Report from Dagstuhl Seminar 11182

Exploiting graph structure to cope with hard problems

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 11182 "Exploiting graph structure to cope with hard problems" which has been held in Schloss Dagstuhl – Leibniz Center for Informatics from May 1st, 2011 to May 6th, 2011. During the seminar experts with a common focus on graph algorithms presented various new results in how to attack NP-hard graph problems by using the structure of the input graph. Moreover, in the afternoon of each seminar's day new problems have been posed and discussed.

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1 Executive Summary

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The organizers of the seminar were Andreas Brandstädt, University of Rostock, Germany; Martin C. Golumbic, University of Haifa, Israel; Pinar Heggernes, University of Bergen, Norway; and Ross McConnell, University of Colorado, USA. The collector of the presented seminar material and the editor of the final report was Christian Hundt, University of Rostock, Germany.

One of the main goals of this Dagstuhl seminar was to gather experts with a common focus on graph algorithms but with various specializations to attack NP-hard graph problems using the structure of the input graph. This goal was achieved to a great extent, as the number of participants of our seminar was above the limit that was given beforehand. The seminar was granted space for 30 participants, and we had 35 participants on site. Still there were many experts and young researchers in the field that would like to come but that could not be invited due to lack of space. The participants that came to our seminar were from the following countries: 9 from Germany, 7 from USA, 5 from France, 4 from Israel, 4 from Norway, 1 from Canada, 1 from Korea, 1 from Taiwan, 1 from Turkey, 1 from Greece, and 1 from Great Britain.

By bringing together experts with backgrounds in graph classes, optimization, width parameters, and parameterized and exact computing, our aim was that several of the hard problems arising in real applications would eventually find practical solutions. The seminar program was divided into two parts each day of the seminar: (1) presentation of new results, and (2) posing and discussions on new problems. Each of the presented new results and the discussed problems will be explained in detail in the sections below. In this section, we briefly summarize the program of the seminar.

On the first day of the seminar, Jesper Nederlof presented new results on solving connectivity problems parameterized by treewidth in single exponential time, Charis Papadopoulos presented new results on characterizing the linear clique-width of a class of graphs by forbidden induced subgraphs, Yngve Villanger presented new kernelization and approximation results on minimum fill-in of sparse graphs, and Martin Milanič presented new results on hereditary efficiently dominatable graphs. During the problem solving session of the first day, Pinar Heggernes posed a problem on dense subgraphs on proper interval graphs, Andreas Brandstädt and Christian Hundt posed problems on k-leaf powers, Dieter Rautenbach posed a problem on 2-domination on strongly chordal graphs, Pavol Hell asked which problems that are hard on general digraphs become efficiently solvable on interval digraphs, and Feodor Dragan introduced and asked questions about the short fill-in problem.

On the second day of the seminar, Ann Trenk presented a survey on the total linear discrepancy of a poset, Elad Cohen presented new results on vertex intersection graphs of paths on a grid, Yahav Nussbaum presented new results on the recognition of probe proper interval graphs, and Tinaz Ekim presented a survey on polar graphs. During the problem solving session of the second day, Daniel Lokshtanov posed a problem on map graphs, Christian Hundt gave more details on his problem on k-leaf powers, Martin Golumbic posed a problem on the recognition of one-bend EPG and VPG graphs, and Sang-il Oum posed a problem on Bott equivalence between directed acyclic graphs.

On the third day of the seminar, Dieter Rautenbach presented new results on unit interval graphs, R. Sritharan presented new results on finding a sun in graphs, Bernard Ries presented new vertices on coloring vertices of triangle-free graphs, and Wen-Lian Hsu presented new results on PC-trees and planar graphs. In the afternoon of the third day, there was an excursion to Trier and a hike in the close by surroundings of Dagstuhl area.

On the fourth day of the seminar, Pavol Hell presented new results on partitioning chordal graphs, Pim van 't Hof presented new results on contracting graphs to paths and trees, Daniel Lokshtanov presented new results on contracting graphs to bipartite graphs, and Frédéric Maffray presented new results on 3-colorable P_5-free graphs. During the problem solving session of the fourth day, Daniel Lokshtanov posed a problem on the parameterized complexity of protein folding, Yngve Villanger posed a question about forbidden induced subgraphs of circular arc graphs, Christophe Paul asked a question on recognizing circle graphs, and Van Bang Le posed a problem on modified circle graphs.

