Report from Dagstuhl Seminar 11501

Visualization and Processing of Tensors and Higher Order Descriptors for Multi-Valued Data

Edited by

Carl-Fredrik Westin[1] and Bernhard Burgeth[2]

1 **Harvard Medical School – Boston, US,** westin@bwh.harvard.edu

2 **Universität des Saarlandes, DE,** burgeth@math.uni-sb.de

Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 11501 "Visualization and Processing of Tensors and Higher Order Descriptors for Multi-Valued Data", taking place December 11–16, 2011. The seminar gathered 26 senior and younger researchers from various countries in the unique atmosphere offered by Schloss Dagstuhl. The focus of the seminar was to discuss modern and emerging methods for analysis and visualization of tensor and higher order descriptors from medical imaging and engineering applications. Abstracts of the talks are collected in this report.

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Edited in cooperation with Lauren O'Donnell

1 Executive Summary

Carl-Fredrik Westin
Bernhard Burgeth

Higher Order Descriptors in Medical Imaging and Engineering

This seminar is the 4th in a series of Dagstuhl Seminars devoted to the visualization and processing of tensor fields and higher order descriptors. Tensor fields play an important role in many different scientific disciplines and application domains such as medical imaging, image processing, fluid dynamics, and structural mechanics. Analysis and visualization of multi-valued data have gained significant importance in scientific visualization and image processing due to rapid advances in medical imaging and and in the engineering industry.

In medical imaging, multi-valued data include diffusion weighted magnetic resonance imaging (dMRI), a medical imaging modality that allows the measurement of water diffusion in tissue *in vivo*. These measurements allow the description of diffusion in living fibrous tissue (e.g., white matter or muscle). The diffusion can be described by a diffusion tensor (i.e., a positive semidefinite 3×3 matrix). It is customary to acquire more complex data than can be described by the tensor model and recently the analysis has been extended to higher-order descriptors (i.e., higher-order tensors or spherical harmonics). There are several open questions how to best analyze and visualize such data.

In addition to tensor data from the medical field, a number of scientific and engineering applications produce tensor fields as a results of simulations of physical phenomena. The tensor concept is essential for the physical description of anisotropic behavior, especially in solid mechanics and civil engineering (e.g. stress-strain relationships, inertia tensors, permittivity tensor). The field of engineering faces many open problems in tensor field visualization and processing and novel technology is needed to address these problems.

Seminar Topics and Breakout Sessions

The emphasis of the seminar was on presenting the recent developments in the multidisciplinary field as well as identifying new challenges. We discussed a broad set of topics and challenges that cover both theoretical and practical issues related to analyzing and visualizing fields of tensors and higher order descriptors. During the workshop we discussed

- Higher-order models in dMRI beyond the diffusion tensor
- Higher-order tensors in image processing
- Computational analysis and visualization of airflow dynamics
- Novel differential geometric approaches to brain connectivity from dMRI
- Connectivity concepts in mathematical morphology for tensor fields
- Tensor concepts in structural mechanics and material science
- Visualization of uncertainty
- dMRI in brain studies for clinical applications

This year we scheduled time for breakout sessions that would foster focused discussions in smaller groups. During the first day of the meeting the group defined a list of important topics and open questions. Three of those were chosen and defined the breakout sessions:

- How do we define a suitable Finsler metric from diffusion data?
- How do we define biologically meaningful metrics from diffusion tensor and higher order model diffusion data?
- What are important questions in engineering that can be answered with visualization?

These breakout sessions turned out to be very successful and the groups scheduled extra time in the evenings for continued discussions. The format of the new breakout sessions fits very well in the Dagstuhl environment promoting discussions and interactions. If we get the chance to organize another meeting at Dagstuhl, the breakout sessions will definitely be a part of the schedule.

Outcomes

The participants all agreed that the meeting was successful and stimulating. Seminar participants are already collaborating on a Springer book summarizing the results of the meeting. The Springer book will have about twenty chapters authored by the meeting participants, and we expect the book to be published in early 2013. The participants expressed interest in documenting the discussions in the breakout session in book chapters, in addition to the science described in their regular presentation.

