chains. Again we discuss two possible options: Changing the nature of the LTS model, or extending it in a conservative fashion. The latter gives rise to the model of interactive Markov chains.

We continue with a discussion algorithmic aspects of model checking for probabilistic extensions of the temporal logic CTL, for the four models introduced above. We first look at discrete-time Markov chains, then probabilistic automata, focusing on the logic PCTL. We then turn our attention to CSL model checking of continuous-time Markov chains and interactive Markov chains.
Finally we review advanced tool support for stochastic model checking, touching upon the tools PASS, INFAMY, PARAM, and mcpta.

Keywords: Model checking, Markov chains, Markov decision processes

Modal Transition Systems with Weight Sets

Line Juhl (Aalborg University, DK)

We propose weighted modal transition systems, an extension to the well-studied specification formalism of modal transition systems that allows to express both required and optional behaviours of their intended implementations. In our extension we decorate each transition with a weight interval that indicates the range of concrete weight values available to the potential implementations. In this way resource constraints can be modelled using the modal approach.

We focus on two problems. First, we study the question of existence/finding the largest common refinement for a number of finite deterministic specifications and we show PSPACE-completeness of this problem. By constructing the most general common refinement, we allow for a stepwise and iterative construction of a common implementation. Second, we investigate generalised model checking and show that a formula in a natural weight extension of the logic CTL is satisfied by a given modal specification if and only if it is satisfied by all its refinements.

Keywords: Modal transition systems, refinement, weighted transition systems, deterministic specifications

Joint work of: Juhl, Line; Larsen, Kim G.; Srba, Jiri

Lemke’s algorithm for discounted games

Marcin Jurdzinski (University of Warwick, GB)

The performance of a pivoting algorithm due to Lemke is studied on linear complementarity problems (LCPs) that arise from discounted games. The algorithm has not been previously studied in the context of infinite games, and it offers an alternative to the classical strategy-improvement algorithms. The algorithm is described purely in terms of discounted games, thus bypassing the reduction from the games to LCPs, and hence facilitating a better understanding of the algorithm when applied to games. A family of discounted games is given on which the algorithm runs in exponential time, indicating that in the worst case it performs no better for discounted games than it does for general P-matrix LCPs.

Keywords: Discounted games, linear complementarity problem

Joint work of: Fearnley, John; Jurdzinski, Marcin; Savani, Rahul
Continuous-Time Stochastic Games with Time-Bounded Reachability

Jan Kretinsky (TU München, DE)

We study continuous-time stochastic (Markov) games with time-bounded reachability objectives. We show that each vertex in such a game has a value (i.e., an equilibrium probability), and we classify the conditions under which optimal strategies exist.

We also show how to compute optimal strategies in finite uniform games, and how to compute epsilon-optimal strategies in finitely-branching games with bounded rates.

Keywords: Continuous time stochastic systems; time bounded reachability; Markov games

Joint work of: Brazdil, Tomas; Forejt, Vojtech; Kral, Jan; Kretinsky, Jan; Kucera, Antonin


Considerations about weighted automata

Werner Kuich (TU Wien, AT)

We give a survey about some important definitions and result from the theory of weighted finite automata.

The first paper which used in an implicit way weighted finite automata seems to be N. Chomsky, G. A. Miller: Finite state languages. Information and Control 1(1958)91-112.

Starting with this paper we discuss results of the theory of weighted automata up to 2009.

Keywords: Weighted automata
Counting CTL

François Laroussinie (University Paris-Diderot, FR)

We will present a range of quantitative extensions for the temporal logic CTL. We enhance temporal modalities with the ability to constrain the number of states satisfying certain sub-formulas along paths. By selecting the combinations of Boolean and arithmetic operations allowed in constraints, one obtains several distinct logics generalizing CTL. We provide an analysis of their expressiveness and of the complexity of their model-checking problem (ranging from P-complete to undecidable).

Joint work of: Laroussinie, François; Meyer, Antoine; Petonnet, Eudes

Priced Timed Automata and Games

Kim Guldstrand Larsen (Aalborg University, DK)

Priced timed automata are emerging as useful formalisms for modeling and analysing a broad range of resource allocation problems. In this talk, we review the formalisms of priced timed automata and games, highlighting recent (un)decidability results as well as point to a number of open problems.