On the fifth day of the seminar, Feodor Dragan presented new results on graph classes, tree decomposition and approximation algorithms, Elias Dahlhaus presented new results on minimal fill-in ordering of planar graphs in linear time, and Christian Hundt presented new results on the dominating induced matching problem for hole-free graphs. The seminar ended with a brief discussion on all the presented results.

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3 Overview of Talks

Monday, May 2nd, 2011

3.1 Solving Connectivity Problems Parameterized by Treewidth in Single Exponential Time

Jesper Nederlof (University of Bergen, NO)

Joint work of Cygan, Marek; Nederlof, Jesper; Pilipczuk, Marcin; Pilipczuk, Michał; van Rooijk, Johan M. M.; Wojtaszczyk, Jakub Onufry
Main reference Marek Cygan, Jesper Nederlof, Marcin Pilipczuk, MichałPilipczuk, Johan van Rooij, Jakub Onufry Wojtaszczyk, "Solving connectivity problems parameterized by treewidth in single exponential time", arXiv:1103.0534v1 [cs.DS]
URL http://arxiv.org/abs/1103.0534v1

For the vast majority of local graph problems (local in the sense that a solution can be verified by checking separately the neighborhood of each vertex) on graphs of small treewidth, standard dynamic programming techniques give $c^{tw(G)}|V|^{O(1)}$ time algorithms, where $tw(G)$ is the treewidth of the input graph $G = (V, E)$ and c is a constant. On the other hand, for problems with a global requirement (usually connectivity) the best known algorithms were naive dynamic programming schemes running in at least $tw(G)^{tw(G)}$ time.

We breach this gap by introducing a technique we named Cut & Count that allows to produce $c^{tw(G)}|V|^{O(1)}$ time Monte Carlo algorithms for most connectivity-type problems, including DIRECTED HAMILTONIAN PATH, STEINER TREE, FEEDBACK VERTEX SET and CONNECTED DOMINATING SET.

This talk will be entirely self-contained. For convenience I will introduce a simplification of the treewidth concept, called $t{*}n$-graphs. Then I will introduce and proof the Isolation Lemma due to Mulmuley et al. (STOC 1987), and finally I will use this lemma by demonstrating the Cut & Count technique and showing how to solve the STEINER TREE problem in $t * n$-graphs in $c^t n^{O(1)}$ time. If time allows I will also discuss some other results from the paper.

3.2 Characterizing the Linear Clique-Width of a Class of Graphs by Forbidden Induced Subgraphs

Charis Papadopoulos (University of Ioannina, GR)

Joint work of Papadopoulos, Charis; Pinar Heggernes; Daniel Meister

We study the linear clique-width of graphs that are obtained from paths by disjoint union and adding true twins. We show that these graphs have linear clique-width at most 4, and we give a complete characterization of their linear clique-width by forbidden induced subgraphs. As a consequence, we obtain a linear-time algorithm for computing the linear clique-width of the considered graphs.

3.3 Minimum Fill-in of Sparse Graphs: Kernelization and Approximation

Yngve Villanger (University of Bergen, NO)

Joint work of Fomin, Fedor V; Philip, Geevarghese; Villanger, Yngve

The Minimum Fill-in problem is to decide if a graph can be triangulated by adding at most k edges. The problem has important applications in numerical algebra, in particular in sparse matrix computations. We develop kernelization algorithms for the problem on several classes of sparse graphs. We obtain linear kernels on planar graphs, and kernels of size $O(k^{3/2})$ in graphs excluding some fixed graph as a minor and in graphs of bounded degeneracy. As a byproduct of our results, we obtain approximation algorithms with approximation ratios $O(\log k)$ on planar graphs and $O(k^{1/2} \log k)$ on H-minor-free graphs. These results significantly improve the previously known kernelization and approximation results for Minimum Fill-in on sparse graphs.