The environment at Schloss Dagstuhl has generated several new scientific collaborations. The work in the engineering breakout session has resulted in a new project of four participants (Stommel, Burgeth, Scheuermann, Hotz) and a submission of a grant proposal to the Landesforschungsförderprogramm (LFFP) des Saarlandes. Meanwhile, the application for the grant has been approved. Three seminar participants who met at the meeting (O'Donnell, Hui Zhang, Schultz), from the USA, Great Britain, and Germany, are collaboratively organizing a workshop on computational diffusion MRI at the conference for Medical Image Computing and Computer-Assisted Intervention 2012.

It was voted that the group will apply for another meeting in this series, and that in addition to the current organizers (Carl-Fredrik Westin, Bernhard Burgeth, Anna Vilanova Bartroli), add Dr. Ingrid Hotz (ZIB – Berlin) as an organizer of the next event.

Acknowledgement

The organizers thank all the attendees for their contributions and extend special thanks to the team of Schloss Dagstuhl for helping to make this workshop a success. As always, we enjoyed the warm atmosphere of the Schloss, which supports both formal presentations as well as informal exchanges of ideas.

2 Table of Contents

3 Overview of Talks

3.1 Operator-algebraic processing of matrix fields: potentials, shortcomings, and perspectives

Bernhard Burgeth (Universität des Saarlandes, DE)

Image processing provides a variety of methods for the filtering and analysis of grayscale or color images. In this talk we report on an operator-algebraic framework that allows us to transfer concepts from PDE- or ordering-based image processing to the setting of matrix fields. We explain the fundamental concepts underlying this framework, such as a symmetric product of symmetric matrices or Loewner ordering, and discuss its virtues, difficulties, shortcomings, and perspectives.

3.2 Cycles of White Matter

Cagatay Demiralp (Brown University – Providence, US)

URL http://www.cs.brown.edu/ cad/

Motivated by sheer curiosity in the question that if there exists a natural categorization of the brain based on white-matter topology, I will talk about our effort to characterize the topology of brain white matter using simplicial homology.

3.3 Enhancement of Crossing Fiber-structures in DW-MRI via the Cartan Connection

Remco Duits (TU Eindhoven, NL)

Main reference **(1)** R. Duits, E. Creusen, A. Ghosh, T. Dela Haije, "Morphological and Linear Scale Spaces on $\mathbb{R}^3 \rtimes S^2$ for Enhancement of Crossing Fibers in DW-MRI," IJCV 2010 vol. 92, pp. 231–264, March 2011.
URL http://www.bmia.bmt.tue.nl/People/RDuits/JMIVDuits2011final.pdf
Main reference **(2)** R. Duits, E. Creusen, A. Ghosh, T. Dela Haije, "Diffusion, Convection and Erosion on $\mathbb{R}^3 \rtimes S^2$ and their Application to the Enhancement of Crossing Fibers," arXiv:1103.0656v4 [math.AP]
URL http://arxiv.org/abs/1103.0656v4

Diffusion-Weighted MRI (DW-MRI) measures local water diffusion in biological tissue, which reflects the underlying fiber structure. In order to enhance the fiber structure in the DW-MRI data we consider both (convection-)diffusions and Hamilton-Jacobi equations (erosions) on the coupled space $\mathbb{R}^3 \rtimes S^2$ of 3D-positions and orientations, embedded as a quotient in the group SE(3) of 3D-rigid body movements. These left-invariant evolutions are expressed in the frame of left-invariant vector fields on SE(3), which serves as a moving frame of reference attached to fiber fragments. The linear (convection-) diffusions are solved by a convolution with the corresponding Green's function, whereas the Hamilton-Jacobi equations are solved by a morphological convolution with the corresponding Green's function.

The underlying differential geometry is induced by a Cartan connection on a principal fiber bundel within SE(3): The evolutions locally take place along auto-parallel curves (exponential curves), which (due to torsion of the Cartan connection) do not coincide with the (sub-Riemannian) geodesics that we derived recently. All methods are tested on DTI-images of the brain. These experiments indicate that our techniques are useful to deal with both the problem of limited angular resolution of DTI and the problem of spurious, non-aligned crossings in HARDI. Finally, we propose new fiber tracking algorithms based on the evolved DW-MRI. The whole framework is a special case in our larger group theoretical framework with various imaging applications.

For more information, see [1, 2, 3, 4] and http://arxiv.org/abs/1103.0656v4. For other instances of our group-theoretical framework see [5, 6, 7] for evolutions on invertible orientation scores and see [8] and http://arxiv.org/abs/1110.6087 for evolutions on Gabor transforms.

