

Theory and Applications of Graph Searching Problems (GRASTA 2011)

Edited by

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Abstract

From February 14, 2012 to February 18, 2012, the Dagstuhl Seminar 11071 “Theory and Applications of Graph Searching Problems (GRASTA 2011)” was held in Schloss Dagstuhl – Leibniz Center for Informatics. During the seminar, participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and open problems are put together in this paper. The first section describes the seminar topics and goals in general. The second section contains the abstracts of the talks and the third section includes the open problems presented during the seminar.

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1 Executive Summary

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Graph searching is often referred to, in a more playful language, as a pursuit-evasion game (or, alternatively, cops and robbers game). This is a kind of game where one part is a set of escaping mobile entities, called evaders (or fugitives), that hide in a graph representing a network, and the other part is a number of chasing agents, called searchers (or pursuers), that move systematically in the graph. The game may vary significantly according to the capabilities of the evaders and the pursuers in terms of relative speed, sensor capabilities, visibility, etc. The objective of the game is to capture the evaders in an optimal way, where the notion of optimality itself admits several interpretations.



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Graph searching revealed the need to express in a formal mathematical way intuitive concepts such as avoidance, surrounding, sense of direction, hiding, persecution, and threatening. There are many variants of graph searching studied in the literature, which are either application driven, i.e. motivated by problems in practice, or are inspired by foundational issues in Computer Science, Discrete Mathematics, and Artificial Intelligence including

- Information Seeking
- Robot motion planning
- Graph Theory
- Database Theory and Robber and Marshals Games
- Logic
- Distributed Computing
- Models of computation
- Network security

The objective of the seminar was to bring researchers from the widest possible variety of disciplines related to graph searching and we will especially encourage the maximum interplay between theory and applications. The meeting initiated the exchange of research results, ideas, open problems and discussion about future avenues in Graph Searching. As a fruit of this encounter new research results, open problems, and methodologies will appeared, especially those of interdisciplinary character.

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3 Overview of Talks

3.1 Cops and Robbers played on random graphs

Pawel Pralat (West Virginia Univ. – Morgantown, US)

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We study the vertex pursuit game of Cops and Robbers, in which cops try to capture a robber on the vertices of the graph. The minimum number of cops required to win on a given graph G is called the cop number of G . We present asymptotic results for the game of Cops and Robbers played on random graph. In particular we show that:

- the Meyniel's conjecture holds a.a.s. for a random d -regular graph $G(n, d)$ as well as a binomial random graph $G(n, p)$ – joint work with Wormald,
- the cop number of $G(n, p)$ as a function of an average degree forms an intriguing zigzag shape – joint work with Luczak,
- almost all cop-win graphs contain a universal vertex – joint work with Bonato and Kemkes.

Other related problems will be mentioned as well.

3.2 Complexity of Cops and Robber Game


Petr Golovach (University of Durham, GB)

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The Cops and Robbers game was defined independently by Winkler-Nowakowski and Quilliot in the 1980s and since that time has been studied intensively. Despite of such a study of the combinatorial properties of the game, almost no algorithmic results on this game are known. Perhaps the main algorithmic result known about Cops and Robbers game is the observation that determining whether the cop number of a graph on n vertices is at most k can be done by a backtracking algorithm which runs in time $n^{O(k)}$ (thus polynomial for fixed k). From the hardness side, Goldstein and Reingold in 1995 proved that the version of the Cops and Robbers game on *directed* graphs is EXPTIME-complete. Also, they have shown that the version of the game on undirected graphs when the cops and the robber are given their initial positions is also EXPTIME-complete. They also conjectured that the game on undirected graphs is also EXPTIME-complete. However, even NP-hardness of the problem was proved only in 2008 by Fomin, Golovach and Kratochvíl. We survey the known complexity results about the Cops and Robbers game and its variants and give a list of open problems.