Keywords: Priced timed automata, priced timed games

A Uniform Framework for the Definition of Stochastic Process Languages

Diego Latella (CNR - Pisa, IT)

In this talk we present how Rate Transition Systems (RTS) can be used as a unifying framework for the definition of the semantics of stochastic process algebras. RTS facilitate the compositional definition of such semantics exploiting operators on the next state functions which are the functional counterpart of classical process algebra operators. We apply this framework to (representative fragments of) major stochastic process calculi including TIPP, PEPA and IML, and show how they solve the issue of transition multiplicity in a simple and elegant way. We, moreover, show how RTS help describing different languages, their differences and their similarities. For each calculus, we also show the formal correspondence between the RTS semantics and the standard SOS one.

Keywords: Stochastic Process Languages

Joint work of: De Nicola, Rocco; Latella, Diego; Loreti, Michele; Massink, Mieke
Statistical Model Checking: An Overview

Axel Legay (University of Liège, BE)

Given a stochastic system (a Markov Chain,...), the probabilistic model checking problem consists in deciding whether this system satisfies some property with a probability greater or equal to a certain threshold. There exist several numerical algorithms (implemented in tools such as PRISM or LIQUOR) for solving such problems.

Unfortunately, those algorithms do not scale up to realistic systems. In this talk, we will show that techniques coming from the area of statistics can be used to solve the probabilistic model checking problem. Contrary to numerical algorithms, those statistic algorithms are applicable to realistic systems. In this talk, we will consider the application of statistical model checking to two families of systems, namely digital/analog circuits and systems biology. We will also briefly introduce Bayesian model checking (particularly suited for systems biology). If time permits, we will also present an application of statistical model checking to a huge industrial case study from EADS.

Weighted Multi Bottom-up Tree Transducers

Andreas Maletti (Universitat Rovira i Virgili -Tarragona, ES)

Synchronous tree substitution grammars (STSG) are a (formal) tree transformation model that is used in the area of syntax-based machine translation. A competitor that is at least as expressive as STSG is proposed and compared to STSG. The competitor is the extended multi bottom-up tree transducer (MBOT), which is the bottom-up analogue with the additional feature that states have non-unary ranks.

Unweighted MBOT have already been investigated with respect to their basic properties, but the particular properties of the constructions that are required in the machine translation task are largely unknown. STSG and MBOT are compared with respect to binarization, regular restriction, and application. Particular attention is paid to the complexity of the constructions.

**Keywords:** Tree transducer, machine translation, translation model, natural language processing
1-clock Timed Automata with Energy Constraints

Nicolas Markey (ENS - Cachan, FR)

We study one-clock priced timed automata in which prices can grow linearly ($\dot{p} = k$) or exponentially ($\dot{p} = kp$), with discontinuous updates on edges, and under "energy constraints". We propose EXPTIME algorithms to decide the existence of controllers that ensure existence of infinite runs or reachability of some goal location with non-negative observer value all along the run. Our algorithms also solve the optimization problem under energy constraints.

This talk should be after Kim Larsen’s and Patricia Bouyer-Decitre’s talks.

Stochastic Games for Verification of Probabilistic Timed Automata

Gethin Norman (University of Glasgow, GB)

Probabilistic timed automata (PTAs) are used for formal modelling and verification of systems with probabilistic, nondeterministic and real-time behaviour. For non-probabilistic timed automata, forwards reachability is the analysis method of choice, since it can be implemented extremely efficiently. However, for PTAs, such techniques are only able to compute upper bounds on maximum reachability probabilities. In this paper, we propose a new approach to the analysis of PTAs using abstraction and stochastic games. We show how efficient forwards reachability techniques can be extended to yield both lower and upper bounds on maximum (and minimum) reachability probabilities. We also present abstraction-refinement techniques that are guaranteed to improve the precision of these probability bounds, providing a fully automatic method for computing the exact values. We have implemented these techniques and applied them to a set of large case studies. We show that, in comparison to alternative approaches to verifying PTAs, such as backwards reachability and digital clocks, our techniques exhibit superior performance and scalability.