References

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3.4 Abstract: “Probing the Human Brain Connectome”

Luc M. J. Florack (TU Eindhoven, NL)

Joint work of Florack, L. M. J.; Fuster, A.

Human brain connectomics is the field of science that aims to integrate data and knowledge about structure and function of the human brain at all levels of scale. The comprehensive description that should result from this study, a.k.a. the connectome by analogy with the genome, is one of the grand challenges of the 21st century. As part of this endeavor we will develop new methods for tractography, the geometric delineation of neural fiber pathways, and for structural brain connectivity, at macroscopic scales (i.e. order of magnitude 1mm and above) that can be probed with state-of-the-art magnetic resonance imaging (MRI). Our methodology relies on the physics of anisotropic diffusion of water in brain white matter and its empirical manifestation under diffusion weighted magnetic resonance imaging.

We stipulate that local anisotropic diffusivities can be “geometrized away” similar to the geometrization of gravitational forces in general relativity. However, it turns out that a Riemannian framework, which has proven powerful in the case of mild anisotropies, is inappropriate for the description of the highly anisotropic diffusivity profiles observed in the brain. We propose to exploit a generalization of Riemannian geometry, viz. Riemann-Finsler geometry, to remove this limitation.

3.5 From diffusion tensor to Riemannian metric tensor

Andrea Fuster (TU Eindhoven, NL)

Diffusion Tensor Imaging (DTI) can be investigated by using geometric methods. An active field of research in the past years has been the study of DTI in a Riemannian framework. The main idea is to associate a Riemannian metric to the diffusion tensor, by identifying the latter with the inverse metric. However, this might not always be the right choice of metric tensor, as already pointed out in the literature. We study this analogy from the point of view of the underlying diffusion equations, in different scenarios.

3.6 Estimation of 4th order tensors with positivity constraint in Diffusion MRI

Aurobrata Ghosh (INRIA Sophia Antipolis, FR)

Joint work of Ghosh, Aurobrata; Deriche Rachid
Main reference A. Ghosh, “High Order Models in Diffusion MRI and Applications,” PhD Thesis, April 2011.
URL http://tel.archives-ouvertes.fr/tel-00645820/fr/

Diffusion MRI, which is sensitive to the Brownian motion of molecules, has become today an excellent medical tool for probing the tissue micro-structure of cerebral white matter in vivo and non-invasively. It makes it possible to reconstruct fiber pathways and segment major fiber bundles that reflect the structures in the brain which are not visible to other non-invasive imaging modalities. Since this is possible without operating on the subject, but by integrating partial information from Diffusion Weighted Images (DWIs) into a reconstructed complete image of diffusion, Diffusion MRI opens a whole new domain of image processing. Here we shall explore the role that Cartesian tensors play in the mathematical model. We shall begin with 2nd order tensors, since these are at the core of Diffusion Tensor Imaging. We shall then explore higher and even ordered symmetric tensors, that can take into account more complex micro-geometries of biological tissues such as axonal crossings in the white matter. We will emphasize the estimation of 4th order diffusion tensors with positivity constraint from DWIs.

3.7 Metrics on Vector- and Tensorbundles

Hans Hagen (TU Kaiserslautern, DE)

Geometry and Topology, especially Vector and Tensorfields are “the basic technologie” for Geometric Modelling and Scientific Visualization. A topological manifold M is locally connected, locally compact and a union of a countable collection of compact subsets. Such a topological space is metrizable! Vector- and Tensor-bundles do have such a manifold structure. We consider a vectorfield (tensorfield) being part of tangent-bundle (tensor-bundle) of a Riemannian manifold with a metric tensor. Special features of such a metric tensor are “used” to visualize features of the vectorfield (tensorfield).

3.8 Analyzing tensor fields to study flow fields

Mario Hlawitschka (Universität Leipzig, DE)

Often, vector fields are “reduced” to scalar fields to study their properties. Opposing that path, I re-define methods based on the tensor field that is the derivative of the given vector field. In that space, we can rewrite known methods in a more-general framework. The methods are presented using the storyline of the analysis of compressible unstable flow around wind turbine fields where the focus lies in studying the effects blade-induced turbulences have on the energy generation as well as mechanical stability of the setup.