3.3 Robotic Pursuit Evasion and Graph Search


Athanasios Kehagias (Aristotle University of Thessaloniki GR)

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Robotic Pursuit Evasion (PE) is a hot research area in the robotics community. Among the various mathematical tools roboticists use to model the PE problem, the Graph Search (GS) theory is a prominent (but not the only) example. In this talk I will present and compare several GS-based approaches to robotic PE. I will point out similarities but also differences between robotic PE and graph search. In particular, I will compare the goals, methodology and outlook of roboticists, pure mathematicians and applied mathematicians who have attacked the problem. I will also present some robotic PE problems which require extensions of the “classical” GS setup and I will briefly discuss models of robotic PE which use graphs but *not* the graph search setup.

3.4 Polygon reconstruction from local observations


Peter Widmayer (ETH Zürich, CH)

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We study the problem of reconstructing an unknown simple polygon from a series of certain local observations, similar in spirit to the reconstruction of an unknown network by exploring it. For mobile agents that move in simple ways inside a polygon, we are interested in understanding what types of local observations carry enough information to allow polygon reconstruction. This is part of a more general effort to understand when and how simple primitives allow mobile agents to draw global conclusions about the environment from local observations.

3.5 The price of connectivity in graph searching games


Dariusz Dereniowski (Gdansk University of Technology, PL)

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In the edge searching problem the goal is to clear a simple graph that is initially entirely contaminated. The task is performed by a team of searchers that are allowed to make three types of moves: a searcher is placed on a vertex, a searcher is removed from a vertex, and a searcher slides from a vertex to one of its neighbors. The fugitive is invisible, fast, and has complete knowledge about the graph and the strategy used by the searchers. The fugitive is considered captured if a searcher reaches his location. We are interested in determining the minimum number of searchers (i.e. the search number) required to search a given graph. In the connected graph searching problem we have an additional restriction: the subgraph that is free of the fugitive is always connected. In this talk we discuss the connection between the search number and the connected search number, including an algorithm that converts a given search strategy using k searchers into a connected one using at most $2k + 3$ searchers.

3.6 On the Fast Edge Searching Problem


Boting Yang (University of Regina, CA)

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In this talk, we consider the problem of finding the minimum number of steps to capture the fugitive. We introduce the fast edge searching problem in the edge search model, we describe relations between the fast edge searching and other searching problems, such as the fast searching and the node searching problems, and we present some recent progress on lower bounds and upper bounds of fast search numbers.

3.7 Algorithms for solving infinite games on graphs


Marcin Jurdzinski (University of Warwick, GB)

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This talk is a selective survey of algorithms for solving a number of infinite-path-following games on graphs, such as parity, mean-payoff, and discounted games. The games considered are zero-sum, perfect-information, and non-stochastic. Several state-of-the-art algorithms for solving infinite games on graphs are presented, exhibiting disparate algorithmic techniques, such as divide-and-conquer, dynamic programming/value iteration, local search/strategy improvement, and mathematical programming, as well as hybrid approaches that dovetail some of the former. While the problems of solving infinite games on graphs are in NP and co-NP, and also in PLS and PPAD, and hence unlikely to be complete for any of the four complexity classes, no polynomial-time algorithms are known for solving them.

3.8 On the complexity of CSP decompositions


Zoltán Miklos (EPFL – Lausanne, CH)

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We give a short overview of the relation of certain graph and hypergraph games (robbers and cops/marshals) and CSP decompositions. We discuss the complexity of these problems, in particular the case of tree decompositions. Finally, we report on some progress about the analogous hypergraph problems.

3.9 Searching Games

Maria Serna (UPC – Barcelona, ES)

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We consider a general multi-agent framework in which a set of n agents are roaming a network (such as internet or social networks) where m valuable and sharable goods (or resources or

services) are hidden in m different vertices of the network. We analyze several strategic situations that arise in this setting by means of game theory. To do so we introduce search games, in those games agents have to select a simple path from a predetermined set of initial vertices. Depending on how the goods are splitted among the agents we consider two search game types: finders-share in which the agents that find a good split among them the corresponding benefit and first-share in which only the agents that first find a good share the corresponding benefit. We show that finders-share games always have pure Nash equilibria (PNE). For obtaining this result we introduce the notion of Nash preserving reduction between strategic games. We show that finders-share search games are Nash reducible to single-source network congestion games. This is done through a series of Nash preserving reductions. For first-share search games we show the existence of games with and without PNE. Furthermore we identify some graph families in which the first-share search game has always a PNE that is computable in polynomial time. We discuss also some variants of searching games and the associated graph parameters.



