Report on Dagstuhl Seminar 10232

Semantics of Information

Organized by

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Summary

There have been recent advances in the *applications* of computer science to several areas of science, including for example,

- new models of classical and quantum physics, computing and information that have emerged from work relying on category theory and domain theory,
- the growth from the turn of this century of the application of process algebra and related techniques from concurrency theory – especially those using stochastic models to biology. This comes under the heading "systems biology, although the term also includes the area of computational science, which uses computers more or less as black box computational devices to generate simulations of biological phenomena, and
- increasing evidence that the work on game semantics and its application to computation has important features in common with economic game theory.

These advances have simultaneously generated interest in areas of mathematics that underlie the areas of computer science that have been leading these new applications. The Dagstuhl Seminar 10232, *Semantics of Information* was devoted to talks by researchers in a wide range of disciplines: mathematics, computer science, systems biology, physics, and economic gam theory, all of which explored the relationship of computer science and its theory to their area. Because the participants came from such diverse backgrounds, the seminar included only four talks each day, each of which was an hour in length, allowing large amounts of time for researchers to interact with one another in an informal setting. In addition, we held a problem session on the closing day at which researchers discussed problems they were working on, or that they wanted the participants to consider. The overall aim was to generate collaborations among the participants, hopefully forming bridges between their disparate areas of expertise. The list of talks given below shows the diversity of interests represented at the meeting.

The seminar was the latest instance of a series of meetings with the same theme (cf. `http://www.math.tulane.edu/~mwm/clifford`, `http://www.math.`

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tulane.edu/~mwm/WIP2008 and http://www.math.tulane.edu/~mwm/WIP2009), all of which have focused on generating similar interactions between researchers actively investigating the applicability and utility of information.

Titles and Abstracts of Talks

- *From Lawvere to Brandenburger-Keisler: interactive forms of diagonalization and self-reference* by Samson Abramsky, University of Oxford

  Abstract: We analyze the Brandenburger-Keisler paradox in epistemic game theory, which is a 'two-person version of Russell's paradox'. Our aim is to understand how it relates to standard one-person arguments, and why the 'believes-assumes' modality used in the argument arises. We recast it as a fixpoint result, which can be carried out in any regular category, and show how it can be reduced to a relational form of the one-person diagonal argument due to Lawvere. We give a compositional account, which leads to simple multi-agent generalizations. We also outline a general approach to the construction of assumption complete models.

- *Observable Implications of Unobservable Variables*, by Adam Brandenburger, New York University

  Abstract: Start with a probability measure e on a space of observable (?empirical?) variables. Can we find an extended space that includes ?hidden? variables, and a probability measure p on this space, where p is required to satisfy certain independence conditions, so that e can be obtained from p? This question can be asked about either classical or quantum-mechanical systems. (In a typical classical case, a negative answer might tell us that two parties Ann and Bob have a communication channel. In a typical quantum-mechanical case, a negative answer might tell us that two spatially separated particles are entangled.) When the space of observables is finite or countable, we give a complete answer to the question: We define a set of inequalities (obtainable from a larger set via Fourier-Motzkin elimination) that describes the probability measures e that can be so analyzed. When the space of observables has a countably generated sigma-algebra, we are able to give a necessary condition.

  This is joint work with H. Jerome Keisler, University of Wsiconsin.

- *Graphical Calculus for Quantum Computing*, by Bob Coecke, University of Oxford

  Abstract: We present a both simple and comprehensive graphical calculus for quantum computing. We axiomatize the notion of an *environment*, which together with the axiomatic notion of classical structure enables us to define classical channels, quantum measurements and classical control. If we moreover adjoin the axiomatic notion of complementarity, we obtain sufficient structural power for constructive representation and correctness derivation of typical quantum informatic protocols.

- *Choquet-Kendall-Matheron Theorems on Non-Hausdorff Spaces.* by Jean Goubault-Larrecq, LSV/ENS Cachan

  Abstract: We establish Choquet-Kendall-Matheron theorems on non-Hausdorff topological spaces. This typical result of random set theory is profitably recast in purely topological terms, using intuitions and tools from domain theory. We obtain three variants of the theorem, each one characterizing distributions, in the form of continuous valuations, over relevant powerdomains of demonic, respectively, angelic, respectively, erratic non-determinism.