3.9 Finding Representative Subsets from 3d Second-Order Stress Tensor Fields

Ingrid Hotz (ZIB – Berlin, DE)

Tensor fields play an important role in many areas ranging from engineering to medicine. Compared to their significance, excluding diffusion tensor imaging, tensors are an underrepresented topic in visualization. This may be due to the fact that first there is no long tradition in tensor field analysis, second the terminology varies from application to application, and third the questions posed onto the data are very fuzzy. An additional challenge arises from the complexity of the tensor data itself. The goal of our work is to overcome some of these challenges creating schematic depictions of the data based on domain specific feature spaces.

The basis of these feature spaces is a decomposition of the tensor information in scalar and directional features. The scalar feature or shape space is a subset of the space spanned by the three eigenvalues, parametrized by invariants that are prevalent in a certain application. Often such invariants are already reflected by commonly used glyphs in the respective application. An example for invariants that are of importance for stress tensor fields in context with failure analysis is the maximum shear stress and the shape factor. Both entities can be nicely represented using Mohr's circles.

Our work can be summarized by the following steps: (1.) find appropriate feature spaces, (2.) structure these feature spaces into clusters, and (3.) find appropriate representatives for these subsets. Additional cluster analysis provides size, variance and directional distributions of the clusters. The resulting atlas like data representation can be used to intuitively interact with the data focusing onto trends or outliers respectively.

3.10 Brain metrics breakout session

Derek K. Jones (Cardiff University, GB)

Obtaining good quality diffusion MRI and making sound and robust inferences from the data is not trivial, however, and involves a long chain of events from ensuring that the hardware is performing optimally, the pulse sequence is carefully designed, the acquisition is optimal, the data quality is maximized while artifacts are minimized, the appropriate post-processing is used, and, where appropriate, the appropriate statistical testing is used, and the data are interpreted correctly. In our breakout session, we discussed methods to compare brain metrics, what meaningful biological metrics do we get from HARDI data, and how can we get a formal probability of connection.

3.11 HOPE: Higher Order Phase Estimation

Hans Knutsson (Linköping University Hospital, SE)

Joint work of Knutsson, Hans; Westin, Carl-Fredrik

Local phase is a powerful concept which has been successfully used in many image processing applications. For multidimensional signals the concept of phase is complex and there is no consensus on the precise meaning of phase. It is, however, accepted by all that a measure of phase implicitly carries a directional reference.

We present a novel representation of multidimensional phase that is shown to be equivalent to an extended Klein bottle. In contrast to previously suggested phase representations it is shown to be globally isometric for the simple signal class.

For 1-dimensional signals the new phase representation reduces to the original definition of amplitude and phase for analytic signals. Phase estimation using classical quadrature filter pairs is based on the analytic signal concept and requires a pre-defined filter direction. The new local phase representation removes this requirement by implicitly incorporating local orientation. The estimation approach uses spherically separable monomial monomial filter of orders 0, 1 and 2 which naturally extends to N dimensions.

3.12 What Summarizes a Diffusion MRI Dataset?

David H. Laidlaw (Brown University – Providence, US)

Diffusion MRI datasets are challenging to comprehend; as a result, their potential has not yet been realized. Comprehension sometimes begins with visualization, but it usually needs to end with a summary – often a single number. What makes a good summary number? How do we know? I'm afraid I don't know the answers, so I hope that some of the participants will help to figure this out. I do have some examples that I hope will illustrate the problem, and I look forward to some stimulating discussion.

3.13 Nonnegative Definite Tensors and Neuroimaging

Lek-Heng Lim (University of Chicago, US)

Joint work of Lim, Lek-Heng; Schultz, Thomas

One of the most important classes of matrices is the symmetric positive definite ones. They arise as covariance, density matrix, inner products, Laplacians, Mercer kernels, etc. So what is the equivalent of positive/nonnegative definiteness for higher order symmetric tensors? It turns out that there are two natural but different notions: One is that the homogeneous polynomial associated with the tensor be nonnegative valued while the other is that this polynomial be expressible as a sum of powers of linear forms. These two notions are in fact dual in an appropriate sense.

We show that both notions arise in diffusion MRI and lead to two methods for extracting nerve fibers crossing. We shall see that deciding nonnegative definiteness (either notions) of a higher-order tensor is an NP-hard problem but that due to a happy coincidence the cases relevant to these MRI applications yield readily computable convex problems.