3.10 An overview of The Firefighter Problem

Margaret-Ellen Messinger (Mount Allison University – Sackville, CA)

The Firefighter Problem is a simplified model for the spread of a fire (or disease or computer virus) in a network. Initially, a fire breaks out at a vertex in a connected graph. At each subsequent time step, firefighters *protect* a fixed number of unburned vertices and then the fire spreads to all *unprotected* neighbors. Since its introduction in 1995, there has been a steady growth of both structural and algorithmic results. One possible objective is to maximize the number of saved vertices: this generally requires a strategy on the part of the firefighters, while the fire itself spreads without any strategy. Another possible objective is to find the number of firefighters needed to save a particular number of, or fraction of, or subset of the vertices. (These objectives are sometimes in conflict.) I will discuss some interesting results as well as variants and open problems.

3.11 Graphs with average degree smaller than $\frac{30}{11}$ are burning slowly

Pawel Pralat (West Virginia Univ. – Morgantown, US)


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We consider the following *firefighter problem* on a finite graph $G = (V, E)$. Suppose that a fire breaks out at a given vertex $v \in V$. In each subsequent time unit, a firefighter protects one vertex which is not yet on fire, and then the fire spreads to all unprotected neighbours of the vertices on fire. Since the graph is finite, at some point each vertex is either on fire or is protected by the firefighter, and the process is finished. The objective of the firefighter is to save as many vertices as possible. Let $\mathbf{sn}(G, v)$ denote the number of vertices in G the firefighter can save when a fire breaks out at vertex $v \in V$, assuming the best strategy is used. The surviving rate $\rho(G)$ of G is defined as the expected percentage of vertices that can be saved when a fire breaks out at a random vertex of G , that is, $\rho(G) = \frac{1}{n^2} \sum_{v \in V} \mathbf{sn}(G, v)$. The main focus of the talk is on sparse graphs. Let $\epsilon > 0$. We show that any graph G on n vertices with at most $(\frac{15}{11} - \epsilon)n$ edges can be well protected, that is, $\rho(G) > \frac{\epsilon}{60} > 0$.

Moreover, a construction of a random graph is proposed to show that the constant $\frac{15}{11}$ cannot be improved.

3.12 Cops and Robbers on Directed Graphs


Jan Obdrzalek (Masaryk University, PL)

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We survey the current status of cops and robber games on directed graphs. After presenting the different variants of the game for the most common digraph measures we ask the following question: Is there a digraph width measure which is powerful (i.e. a big class of problems is decidable in linear/polynomial time if this measure is bounded), significantly different from tree-width and yet, at the same time, characterizable by a variant of the cops-and-robber game for tree-width? We show that, under some standard complexity theory assumption, this is not so. We also show a new improvement of this result: That we do not need the measure to be efficiently orientable for our theorem to hold.

3.13 Cop and robber games when the robber can hide and ride

Nicolas Nisse (INRIA Sophia Antipolis, FR)

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In the classical cop and robber game, two players, the cop C and the robber R , move alternatively along edges of a finite graph $G = (V, E)$. The cop captures the robber if both players are on the same vertex at the same moment of time. A graph G is called cop win if the cop always captures the robber after a finite number of steps. Nowakowski, Winkler (1983) and Quilliot (1983) characterized the cop-win graphs as graphs admitting a dismantling scheme. In this talk, we characterize in a similar way the cop-win graphs in the game in which the cop and the robber move at different speeds s' and s , $s' \leq s$. We also investigate several dismantling schemes necessary or sufficient for the cop-win graphs in the game in which the robber is visible only every k moves for a fixed integer $k > 1$. We characterize the graphs which are cop-win for any value of k .