3.4 On Hereditary Efficiently Dominatable Graphs

Martin Milanič (University of Primorska, SI)

Main reference Martin Milanič, "A hereditary view on efficient domination,' to appear in the Proceedings of the 10th Cologne-Twente Workshop on Graphs and Combinatorial Optimization (CTW 2011).

An efficient dominating set (or perfect code) in a graph is a set of vertices the closed neighborhoods of which partition the graph's vertex set. Determining whether a given graph contains an efficient dominating set is NP-complete in general. We study the class of hereditary efficiently dominatable graphs, that is, graphs every induced subgraph of which contains an efficient dominating set.

Based on a decomposition theorem for (bull, fork, C_4)-free graphs, we derive the forbidden induced subgraph characterization of hereditary efficiently dominatable graphs. We also present a polynomial time algorithm to find an efficient dominating set in a hereditary efficiently dominatable graph.

Tuesday, May 3rd, 2011

3.5 The Total Linear Discrepancy of a Poset

Ann Trenk (Wellesley College, US)

Main reference Howard, David; Shull, Randy; Streib, Noah; Trenk, Ann; "The total linear discrepancy of an ordered set," Discrete Math. 310(2010) 1022-1025.

In this talk we discuss the total linear discrepancy of a poset. If L is a linear extension of a poset P, and x, y is an incomparable pair in P, the height difference between x and y in L is $|L(x) - L(y)|$. The total linear discrepancy of P in L is the sum over all incomparable pairs of these height differences. The total linear discrepancy of P is the minimum of this sum taken over all linear extensions L of P. While the decision problem of determining whether the (ordinary) linear discrepancy of a poset is at most k is NP-complete, the total linear discrepancy can be computed in polynomial time. In this talk we characterize those linear extensions that are optimal for total linear discrepancy. The characterization provides an easy way to count the number of optimal linear extensions.

3.6 Intersection Graphs of Paths on a Grid

Elad Cohen (University of Haifa, IL)

Joint work of Asinowski, Andrei; Golumbic, Martin C.; Limouzy, Vincent; Lipshteyn, Marina; Stern, Michal

We investigate the class of vertex intersection graphs of paths on a grid, and specifically consider the subclasses that are obtained when each path in the representation has at most k bends (turns). We call such a subclass the B_k-VPG graphs, $k \geq 0$.

We present a complete hierarchy of VPG graphs relating them to other known families of graphs. String graphs are equivalent to VPG graphs. The grid intersection graphs are shown to be equivalent to the bipartite B_0-VPG graphs.

Chordal B_0-VPG graphs are shown to be exactly Strongly Chordal B_0-VPG graphs. We prove the strict containment of B_0-VPG and circle graphs into B_1-VPG. Planar graphs are known to be in the class of string graphs, and we prove here that planar graphs are B_3-VPG graphs.

In the case of B_0-VPG graphs, we observe that a set of horizontal and vertical segments have strong Helly number 2. We show that the coloring problem for B_k-VPG graphs, for $k \geq 0$, is NP-complete and give a 2-approximation algorithm for coloring B_0-VPG graphs. Furthermore, we prove that triangle-free B_0-VPG graphs are 4-colorable, and this is best possible.

This work was presented at LAGOS 2011.

3.7 Recognition of Probe Proper Interval Graphs

Yahav Nussbaum (Tel Aviv University, IL)

In a probe graph the vertex set is partitioned into probes and non-probes. A probe proper interval graph is an intersection graph of intervals on the line such that every vertex is mapped to an interval, no interval contains another, and two vertices are adjacent if and only if the corresponding intervals intersect and at least one of them is a probe.

In this talk I will present the first linear-time algorithm that determines whether the input graph is a probe proper interval graph, and if the answer is positive then the algorithm constructs a corresponding set of intervals.

3.8 Polar Graphs

Tinaz Ekim (Bogaziçi University – Istanbul, TR)

G is polar if its vertex set admits a partition (A, B) where $G[A]$ is complete multipartite and $G[B]$ is a disjoint union of cliques. We distinguish a special case, called monopolar, when A is a stable set. The recognition of polar and monopolar graphs are NP-complete in general, but they become polynomially solvable in some classes of graphs. In this talk, we survey the recognition and the forbidden subgraph characterization of polar and monopolar graphs in cographs, in chordal graphs, in permutation graphs, in line graphs and in claw-free graphs. We also discuss some related research topics.