3.14 Interactive Exploration of Stress Tensors Used in Computational Turbulent Combustion

Georgeta Elisabeta Marai (University of Pittsburgh, US)

Joint work of Marai, Georgeta Elisabeta; Yilmaz, Levent; Nik, Mehdi; Luciani, Timothy; Maries, Adrian; Haque, Abedul

Simulation and modeling of turbulent flow, and of turbulent reacting flow in particular, involves solving for and analyzing time-dependent and spatially dense tensor quantities, such as turbulent stress tensors. The interactive visual exploration of these tensor quantities can effectively steer the computational modeling of combustion systems. In this chapter, we discuss the challenges in dense symmetric tensor visualization applied to turbulent combustion calculation, and analyze the feasibility of using several established tensor visualization techniques in the context of exploring space-time relationships in computationally-simulated combustion tensor data. To tackle the pervasive problems of occlusion and clutter, we combine a detailed 3D inspection view based on volume rendering with glyph-based representations, used as 2D probes, while leveraging interactive filtering and flow salience cues to clarify the structure of the tensor datasets. Side-by-side views of multiple timesteps facilitate the analysis of time-space relationships. The result is a visual analysis tool to be utilized in debugging, benchmarking, and verification of models and solutions in turbulent combustion. We demonstrate this analysis tool on three example configurations and report feedback from combustion researchers.

3.15 Tensor voting with vote clustering

Rodrigo Moreno (Linköping University Hospital, SE)

Tensor voting is a robust technique to propagate and aggregate local information encoded through 2nd order tensors. Traditionally, tensor summation is used to aggregate the votes, which can be inappropriate in some applications. In this talk, I discuss the use of clustering-based aggregation for analyzing the votes cast by tensor voting. Two possible applications of this methodology are context-based tensor decomposition and extracting higher-order tensor fields from 2nd-order tensor fields.

3.16 Tractography in the clinic: What works and what is missing?

Lauren O'Donnell (Harvard Medical School – Boston, US)

In this talk I review tractography methods used in clinical applications and discuss open issues. By measuring water diffusion in the brain, diffusion tensor MRI (DTI) gives information about the orientation and integrity of fiber tracts, the major neural connections in the white matter. DTI tractography follows directions of maximal water diffusion to estimate the trajectories of the fiber tracts. We summarize the current published state of the art of clinical tractography, focusing on correspondence with known anatomy, correspondence with electrical stimulation, and tractography's effect on measurable clinical endpoints. We focus on three tracts of interest for neurosurgery: the corticospinal tract, the arcuate fasciculus, and the optic radiation. We discuss the technology that is missing, including automated higher-level analyses of use to a doctor, such as answering the question "is this white matter normal?"

3.17 Estimation of Free-Water Corrected Diffusion Tensors

Ofer Pasternak (Harvard Medical School – Boston, US)

In Diffusion tensor Imaging, when partial volume of brain tissue and free-water occurs, the estimated tensor describes a mixture of both compartments. As a result, tensor indices such as FA and MD become less specific to tissue microstructure. We discuss tensor-regularization based approaches that provide estimations of free-water corrected tensors, and the implications of applying free- water correction in various case studies.

3.18 Human cortical connectome reconstruction from diffusion weighted MRI: The effect of tractography algorithm

Alard Roebroeck (Maastricht University, NL)

Reconstructing the macroscopic human cortical connectome by Diffusion Weighted Imaging (DWI) is a challenging research topic that has recently gained a lot of attention. In the present work, we investigate the effects of intra-voxel fiber direction modeling and tractography algorithm on derived structural network indices (e.g. density, small-worldness and global efficiency). The investigation is centered on three semi-independent distinctions within the large set of available diffusion models and tractography methods: i) single fiber direction versus multiple directions in the intra-voxel diffusion model, ii) deterministic versus probabilistic tractography and iii) local versus global measure-of-fit of the reconstructed fiber trajectories. We discuss interactions in the combined effects of these methods, considerations of tractography sensitivity and specificity, and implications for future studies. It is concluded that the choice of tractography algorithm along the three dimensions and thresholds (FA, angle, probabilistic) can affect structural network indices dramatically, which is crucial for the sensitivity of any human structural network study and for the validity of study comparisons.