3.14 Complexity of the cop and robber guarding game

Tomas Valla (Charles University – Prague, CZ)


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The guarding game is a game in which several cops has to guard a region in a (directed or undirected) graph against a robber. The robber and the cops are placed on vertices of the graph; they take turns in moving to adjacent vertices (or staying), cops inside the guarded region, the robber on the remaining vertices (the robber-region). The goal of the robber is to enter the guarded region at a vertex with no cop on it. The problem is to determine whether

for a given graph and given number of cops the cops are able to prevent the robber from entering the guarded region. The problem is highly nontrivial even for very simple graphs. It is known that when the robber-region is a tree, the problem is NP-complete, and if the robber-region is a directed acyclic graph, the problem becomes PSPACE-complete [Fomin, Golovach, Hall, Mihalák, Vicari, Widmayer: How to Guard a Graph? *Algorithmica*, DOI: 10.1007/s00453-009-9382-4]. We solve the question asked by Fomin et al. and we show that if the graph is arbitrary (directed or undirected), the problem becomes ETIME-complete.

3.15 Multi-target ray searching problems


Spyros Angelopoulos (CNRS – Paris, FR)

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We consider the problem of exploring m concurrent rays (i.e., branches) using a single searcher. The rays are disjoint with the exception of a single common point, and in each ray a potential target may be located. The objective is to design efficient search strategies for locating t targets (with $t \leq m$). This setting generalizes the extensively studied ray search (or star search) problem, in which the searcher seeks a single target. In addition, it is motivated by applications such as the interleaved execution of heuristic algorithms, when it is required that a certain number of heuristics have to successfully terminate. We study the problem under two different cost measures, and show how to derive optimal search strategies for each measure.

3.16 Characterizations of k -cop win graphs


Nancy Clarke (Acadia University – Wolfville, CA)

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We give two characterizations of the graphs on which k cops have a winning strategy in the game of Cops and Robber. These generalize the corresponding characterizations that are known in the one cop case. In particular, we give a relational characterization of k -copwin graphs, for all finite k , and then use this characterization to obtain a vertex elimination order characterization of such graphs. Instead of the elimination order being of the vertices of the given graph G as in the one cop case, it is an ordering of the vertices of the $(k + 1)$ -fold categorical product of G with itself. Most of our results hold for variations of the game and some of them extend to infinite graphs.

3.17 Some thoughts on constrained cops-and-robbers


Gena Hahn (Université de Montréal, CA)

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This talk is essentially about open questions. First we propose a more general setting for cops-and-robbers games on graphs. Next, we suggest a way to model position and move constraints for the games and observe that there is a partially ordered set of constraints. We then ask what the structure of the poset might be, having observed that the theorem of Nowakowski and Winkler that characterizes cop-win graphs via a binary relation on the set of vertices carries over to the general setting. We close by suggesting that graphs that have some, but not all, loops should be studied, as well as tournaments, and propose a few problems.

3.18 Hypergraph searching as notion justification


Andrei Krokhin (University of Durham, GB)

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We discuss a class of hypergraphs that appeared recently in the study of the constraint satisfaction problem. We show that this class can be described by a natural variant of the hypergraph searching game.

3.19 Monitoring on a Grid


Dieter Mische (UPC – Barcelona, ES)

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We consider a set of g walkers W moving on the $n \times n$ integer grid. Initially, each walker chooses a vertex u.a.r., and in each step, each walker chooses u.a.r. and independently from the other walkers, one neighboring vertex. Moreover, we are given a set D of fixed devices, which are also placed on the integer grid. The devices are used to read data from walkers, and a device can read data of a walker if the walker is within a certain grid distance. We give bounds on the expected number of steps it takes to read data from all walkers for the case where all devices are put onto the halving line of the grid and for the case where all devices are regularly spread on the grid (in a grid-like way).