Wednesday, May 4th, 2011

3.9 Unit Interval Graphs — A Story with Open Ends

Dieter B. Rautenbach (Universität Ulm, DE)

Joint work of Rautenbach, Dieter B.; Szwarcfiter, Jayme L.

We give two structural characterizations of the class of finite intersection graphs of the open and closed real intervals of unit length. This class is a proper superclass of the well known unit interval graphs.

3.10 Finding a Sun in Graphs

Sritharan, R (University of Dayton, US)

Joint work of Eschen, Elaine; Hoang, Chinh; Spinrad, Jeremy; Sritharan, R

A sun is a graph with a Hamiltonian cycle $(x_1, y_1, ..., x_n, y_n), n >= 3$, where each x_i has degree two and the y_i vertices form a clique. Deciding whether an arbitrary graph contains a sun is NP-complete, while for some graph classes (e.g. chordal, hhd-free) the problem is solvable in polynomial time. We give a polynomial-time algorithm to test for suns in building-free graphs. Building-free graphs generalize Meyniel graphs (and hence, hhd-free, i-triangulated, and parity graphs). We also discuss an elimination scheme of vertices for graphs that do not contain any gem or a building.

3.11 Coloring Vertices of Triangle-free Graphs

Bernard Ries (Université Paris-Dauphine, FR)

The vertex coloring problem is known to be NP-complete in the class of triangle-free graphs. Moreover, it remains NP-complete even if we additionally exclude a graph F which is not a forest. We study the computational complexity of the problem in (K_3, F)-free graphs with F being a forest. From known results it follows that for any forest F on 5 vertices, the vertex coloring problem is polynomial-time solvable in the class of (K_3, F)-free graphs. In the present paper, we study the problem for (K_3, F)-free graphs with F being a forest on 6 vertices. It is known that in the case when F is the star $K_{1,5}$, the problem is NP-complete. It turns out that this is the only hard case.

3.12 PC-Trees and Planar Graphs

Wen-Lian Hsu (Academica Sinica – Taipei, TW)

Linear time planarity test was first established by Hopcroft and Tarjan in 1974 based on a path addition approach. A vertex addition approach, originally developed by Lempel, Even and Cederbaum, was later improved by Booth and Lueker in 1976 to run in linear time using a data structure called "PQ-tree". PQ-tree can also be used to test the consecutive ones property and to recognize interval graphs. Shih and Hsu developed a linear time planarity test based on PC-trees. PC-tree, a generalization of PQ-tree [W.L. Hsu and R. McConnell, PC-trees and circular-ones arrangement, Theoretical Computer Science 296(1), 2003, pp. 99-116], is more natural in representing the relationships between biconnected components and nodes in planar graphs. An earlier version of Shih and Hsu has been referred to as the simplest linear time planarity test by Thomas in his lecture notes. In this talk we shall describe an ultimate version of planarity test based on PC-trees, which is much simpler than any previous version. Moreover, we shall describe how to extend this algorithm naturally to find maximal planar subgraphs in linear time for arbitrary graphs.

Thursday, May 5th, 2011

3.13 Graph Partitions

Pavol Hell (Simon Fraser University – Burnaby, CA)

Some partition problems for chordal graphs have forbidden induced subgraph characterizations and others don't. I will describe some examples and classify all partitions with up to three parts. The main problem of classification is open. The talk will include joint work with Tomas Feder, Shekoofeh Nekooei-Rizi, Juraj Stacho, Geordie Schell, and Wing Xie.

3.14 Contracting Graphs to Paths and Trees

Pim van 't Hof (University of Bergen, NO)

The problems Path-Contractibility and Tree-Contractibility take as input an undirected graph G and an integer k, and ask whether we can obtain a path or a tree, respectively, by contracting at most k edges of G. Both problems are NP-complete, and fixed parameter tractability follows from Courcelle's Theorem. We present algorithms with running time $c^k n^{O(1)}$ with small constants $c < 5$ for both problems. Furthermore, we show that Path-Contractibility has a kernel with at most $5k + 3$ vertices, while Tree-Contractibility does not have a polynomial kernel unless coNP is a subset of NP/poly. Interestingly, Feedback Vertex Set, which can be seen as the vertex deletion variant of Tree-Contractibility, is known to have a kernel with $O(k^2)$ vertices.