3.19 Tensor lines: A good concept for solid mechanics applications?

Gerik Scheuermann (Universität Leipzig, DE)

Tensorlines are among the very few known continuous techniques to visualize tensor fields. In addition, the whole area of tensor topology relies on this concept. Unfortunately, it can be observed that some engineers question their expression power for their application in solid mechanics. It is understood that tensorlines in solid mechanics (as already noted by Dickinson 1989 in the first paper on tenosr lines) describe the principle stress directions. However, these directions may not give much information about material failure which is often a central concern of the engineer. The talk discusses the use of tensor lines in solid mechanics by looking at the history and comparing it to successfull applications in neuroscience, liquid crystals and medical applications.

3.20 Learning Higher Order Tensor Rank Estimates the Number of Fiber Compartments in Diffusion MRI

Thomas Schultz (MPI für Intelligente Systeme – Tübingen, DE)

The need to decide on the number of fiber compartments is a fundamental limitation of multi-compartment models in high angular resolution diffusion imaging. This talk clarifies that when using higher-order tensor based spherical deconvolution to estimate multiple fiber directions, the number of compartments amounts to the numeric rank of the fODF tensor. Unfortunately, few practical results on finding numeric tensor rank are available. As a pragmatic alternative, we demonstrate that support vector machines, a standard tool from machine learning, can be used successfully to estimate tensor rank (and therefore fiber number) in this application.

3.21 Application of Tensors to Model the Mechanical Properties of Short Fiber Reinforced Plastics

Markus Stommel (Universität des Saarlandes, DE)

This contribution will focus on the application of tensors and tensors fields from an engineering perspective using the example of composite materials. Composite materials consist of at least two materials differing in physical properties and shape. Therefore, on the microscale the material is inhomogeneous and consists of two or more phases like a matrix material that is reinforced by a second fibroid material. These materials are increasingly used in advanced technical applications which implicate the need for demanding simulation techniques.

The simulation of technical parts made of composites implies the use of continuum mechancis approaches. They usually require the homogenization of the composites micro-structure toward a "smeared" continuum on a macroscale. This lecture deals with the past and ongoing effort in homogenization techniques that are used in engineering to simulate composites by the finite element method (FEA). It will demonstrate the basic procedure in performing FEA on short fiber reinforced plastics including the process and structural simulation concepts and their interaction. Afterwards some tensor related topics will be addressed that are open to further development of the homogenization methods.

3.22 Towards population studies with HARDI

Ragini Verma (University of Pennsylvania, US)

Joint work of Verma, Ragini; Bloy, Luke; Ingalhalikar, Madhura; Smith, Alex

With the increase in HARDI studies, there has been a need for developing methods for analysis and processing of HARDI data. The talk will cover methods essential for HARDI analysis, motivated towards large population studies. The basic methods for doing population studies include voxel-based analysis of the brain (which needs the development of metrics), region-based analysis (which needs data-based clustering and atlas building), pattern classification and connectivity analysis. In addition to discussing these methods required to facilitate subsequent analysis, the talk will raise issues related to planning HARDI studies – do we need it, what do we need from it, how to compare it with DTI, and how to harness the unknown world of "connectivity".

3.23 Rotationally invariant sampling and tensor Coulomb forces

Carl-Fredrik Westin (Harvard Medical School – Boston, US)

Joint work of Westin, Carl-Fredrik; Knutsson, Hans

Minimizing the error propagation that a diffusion MRI (dMRI) gradient scheme introduces is an important task in the design of robust and un-biased experiments. Previous studies define the optimal single-shell scheme with respect to various parameters that include: 1) the angular distance between neighboring samples, minimized using an electrostatic optimization 2) the condition number, which estimates the effect of noise, and 3) rotational invariance, so the scheme produces rotationally unbiased estimates. Among the previously proposed schemes, the electrostatic optimization has been shown to produce the most balanced schemes.

It is common that acquisitions schemes are composed of a single b-value shell providing angular sampling of the diffusion profile. But the newer methods require, in addition, a radial sampling in order to observe phenomena such as restriction and hindrance. We propose two schemes for the construction of rotationally invariant multiple- shells. The first is a dual frame method that optimizes the rotation invariance of any set of samples. The second uses a subset of the icosahedral set that can intuitively be used for nested rotationally invariant schemes with pre-defined number of samples.