3.20 LIFO-search

Dimitrios M. Thilikos (National and Kapodistrian University of Athens, GR)

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We study a variant of classic fugitive search game called LIFO-search where searchers are ranked, i.e. are assigned different numbers. The additional rule is that a searcher can be removed only if no searchers of lower rank are in the graph at that moment. We introduce the notion of shelters in graphs and we prove a min-max theorem implying their equivalence with the tree-depth parameter. As shelters provide escape strategies for the fugitive, this implies the the LIFO-search game is monotone and that the LIFO-search parameter is equivalent with the one of tree-depth.

4 Open problems

4.1 Cops and Robbers, parameterized algorithms


Fedor V. Fomin and Petr Golovach

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By making use of backtracking algorithm, it is possible to decide if k cops can win on an n -vertex graph in time $n^{O(k)}$. It is easy to show that if the treewidth a graph is at most t , then the cop number of G is at most $t + 1$. Thus on graphs of constant treewidth computing the minimum number of cops sufficient to win can be done in polynomial time. What is the parameterized complexity of the problem parameterized by the treewidth of the graph? Similar questions can be asked about the parameterization by the clique-width, the genus, and by the size of the excluded minor. The cop number of a graph is bounded by functions of these parameters.

4.2 Computing edge and nodes search numbers on special graph classes

Pinar Heggermes


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Let $es(G)$ and $ns(G)$ be the edge and node search numbers of a graph G , respectively. These parameters are closely related: $ns(G) - 1 \leq es(G) \leq ns(G) + 1$. In general, both parameters are NP-hard to compute, but there are families of graphs, like interval graphs, split graphs, and cographs, on which both parameters can be computed in polynomial time. Is there there a class \mathcal{F} of graphs such that $ns(G)$ can be computed in polynomial time for every graph $G \in \mathcal{F}$, whereas computing the edge search number is NP-hard on \mathcal{F} ? Natural candidate classes to look at are those on which the computation of node search number, or equivalently pathwidth, can be done in polynomial time, but no results are known on the computation of their edge search number. As a first case to consider, $ns(G)$ can be computed in polynomial

(even linear) time if G is a permutation graph. Is the edge search number of permutation graphs computable in polynomial time?

4.3 Connected node search number

Dimitrios M. Thilikos

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Let $\mathbf{ns}(G)$ and $\mathbf{cns}(G)$ be the node and connected node search numbers of graph G . We denote by $\mathbf{mcns}(G)$, the monotone connected search number. It was recently shown that

$$\mathbf{ns}(G) \leq \mathbf{cns}(G) \leq \mathbf{mcns}(G) \leq 2 \cdot \mathbf{ns}(G).$$


Since deciding if $\mathbf{ns}(G) \leq k$ is fixed parameter tractable parameterized by k , this gives an FPT approximation algorithm for connected search number. Is deciding $\mathbf{cns}(G) \leq k$ or $\mathbf{mcns}(G) \leq k$ in FPT?

It is known that numbers $\mathbf{cns}(G)$ and $\mathbf{mcns}(G)$ can be different. How much can they be different? Is it correct that for almost all graphs $\mathbf{cns}(G)/\mathbf{mcns}(G) \rightarrow 1$ as the size of G goes to infinite?

It is believed that the parameter $\mathbf{cns}(G)$ is closed under contractions, i.e., contractions of edges do not make the parameter increase. Is there a formal proof of this? Is deciding $\mathbf{cns}(G) \leq k$ in NP?

4.4 Span-width


Isolde Adler

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We say that a graph G has a span-width at most k , if there is a tree decomposition of G of width k such that every vertex belongs to at most $k + 1$ bags. What is the parameterized complexity of deciding if the span-width of a graph is at most k , parameterized by k ? Similar question for tree-spanners.

4.5 Cop number of toroidal graphs

Gena Hahn


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The long standing conjecture of Schroeder is that the cop number of a graph of genus g is at most $g + 3$.

It is known that for toroidal graph this number is at most 4. Do toroidal graphs have cop number at most 3 as conjectured by Andreae in 1986? Or is there a toroidal graph that actually needs 4 cops?