Full Paper:
http://www.springerlink.com/content/29320k02k073741/


Compositional Design Methodology with Constraint Markov Chains

Mikkel Larsen Pedersen (Aalborg University, DK)

Notions of specification, implementation, satisfaction, and refinement, together with operators supporting stepwise design, constitute a specification theory.
We construct such a theory for Markov Chains (MCs) employing a new abstraction of a Constraint MC. Constraint MCs permit rich constraints on probability distributions and thus generalize prior abstractions such as Interval MCs. Linear (polynomial) constraints suffice for closure under conjunction (respectively parallel composition). This is the first specification theory for MCs with such closure properties. We discuss its relation to simpler operators for known languages such as probabilistic process algebra. Despite the generality, our operators and relations are all computable.

Joint work of: Pedersen, Mikkel Larsen; Caillaud, Benoit; Delahaye, Benoit; Larsen, Kim Guldstrand; Legay, Axel; Wasowski, Andrzej

Weighted Timed Automata

Karin Quaas (Universität Leipzig, DE)

We present a general model of weighted timed automata that allows for the modelling of continuous resource consumption of real-time systems. A timed automaton is a finite automaton extended with finitely many real-valued clocks that measure the time. We additionally equip the transitions of a timed automaton with weights coming from a semiring. Also, the states are assigned a weight function determining the weight that arises while being in a state. A weighted timed automaton recognizes a timed series, i.e., a function that maps each timed word to a coefficient in the semiring, namely its weight. We present closure properties of recognizable timed series and investigate weighted extensions of classical decidability problems like the emptiness or equivalence problem. Also we provide characterizations of recognizable timed series in terms of logic and generalizations of the well-known regular expressions.

Hybrid Petri Nets with general one-shot transitions

Anne Remke (University of Twente, NL)

A new Hybrid Petri net formalism that allows deterministic, generally distributed and fluid transitions is introduced. Models in such formalism are analyzed with Parametric Reachability Analysis, by computing all reachable locations, and by separating the deterministic and the stochastic evolution of the system. Several performance metrics, such as the distribution of fluid over time, can be derived by deconditioning according to arbitrary continuous probability distributions. This efficient concept allows for the analysis of an arbitrary number of fluid places, as opposed to classical stochastic hybrid petri net approaches. Moreover validation of our results against the FSPN tool shows that parametric reachability analysis provides much sharper results. A case study motivates and shows the feasibility of our approach.
Jacques Sakarovitch (ENST - Paris, FR)

In this talk, I shall present the proof of the following result:

If two regular languages $L$ and $K$ have the same generating functions, that is, for every integer $n$ they have the same number of words of length $n$, there exists a rational bijection realised by a letter-to-letter transducer that maps $L$ onto $K$.

This statement is a consequence of a refinement of the decidability of the equivalence of two automata with multiplicity in $\mathbb{N}$: two $\mathbb{N}$-automata are equivalent if and only if there are conjugate (by matrices with entries in $\mathbb{N}$) to a same third one, and of the interpretation of conjugacy as a sequence of a state splitting and a state amalgamation.

Or, stated otherwise, it is an instance of the fact that if two $\mathbb{N}$-automata $A$ and $B$ over $A^*$ are equivalent, that is, every word of $A^*$ is given the same weight by $A$ and $B$, then one can effectively construct an automaton $C$ which maps homomorphically onto $A$ and $B$, yielding thus a 1-to-1 correspondence between the computations of $A$ and $B$.

This is a joint work with Marie-Pierre Béal and Sylvain Lombardy.

Keywords: Weighted automata; conjugacy

See also: Conjugacy and equivalence of weighted automata and functional transducers, Proc. of CSR’06, LNCS 3967, 58-69.

Lossy Counter Machines

Philippe Schnoebelen (ENS - Cachan, FR)

Lossy counter machines (LCM’s) are a variant of Minsky counter machines based on weak (or unreliable) counters in the sense that they can decrease non-deterministically and without notification. This model, introduced by R. Mayr [TCS 297:337-354 (2003)], is not yet very well known, even though it has already proved useful for showing hardness results.

Our purpose with this talk is to survey the basic theory of LCM’s and their verification problems. Our aim is to make this theory both easier to understand and more popular in our community.

Keywords: Unreliable counters, well-quasi-orderings, well-structured transition systems
Automatic generation of SPA models for dependability / safety analysis

Markus Siegle (Univ. der Bundeswehr - München, DE)

Writing a Stochastic Process Algebra (SPA) specification is an error-prone task, even for experts.

We therefore present a method for the automatic generation of SPA specifications from a high-level dependability modelling formalism. Since immediate actions are used extensively in the generated specification, we rely on an efficient symbolic algorithm for the elimination of vanishing states.