3.24 Recent Developments in Visualization of Diffusion Tensor Data

Alexander Wiebel (ZIB – Berlin, DE)

Many results of studies using diffusion tensor imaging can be summarized with some simple numbers or a validation of a hypothesis. For an examination of a single patient, however, medical practitioners and scientists often have to inspect the data itself or derived data like reconstructed fiber tracts and their bundles. This talk discusses two visualization techniques that can support such an inspection: one visualizes diffusion parameters of fiber bundles and one provides an illustrative rendering for probabilistic tractography data. Additionally, a short excursion to the use of diffusion MRI for monitoring inflammatory bowel diseases is given.

3.25 Tensor Field Analysis for Geometry Processing

Eugene Zhang (Oregon State University, US)

URL http://web.engr.oregonstate.edu/ zhange

Tensor field analysis has been a subject of interest in many scientific and engineering applications. In this talk I will review applications in geometry processing for which tensor fields play a prominent role. In addition, I will discuss how existing tensor field analysis can benefit these applications and what challenges remain in applying tensor field processing to geometry-related applications.

3.26 In vivo imaging of brain microstructure using diffusion MRI

Gary Hui Zhang (University College London, GB)

Diffusion MRI is an imaging technique that provides unique insight into tissue microstructure. It makes measurements that are sensitive to the displacement pattern of water molecules undergoing diffusion. Because the tissue microstructure determines this displacement pattern, its properties can thus be deduced from the diffusion MRI measurements using appropriate computational techniques. This talk will review basics of diffusion MRI and discuss some of the advanced diffusion modeling techniques developed at UCL with a focus on neuroimaging applications.

4 List of previous meetings in this workshop series

- The 2004 Dagstuhl Perspective Workshop "Visualization and Processing of Tensor Fields" (Seminar 04172, April 2004, Organizers: Hans Hagen and Joachim Weickert) was the first international forum where leading experts on visualization and processing of tensor fields had the opportunity to meet, many for the first time. This workshop identified several key issues and triggered fruitful collaborations that have also led to the first book in this area. Springer book published in 2006: ISBN 978-3-540-25032-6.
- The 2007 Dagstuhl meeting "Visualization and Processing of Tensor Fields" (Seminar 07022, January 2007, Organizers: David Laidlaw and Joachim Weickert) was equally successful and the progress reported in a second book published with Springer published in 2009: ISBN 978-3-540-88377-7.
- The 2009 Dagstuhl meeting "New Developments in the Visualization and Processing of Tensor Fields" (Seminar 09302, July 2009, Organizers: Bernhard Burgeth and David Laidlaw) saw a shift in focus, and in addition to diffusion imaging, paid attention to engineering applications of tensors in fluid mechanics, material science, and elastography. Springer has also published a third book in this series: ISBN 978-3-642-27342-1 (*to appear*).

5 Schedule

Monday		Presenter
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09:40-10:20	Probing the Human Brain Connectome	Luc M. J. Florack, TU Eindhoven
10:20-10:40	Coffee break	
10:40-11:20	In vivo imaging of brain microstructure using diffusion MRI	Gary Hui Zhang, University College London
11:20-12:00	Human cortical connectome reconstruction from diffusion weighted MRI: The effect of tractography algorithm	Alard Roebroeck, Maastricht University
12:15-13.30	Lunch Break	
13:30-14:10	Rotationally invariant sampling and tensor Coulomb forces	Carl-Fredrik Westin, Harvard Medical School – Boston
14:10-14.50	Tensor lines: A good concept for solid mechanics applications?	Gerik Scheuermann, Leipzig University
14:50-15.50	Coffee break	
15:50-16.30	Application of Tensors to Model the Mechanical Properties of Short Fiber Reinforced Plastics	Markus Stommel, Saarland University
16:30-17.30	Breakout sessions	
18:00	Dinner	

Tuesday		Presenter
07:30-08:40	Breakfast	
09:00-09:40	Tractography in the clinic: What works and what is missing?	Lauren O'Donnell, Harvard Medical School
09:40-10:20	Towards population studies with HARDI	Ragini Verma, University of Pennsylvania
10:20-10:40	Coffee break	
10:40-11:20	Estimation of Free-Water Corrected Diffusion Tensors	Ofer Pasternak, Harvard Medical School
11:20-12:00	Cycles of White Matter	Cagatay Demiralp, Brown University – Providence
12:15-13.30	Lunch Break	
13:30-14:10	HOPE: Higher order phase estimation	Hans Knutsson, Linköping University Hospital
14:10-14.50	Tensorbundles as a visualization tool?	Hans Hagen, TU Kaiserslautern
14:50-15.50	Coffee break	
15:50-16.30	Tensor Field Analysis and Processing for Geometry Processing	Eugene Zhang, Oregon State University
16:30-17.30	Breakout sessions	
18:00	Dinner	