4.6 Kelly-width

Paul Hunter

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The Kelly-width of a digraph D is defined as the minimum number of searchers required to catch an invisible, inert fugitive with a (fugitive-)monotone strategy. “Inert” means the fugitive is unable to move from a vertex unless a searcher is about to land on that vertex. When the fugitive is able to move, he may move along any directed path not occupied by a searcher. For a more precise definition, see Hunter & Kreutzer [4]. Digraphs of Kelly-width 1 are precisely the acyclic digraphs, and there is a known polynomial time algorithm for deciding if a digraph has Kelly-width at most 2, see [6].

Is deciding if the Kelly-width of digraph D is at most k in PTIME for any fixed $k \geq 3$?

4.7 Ray searching


Spyros Angelopoulos

In the m -lane *ray search* problem we are given a set of m semi-infinite lanes with a common origin O . A target is placed at an unknown ray at distance d from the origin. We seek a strategy that minimizes the worst-case ratio $cost/d$, where $cost$ denotes the overall distance traversed by the searcher up to the point it locates the target.

This problem has been solved in its deterministic variant by Gal [3]. The question of finding randomized strategies that minimize the worst-case ratio $E[cost]/d$ is not quite settled. In [5] a randomized strategy is presented which however is optimal only in the class of round-robin strategies. Can we find optimal randomized strategies without any restrictive assumptions?

4.8 Ratio of monotonicity

Stephan Kreutzer

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Consider two version of searching on a directed graph:

- Inert invisible fugitive game.
- Visible fugitive game.

Both problems are known to be non-monotone. Is there a number d such that the ratio between monotone and non-monotone versions of these games is at most d ? More generally, is there an FPT approximation of non-monotone via monotone parameters?

4.9 Best strategy to catch the drunk robber

Dimitrios M. Thilikos

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So far, in all cop and robber game settings, cops were considered to be omniscient and lucky in the sense that they will always take the best decision in order to avoid, or delay, capture. An interesting topic would be to study the setting where the robber moves randomly and the cops are clever. This induces a mix between classic graph searching and random walks. An example is given below:

Given a graph such as a line, a cycle, or a (toroidal) grid, or a 3-regular graph, consider a robber that chooses randomly its first position and then moves randomly in neighbor nodes of the graph. Assume also that there are so many cops as the searching number of the graph. The cops play first, may move simultaneously. The two parts play in rounds. The objective here is to compute the minimum, over all cop strategies, expected time of arrest of the (drunk) robber.


Question 1. Are all the optimal strategies monotone in the sense that the expected capture time does not change if cop strategies are restricted to those that do not visit again an already searched location? (The question has some meaning even when the number of cops is smaller than the search number of the graph.)

Question 2. How the expected capture time changes when there are less cops than the cop number?

Question 3. What is the ratio between the expected capture time for a drunk robber and the maximum capture time a “sober” robber (i.e., one that makes its best to avoid capture). Is this ratio common for many (or even all) graphs? Is it a constant such as 2?

4.10 Escaping from random cops

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The paper [1] analyzes a randomized cop-and-robber game on graphs. The cop and the robber do not see each other, unless they are on the same node, in which case the robber is caught. They both move along the edges of the graph, one edge per round, playing in turn. Given a randomized cop strategy, the escape length for that strategy is the worst case expected number of rounds it takes the cop to catch the robber, where the worst case is with regards to all (possibly randomized) robber strategies. Adler et al. [1] proposes a cop strategy with an escape length of $O(n \log D)$ in n -node diameter- D graphs. On the other hand, there is a trivial $\Omega(n)$ lower bound on the escape length.

Open problem: close the gap between the two bounds.

One restricted case that may deserve attention is the case where the cop is bounded to apply simple random walk. In that case, is the best strategy for the robber the one consisting in placing itself at the node with lowest steady state probability, and stay idle? Or, if the initial position of the cop given, is the best strategy for the robber the one consisting in placing itself at the node with highest hitting time, and stay idle?

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