Keywords: Dependability analysis, stochastic process algebra

Rich Interfaces for dependability analysis

Marielle Stoelinga (University of Twente, NL)

We extend the classical system relations of trace inclusion, trace equivalence, simulation, and bisimulation to a quantitative setting in which propositions are interpreted not as boolean values, but as elements of arbitrary metric spaces. Trace inclusion and equivalence give rise to asymmetrical and symmetrical linear distances, while simulation and bisimulation give rise to asymmetrical and symmetrical branching distances. We study the relationships among these distances, and we provide a full logical characterization of the distances in terms of quantitative versions of LTL and pi-calculus. We show that, while trace inclusion (resp. equivalence) coincides with simulation (resp. bisimulation) for deterministic boolean transition systems, linear and branching distances do not coincide for deterministic metric transition systems. Finally, we provide algorithms for computing the distances over finite systems.

Keywords: Quantitative logics, quantitative system models, model checking


A Quantitative Characterization of Weighted Kripke Structures in Temporal Logic

Claus Thrane (Aalborg University, DK)

We extend the usual notion of Kripke Structures with a weighted transition relation, and generalize the usual Boolean satisfaction relation of CTL to a map which assigns to states and temporal formulae a real-valued distance describing the degree of satisfaction. We describe a general approach to obtaining quantitative interpretations for a generic extension of the CTL syntax, and show that, for one such interpretation, the logic is both adequate and expressive with respect to quantitative bisimulation.
A multioperator monoid $A$ is a commutative monoid with additional operations on its carrier set. In general, these operations are not required to distribute over the monoid operation. A weighted tree automaton over $A$ is a finite state tree automaton of which each transition is equipped with an operation of $A$.

We show several examples and give a sketch of the Nivat-Engelfriet-like decomposition and the Kleene-result for this automaton model.

**Keywords:** Weighted tree automata and tree transducers, semirings, multioperator monoids

**Joint work of:** Zoltán Fülöp, Andreas Maletti, Torsten Stüber, Heiko Vogler

---

A specification theory combines notions of specifications and implementations with a satisfaction relation, a refinement relation and a set of operators supporting stepwise design.

We develop a complete specification framework for real-time systems using Timed I/O Automata as the specification formalism, with the semantics expressed in terms of Timed I/O Transition Systems. We provide constructs for refinement, consistency checking, logical and structural composition, and quotient of specifications – all indispensable ingredients of a compositional design methodology.

The theory is implemented on top of an engine for timed games, TIGA, and illustrated with a small case study.

**Joint work of:** Alexandre David, Kim Larsen, Axel Legay, Ulrik Nyman, Andrzej Wasowski

Combinatorial Hybrid Systems

Rafael Wisniewski (Aalborg University, DK)

The talk introduces the concept of a combinatorial control system. It is a discrete abstraction of a hybrid- and thus in particular a continuous control system. I associate to it notions of a flow map, a Lyapunov function and feedback. The state space is partitioned into polyhedral sets and the control action forces a shift from a polyhedron to its neighbor. Locally in each polyhedral set the system is approximated by an affine model. The concept is particularly useful for combined guidance and control of e.g. autonomous robots and satellites.

Keywords: Hybrid systems, control, combinatorial vector fields

Measuring Progress of Probabilistic Model-Checkers

Franck van Breugel (York University - Toronto, CA)

Verification of the source code of a probabilistic system by means of an explicit-state model-checker is challenging. In most cases, the model-checker will either run out of memory or will simply not terminate within any reasonable amount of time. In this talk, we introduce the notion of a progress measure for such a model-checker.

The progress measure returns a number in the interval $[0, 1]$. This number provides us a quantitative measure of the amount of progress the model-checker has made with verifying a particular linear time property. The larger the number, the more progress the model-checker has made. We also discuss how to compute the progress measure for checking invariants.

Java PathFinder (JPF) in an explicit-state model-checker for Java bytecode. We have extended JPF to a probabilistic model-checker.

JPF can traverse the state space in different ways, including depth-first and breadth-first. With the additional probabilistic information available to JPF, new traversal strategies can be added to JPF. We present a few simple traversal strategies that take the probabilities into account. We have extended JPF so that it keeps track of the amount of progress it has made. We show the difference in progress of the different traversal strategies for a few examples.

This talk is based on joint work with Xin Zhang.