Wednesday		Presenter
07:30-08:40	Breakfast	
09:00-09:40	What Summarizes a Diffusion MRI Dataset?	David H. Laidlaw, Brown University – Providence
09:40-10:20	What is a positive definite tensor and how it can be used in neuroimaging	Lek-Heng Lim, University of Chicago
10:20-10:40	Coffee break	
10:40-11:20	From diffusion tensor to Riemannian metric tensor	Andrea Fuster, TU Eindhoven
11:20-12:00	Breakout sessions	
12:15-13.30	Lunch Break	
13:30-14:00	Group photo	
14:00-15:00	Bus to Trier	
15:00-18:00	Visit Trier and its Christmas Market	
18:00-20:00	Dinner in Trier at nearby wine-cave	
20:00-21:00	Returning to Schloss Dagstuhl by buss	

Thursday		Presenter
07:30-08:40	Breakfast	
09:00-09:40	Estimation of 4th order tensors with positivity constraint in diffusion MRI	Aurobrata Ghosh, INRIA Sophia Antipolis
09:40-10:20	Enhancement of Crossing Fiber-structures in DW-MRI via the Cartan Connection	Remco Duits, TU Eindhoven
10:20-10:40	Coffee break	
10:40-11:20	Studying Tensors to Analyze Flow	Mario Hlawitschka, University of Leipzig
11:20-12:00	Learning Higher Order Tensor Rank Estimates the Number of Fiber Compartments in Diffusion MRI	Thomas Schultz , MPI Tübingen
12:15-13.30	Lunch Break	
13:30-14:10	Finding Representative Subsets from 3D Second-Order Stress Tensor Fields	Ingrid Hotz, ZIB – Berlin
14:10-14.50	Exploration of Stress Tensors Used in Computational Turbulent Combustion	G. Elisabeta Marai, University of Pittsburgh
14:50-15.50	Coffee break	
15:50-16.30	Operator-algebraic approach for matrix fields: potentials, shortcomings, and perspectives	Bernhard Burgeth, Saarland University
16:30-17.30	Results from breakout sessions	
18:00	Dinner	

Friday		Presenter
07:30-08:40	Breakfast	
09:00-09:40	Tensor Voting with Vote Clustering	Rodrigo Moreno, Linköping University Hospital
09:40-10:20	Recent Developments in Visualization of Diffusion Tensor Data	Alexander Wiebel, ZIB – Berlin
10:20-10:40	Coffee break	
10:40-12:00	Book, future seminar, and farewell	
12:15-13.30	Lunch and Departure	

Participants

- Bernhard Burgeth
Universität des Saarlandes, DE
- Cagatay Demiralp
Brown Univ. – Providence, US
- Remco Duits
TU Eindhoven, NL
- Luc M. J. Florack
TU Eindhoven, NL
- Andrea Fuster
TU Eindhoven, NL
- Aurobrata Ghosh
INRIA Sophia Antipolis, FR
- Hans Hagen
TU Kaiserslautern, DE
- Mario Hlawitschka
Universität Leipzig, DE
- Ingrid Hotz
ZIB – Berlin, DE
- Derek K. Jones
Cardiff University, GB
- Hans Knutsson
Linköping University, SE
- David H. Laidlaw
Brown Univ. – Providence, US
- Lek-Heng Lim
University of Chicago, US
- Georgeta Elisabeta Marai
University of Pittsburgh, US
- Rodrigo Moreno
Linköping Univ. Hospital, SE
- Lauren O'Donnell
Harvard Medical School – Boston, US
- Ofer Pasternak
Harvard Medical School – Boston, US
- Alard Roebroeck
Maastricht University, NL
- Gerik Scheuermann
Universität Leipzig, DE
- Thomas Schultz
MPI für Intelligente Systeme – Tübingen, DE
- Markus Stommel
Universität des Saarlandes, DE
- Ragini Verma
University of Pennsylvania, DE
- Carl-Fredrik Westin
Harvard Medical School – Boston, US
- Alexander Wiebel
ZIB – Berlin, DE
- Eugene Zhang
Oregon State University, US
- Gary Hui Zhang
University College London, GB