  This is joint work with Klaus Keimel, Technische Universität Darmstadt.

- *Quantum Oracles for Space-Bounded Turing Machines*, by Peter Hines, University of York

  Abstract: This talk describes how coherent (i.e. superposition-preserving) oracles may be constructed within the quantum circuit paradigm for computations defined by conditional iteration and halting . The concrete example presented is that of computations defined by space-bounded Turing machines, but the method is entirely general. The process presented may thus be thought of as a compilation procedure that takes computations presented within a relatively high-level computation paradigm (e.g. space-bounded Turing machines) and translates them into low-level computations (i.e. within the quantum circuit paradigm).

  The procedure is based on three distinct results: a close connection (based on domain theory) between conditional iteration and categorical traces, a 'dualising' procedure, based on inverse semigroup theory, that translates a computation based on conditional iteration into a cyclic computation, and a decomposition of the (particle-style) categorical trace in the quantum circuit paradigm, based on game semantics and related concepts.

  For a bounded Turing machine with worst-case performance of T primitive steps, the resulting quantum circuit requires an ancilla of $\log(T)$ qubits, and takes $O(T)$ primitive steps. Thus the translation from space=bounded Turing machines to quantum circuits is efficient.

- *Haar Measure on Compact Groups – Wendel's Proof Revisited* by Karl Hofmann, Technische Universität Darmstadt

  Abstract: Since there is renewed interest in compact affine semigroups we survey their basic concepts and one of their most significant manifestation: The convolution semigroup of probability measures on a compact semigroup and notably on a compact group. The latter yields a classical proof of the existence and uniqueness of Haar measure on a compact group due to Wendel.

- *Topological Quantum Information, Khovanov Homology and the Jones Polynomial*, by Louis H. Kauffman, UIC

Abstract: In this talk we use the state summation structure of the Kauffman bracket polynomial to give a quantum algorithm for computing the Jones polynomial and relate the state space of the bracket with Khovanov homology. Our strategy is to structure the state space for the Kauffman bracket so that each state makes a monomial contribution. Then we construct a Hilbert space whose basis is in one-to-one correspondence with these states. The payoff is that there is a unitary transformation U on the Hilbert space whose trace is the bracket polynonmial evaluated at points on the unit circle in the complex plane, and the Hilbert space is exactly the chain complex for the Khovanov homology with the unitary U anticommuting with the boundary operator. This anticommutation property is key to understanding how the Jones polynomial is a graded Euler characteristic of the Khovanov homology. Thus we see that the quantum infomation strategy of using a Hibert space to display superpositions of information is conceptually tied to the use of linear spaces to find homological information. The natural quantum mechanical strategy of forming complex linear combinations of basis states fits directly with the notion of a chain in a chain complex as a linear combination of basis elements. We intend to go further in this investigation of the relationship of homological and quantum mechanical concepts. This talk is based on the following paper: arXiv:1001.0354 Topological Quantum Information, Khovanov Homology and the Jones Polynomial. Louis H. Kauffman. math.GT (physics.math-ph).

- *Directed Homology* by Sanjeevi Krishnan, NRL

- *Epigenetic regulation - Symmetry, reversibility and development*, by Jean Krivine, PPS, Universty of Paris VII

Abstract: Without committing to any controversial definition, one may describe aging as a continuous and irreversible process by which molecules, cells or organs accumulate changes in time. With this simple definition, the process by which embryonic stem cells progressively loose their pluripotency by differentiation and acquire fixed functions in various tissues can be viewed as an instance of the aging process. From a modeling perspective there is a need to find a mechanistic molecular implementation of this phenomenon with the hope to better understand age related deceases or cancer. This model of cell development is strongly constrained by the fact that each cell of a single organism, be it a neuron or a stem cell, shares a copy of the same genetic code. It results that the "program" that governs the function of a cell cannot be solely contained in the genes and has to be hidden in some "meta" instructions on top of these genes.