3.15 Contraction to Bipartite Graphs is Fixed Parameter Tractable

Daniel Lokshtanov (University of California – San Diego, US)

Main reference Pinar Heggernes, Pim van 't Hof, Benjamin Lévêque, Daniel Lokshtanov, Christophe Paul, "Contracting Graphs to Paths and Trees," arXiv:1104.3677v1 [cs.DS]
URL http://arxiv.org/abs/1104.3677v1

We initiate the study of the Bipartite Contraction problem from the perspective of parameterized complexity. In this problem we are given a graph G and integer k, and the task is to determine whether we can obtain a bipartite graph from G by a sequence of at most k edge contractions. Our main result is that Bipartite Contraction admits a $f(k)n^{O(1)}$ time algorithm.

Despite a strong resemblance between Bipartite Contraction and the classical Odd Cycle Transversal (OCT) problem, the methods developed to tackle OCT do not seem to be directly applicable to Bipartite Contraction. Our algorithm is based on a novel combination of the irrelevant vertex technique introduced by Robertson and Seymour and the concept of important separators. Both techniques have previously been used as key components of cornerstone theorems in parameterized complexity.

However, to the best of our knowledge, this is the first time the two techniques are applied in unison.

3.16 On the Structure of 3-colorable P_5-free Graphs

Frédéric Maffray (CNRS – Grenoble, FR)

Joint work of Maffray, Frédéric; Morel, Grégory
Main reference Maffray Frédéric, Morel Grégory, "On 3-colorable P5-free graphs," Cahiers Leibniz Research Report 191 (Laboratoire G-SCOP, Grenoble, France), March, 2011.
URL http://www.g-scop.inpg.fr/CahiersLeibniz/

We consider the class of 3-colorable P_5-free graphs. We give a complete description of the structure of the graphs in that class.

Our main result is as follows. If G is a connected 3-colorable P_5-free graph, then either G has a homogeneous set (that induces a bipartite subgraph), or G is the complement of a chordal graph, or G has one among four possible types of well-defined structures.

From this structural description of 3-colorable P_5-free graphs, we can derive a linear time algorithm that tests membership in the class, and a linear-time algorithm that finds a maximum weight stable set.

Friday, May 6th, 2011

3.17 An Approximation Algorithm for the Tree t-Spanner Problem on Unweighted Graphs via Generalized Chordal Graphs

Feodor F. Dragan (Kent State University, US)

A spanning tree T of a graph G is called a tree t-spanner of G if the distance between every pair of vertices in T is at most t times their distance in G. In this talk, we present an algorithm which constructs for an n-vertex m-edge unweighted graph G:

- a tree $(2\lfloor \log_2 n \rfloor)$-spanner in $O(m \log n)$ time, if G is a chordal graph (i.e., every induced cycle of G has length 3);
- a tree $(2\rho\lfloor \log_2 n \rfloor)$-spanner in $O(mn \log^2 n)$ time or a tree $(12\rho\lfloor \log_2 n \rfloor)$-spanner in $O(m \log n)$ time, if G is a graph admitting a Robertson-Seymour's tree-decomposition with bags of radius at most ρ in G; and
- a tree $(2\lceil t/2 \rceil \lfloor \log_2 n \rfloor)$-spanner in $O(mn \log^2 n)$ time or a tree $(6t\lfloor \log_2 n \rfloor)$-spanner in $O(m \log n)$ time, if G is an arbitrary graph admitting a tree t-spanner.

For the latter result we use a new necessary condition for a graph to have a tree t-spanner: if a graph G has a tree t-spanner, then G admits a Robertson-Seymour's tree-decomposition with bags of radius at most $\lceil t/2 \rceil$ and diameter at most t in G.

3.18 Minimal Fill-in Ordering of Planar Graphs in Linear Time

Elias Dahlhaus (DB Systems GmbH – Frankfurt, DE)

We present an alternative linear time algorithm for ordering the vertices of a planar graph such that the set of fill-in edges is minimal with respect to the subset relation. The algorithm is simpler than the algorithm in [E. Dahlhaus, Minimal Elimination of Planar Graphs, Algorithm Theory – SWAT'98, LLNCS 1432 (1998), pp. 210–221] and is easily parallelizable, as it does not rely on the computation of a breadth-first search tree.

3.19 Efficient Edge Domination on Hole-free Graphs

Christian Hundt (Universität Rostock, DE)

Joint work of Brandstädt, Andreas; Hundt, Christian; Nevries, Ragnar
Main reference Andreas Brandstädt, Christian Hundt and Ragnar Nevries. Efficient Edge Domination on Hole-free Graphs in Polynomial Time. In Proc. 9th Latin American Theoretical Informatics Symposium, LATIN 2010, LNCS, Springer-Verlag 2010, pp. 650-661.
URL http://dx.doi.org/10.1007/978-3-642-12200-2_56

This talk deals with the *Efficient Edge Domination* Problem (*EED*, for short), also known as *Dominating Induced Matching Problem*, which asks, given an undirected graph $G = (V, E)$, for an induced matching $M \subseteq E$ that simultaneously dominates all edges of G. Thus, the distance between edges of M is at least two and every edge in E is adjacent to an edge of M. EED is related to parallel resource allocation problems, encoding theory and network routing. The problem is NP-complete even for restricted classes like planar bipartite and bipartite graphs with maximum degree three.

However, the complexity has been open for chordal bipartite graphs. We show that EED can be solved in polynomial time on hole-free graphs. Moreover, we provide even linear time for chordal bipartite graphs. Finally, we strengthen the NP-completeness result to planar bipartite graphs of maximum degree three with girth k for every fixed k.

4 Open Problems

Monday, May 2nd, 2011

4.1 Dense Subgraphs in a Proper Interval Graph

Pinar Heggernes (University of Bergen, NO)

Dense Subgraph in a Proper Interval Graph
Given: a proper interval graph $G = (V, E)$ and an integer $k \in \mathbb{N}$
Find: an induced subgraph of k vertices in G that has the largest possible number of edges

Is this problem polynomial-time solvable or NP-hard?

4.2 Structure and Recognition of k-Leaf Powers

Andreas Brandstädt (Universität Rostock, DE)

A graph $G = (V, E)$ is a *k-leaf power* for a $k \in \mathbb{N}$ if there is a tree T such that V is exactly the set of leaves of T and $uv \in E$ if and only if $d_T(u, v) \le k$. A graph G is a *leaf power* if it is a k-leaf power for some $k \in \mathbb{N}$. For instance, the set of 3-leaf powers forms precisely the class of (bull, dart, gem)-free chordal graphs. Moreover, some results are known for $k = 4$ and $k = 5$.

What is the structure of leaf powers, respectively of k-leaf powers for $k \ge 6$? How complex are the corresponding recognition problems?

4.3 The Complexity of 2-Domination on Strongly Chordal Graphs

Dieter B. Rautenbach (Universität Ulm, DE)

The *dominating set problem* on chordal graphs is NP-hard. But it is computable in polynomial time on strongly chordal graphs.

2-Domination

Given: a graph $G = (V, E)$

Find: a smallest set $D \subseteq V$ such that the neighborhood $N(v)$ of all $v \in V \setminus D$ fulfills $|N(v) \cap D| \ge 2$

Is this problem solvable in polynomial time on strongly chordal graphs, too? Is the problem's complexity known for interval graphs? (R. Sritharan)

4.4 Problems Becoming Feasible on Classes of Interval Digraphs

Pavol Hell (Simon Fraser University – Burnaby, CA)

Directed interval graphs $G = (V, E)$ are the graphs that have an interval model where every vertex v is represented by two intervals (I_v, J_v) and there is a directed edge $uv \in E$ if and only if I_u intersects J_v. What problems that are intractable on general directed graphs become tractable on interval digraphs?

The same question is also interesting for the following subclasses of interval digraphs:

Interval catch digraphs, the digraphs $G = (V, E)$ that have a model where for all $v \in V$ the intervals J_v consist of just one point contained in the interval I_v. They have a forbidden subgraph characterization that resembles AT-freeness and they also exhibit an ordering characterization.