Recent technological breakthrough in molecular biology are starting to reveal the structure of this meta-code that biologists call epigenetic. In this talk we will survey this revolutionary view of genes, cells and function with a computer-science oriented spirit, having in mind that we are facing an evolution-based toolkit for the treatment of genetic information that

should most likely beget interesting concepts for computer science and informatics.

- *Hilbert-Space Methods in Decision Theory*, by Pierfrancesco La Mura, Handelshochschule Leipzig

  Abstract: Most work in game theory is conducted under the assumption that the players are expected utility maximizers. Expected utility is a very tractable decision model, but is prone to wellknown paradoxes and empirical violations (Allais 1953, Ellsberg 1961), which may induce biases in game-theoretic predictions. La Mura (2009) introduced a projective generalization of expected utility (PEU) which avoids the dominant paradoxes, while remaining quite tractable. We show that every nite game with PEU players has an equilibrium, and discuss several examples of PEU games.

- *Inner/Outer Pavings, Stably Compact Spaces, and Capacities* by Jimmie Lawson, LSU

  Abstract: We introduce some recent significant new lines of investigation and application involving stably compact spaces, primarily in the theoretical computer science community. There are three primary themes that are developed: firstly the property of stable compactness is preserved under a large variety of powerdomain or hyperspace constructions, secondly the underlying de Groot duality of stably compact spaces is reflected by duality theorems involving the powerdomain constructions, and thirdly the measure-theoretic idea of input/output pavings provides a convenient framework for many of these considerations.

  This is joint work with Klaus Keimel, Technische Universität Darmstadt.

- *The Algebraic Theory of Effects* by Gordon Plotkin, University of Edinburgh

  Abstract: In this talk we survey the algebraic theory of effects. This theory builds on Moggi's idea of monads as notions of computation, but specialises to monads given by algebraic theories. Computationally, the theory operators are the ways in which the effects arise. By making use of the extra structure thereby provided, one can hope to obtain deeper and more applicable results than hold for arbitrary monads.

  We begin by showing, via the two examples of ordinary and probabilistic choice, that the theory equations arise naturally from an operational point of view. Then, in the context of interactive I/O we exemplify generic effects. These are in 1-1 correspondence with the operations definable from the theory operations; in some cases such as interactive I/O or side-effects they are more natural for programmers than the operations.

  We further show that there is a reasonable general view of operational semantics, illustrating this with a version of Moggi's computational lambda-calculus extended with algebraic operators. The general operational semantics is normally reasonably close to the ones standardly given for particular effects, However this is not the case for side-effects.

Here we discuss coalgebras and when they satisfy an algebraic theory. It turns out that the set of states can be specified as a final such coalgebra and the configurations used in the operational semantics of state can arise as tensors of this coalgebra and algebras for the other effects. This gives a prospect of a more inclusive general theory of operational semantics.

One can think of the algebraic operators as effect constructors, as they give rise to the effects at hand. There is a corresponding notion of effect deconstructor. A notable example is that of exception handlers, where the Benton-Kennedy exception handler is paradigmatic. We explain how exception handlers can be explained in terms of homomorphisms from free algebras to algebras defined by the programmer, or language designer, and discuss a suitable general version of the Benton-Kennedy construct.

A given programming language normally has several computational effects and Moggi showed how monads for several effects could be generated from monads operators for individual ones. Algebraically there are various natural ways of combining two theories, for example their sum and their Kronecker tensor product. By instantiating one argument of these binary operations on theories to particular effects one obtains all the monad operators Moggi gave for various algebraic effects. We also discuss a third combination, the distributive combination of theories; this seems to be the right one for the combination of ordinary and probabilistic nondeterminism.

Finally we sketch a general logic of effects. The aim here is ambitious: to obtain a single programming logic that covers all the standard ones, such as Scott's LCF, Hoare logic and various modal logics. While we are largely successful with LCF, modal logic, and evaluation logic, we do not succeed with Hoare logic: it may be that here too coalgebras will play a role.

There remains much else to do in this area. Prominent are: locality, e.g., local variables; concurrency, where parallel operators can be thought of as deconstructors, but are less easy to deal with than those discussed above; and effects in the sense of Gifford, Talpin, etc.