Adjusted interval digraphs, the digraphs $G = (V, E)$ that have a model where for all $v \in V$ it is true that the intervals I_v and J_v have the same left endpoint. Such graphs have another ordering characterization.

Adjusted interval catch digraphs, the intersection of the previous two classes. The graphs in this class can be recognized in linear time. Moreover, they can be characterized in the following way: In an adjusted interval catch digraph $G = (V, E)$ it is true for all $u, v \in V$ that there is an edge uv if and only if J_u and I_v intersect and I_u starts earlier than I_v.

4.5 Structure and Complexity of Short Fill-in for Strongly Chordal Graphs and Tree Powers

Feodor F. Dragan (Kent State University, US)

Short Fill-in with Property Π
Given: a graph $G = (V, E)$
Find: the smallest number $d \in \mathbb{N}$ that permits an edge set E' between vertices of G such that $G' = (V, E \cup E')$ has property Π and for all $uv \in E'$ it is true $d_G(u, v) \leq d$

If Π is the property of G' being chordal, then the problem is equivalent to finding the tree length of G. What is the structure and complexity of the problem, if Π is the property of G' being strongly chordal or the power of a tree?

The decision version of the problem is related to sandwich problems, because, if d is given, then G' is a graph between G and the d'th power G^d of G.

Tuesday, May 3rd, 2011

4.6 Fast and Simple Recognition of Map Graphs

Daniel Lokshtanov (University of California – San Diego, US)

Map graphs are a generalization of planar graphs that can be defined as the graphs $G = (V, E)$ that permit a planar bipartite graph $H = (V \cup R, E')$ having a bipartition into the vertex set V of G and an additional vertex set R such that two vertices $u, v \in V$ are adjacent in G, i. e., $uv \in E$, if and only if u and v have a common neighbor in H, hence, if there is a vertex $w \in R$ with $uw, vw \in E'$. This means that $G = H^2[V]$.

The recognition of map graphs can be done in $O(|V|^{120})$ time by an algorithm of Mikkel Thorup in 1998. Is it possible to realize a faster recognition of map graphs with a nice and simple algorithm?

4.7 The Leaf Rank of Proper Interval Graphs

Christian Hundt (Universität Rostock, DE)

Leaf Rank of Proper Interval Graphs
Given: a proper interval graph $G = (V, E)$
Find: the smallest possible number $k \in \mathbb{N}$ such that G is a k-leaf power

Proper interval graphs $G = (V, E)$ without true twins have a unique (up to reversal) ordering of their vertices V. In particular, such an ordering $(v_1, \ldots, v_n)$, $n = |V|$ fulfills for all $i < j \in \{1, \ldots, n\}$ that $v_i v_j \in E$ implies $v_i v_r \in E$ and $v_r v_j \in E$ for all $i < r < j$. Based on this ordering the above problem is equivalent to the following one:

Weighted Linear Steiner Root of a Proper Interval Graph
Given: a proper interval graph $G = (V, E)$ without true twins where $n = |V|$
Find: the smallest integer s such that there exist integers $x_1, \ldots, x_n, w_1, \ldots, w_{n-1} \in \mathbb{N}_0$ which fulfill for all $i < j \in \{1, \ldots, n\}$ that $v_i v_j \in E$ if and only if $x_i + w_i + \ldots + w_{j-1} + x_j \leq s$ where $(v_1, v_2, \ldots, v_n)$ is the unique ordering of the vertices V

In particular, the leaf rank of a given proper interval graph G is k if and only if for $s = k - 2$ there is a s-weighted linear Steiner root for G', the graph that emerges from G by removing all true twins. How complex are the two problems above?

4.8 The Recognition Problem of B_1-EPG and B_1-VPG Graphs

Martin Charles Golumbic (Haifa University, IL)

The class *EPG* consists of all graphs $G = (V, E)$ where every vertex $v \in V$ corresponds to a path $P(v)$ in a grid $\mathcal{G}$ and for all $u, v \in V$ the edge uv belongs to E if and only if the paths $P(u)$ and $P(v)$ intersect in at least one grid edge. Analogously, the graphs $G = (V, E)$ in *VPG* correspond to collections of paths in a grid where $uv \in E$ if and only if $P(u)$ and $P(v)$ intersect in at least one grid point. If all paths may have at most one bend, i. e., direction change, then the described graph classes are referred to as B_1-*EPG* and B_1-*VPG*, respectively.

No characterization is known for B_1-EPG graphs or for B_1-VPG graphs, and the recognition problem for both classes is open. Is it NP-hard to recognize B_1-EPG graphs, respectively B_1-VPG graphs, or are there polynomial time algorithm for these problems?

The same question is also interesting for some subclasses. We call a set of single bend paths *boundary generated* if the grid has a rectangular boundary and the endpoints of all paths are on the boundary. If for instance paths are restricted to start from the left y-axis and bend down to the x-axis, then B_1-VPG becomes exactly the well known class of permutation graphs. Under the same restriction, the B_1-EPG graphs can be shown to be the graphs whose edge set can be covered by two clique partitions of G. In this case it is known that these graphs are recognizable in polynomial time. In general, what happens to the complexity of the recognition problems for boundary generated B_1-EPG graphs, and respectively B_1-VPG graphs, for each subset of the four possible direction changes?

4.9 Bott-Equivalent and Diffeomorphilic Directed Acyclic Graphs

Sang-il Oum (KAIST – Daejeon, KR)

Real Bott manifolds can be described in terms of directed acyclic graphs. Given two directed acyclic graphs H and G, the graph H is *Bott-equivalent* to G if H can be obtained from G by the operations of (1) local complementation and (2) slide. Is it possible to decide in polynomial time whether two given directed acyclic graphs H and G are Bott-equivalent?

Two real Bott manifolds are *diffeomorphilic* if and only if the corresponding graphs H and G fulfill that H is isomorphic to a graph that is Bott-equivalent to G. How complex is the decision of this problem?

Thursday, May 5th, 2011

4.10 Fixed Parameter Tractability of Protein Folding

Daniel Lokshtanov (University of California – San Diego, US)

How complex is it to decide if a given graph G is a subgraph of a grid? Is there an FPT algorithm with parameter k, if G is a path with k additional edges?

This problem is related to the *protein folding problem.* Hence, is protein folding in FPT when parameterized by the number of points?

4.11 Forbidden Subgraph Characterization of Circular-Arc Bipartite Graphs

Yngve Villanger (University of Bergen, NO)

Circular arc graphs can be recognized in $O(n+m)$ time due to an algorithm by R. McConnell. What is the forbidden induced subgraph characterization of circular arc graphs?

A bipartite graph H, with a fixed bipartition (X, Y), is an interval bigraph if the vertices of H can be represented by a family of intervals $I_v, v \in X \cup Y$, so that, for $x \in X$ and $y \in Y$, x and y are adjacent in H if and only if I_x and I_y intersect. A circular-arc bigraphs is, the generalization of interval bigraphs, where the intervals of the models are replaced by arcs of a circle. What is the forbidden induced subgraph characterization of circular-arc bigraphs graphs?

4.12 Linear Time Recognition of Circle Graphs

Christophe Paul (CNRS, UniversitéMontpellier II, FR)

Recognizing circle graphs works in $O(n^2)$ time via split decomposition. Split decomposition itself can be realized in linear time $O(n+m)$ but in this way it cannot be used to recognize circle graphs. Recent split decomposition algorithms that work in an incremental fashion, run in $O((n+m)\alpha(n,m))$ time where α is the inverse of the Ackermann function, and they can be used to recognize circle graphs in $O((n+m)\alpha(n,m))$ time. Is it possible to get rid of the $\alpha(m,n)$ factor in the running time, i. e., can circle graphs be recognized in linear time?

4.13 Characterization and Recognition of a Circle Graph Generalization

Van Bang Le (Universität Rostock, DE)

Consider the generalization of circle graphs that have intersection models consisting of (not necessarily straight) lines where at least one point (but not necessarily two) is situated on a circle. Can these graphs be recognized in polynomial time? Particularly, are there graphs that do not belong to this class? (E. Köhler) Is it possible to find the independence number for such graphs if the intersection model is given?

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