Report from Dagstuhl Seminar 12241

Data Reduction and Problem Kernels

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 12241 "Data Reduction and Problem Kernels". During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

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1 Executive Summary

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Preprocessing (data reduction or kernelization) is used universally in almost every practical computer implementation that aims to deal with an NP-hard problem. The history of preprocessing, such as applying reduction rules to simplify truth functions, can be traced back to the origins of Computer Science — the 1950's work of Quine, and much more. A modern example showing the striking power of efficient preprocessing is the commercial integer linear program solver CPLEX. The goal of a preprocessing subroutine is to solve efficiently the "easy parts" of a problem instance and reduce it (shrinking it) to its computationally difficult "core" structure (the *problem kernel* of the instance).

How can we measure the efficiency of such a kernelization subroutine? For a long time, the mathematical analysis of polynomial time preprocessing algorithms was neglected. The basic reason for this anomalous development of theoretical computer science, was that if we seek to start with an instance I of an NP-hard problem and try to find an efficient (P-time) subroutine to replace I with an equivalent instance I' with $|I'| < |I|$ then success would imply P=NP — discouraging efforts in this research direction, from a mathematically-powered point of view.

The situation in regards the systematic, mathematically sophisticated investigation of preprocessing subroutines has changed drastically with advent of parameterized complexity, where the issues are naturally framed. More specifically, we ask for upper bounds on the reduced instance sizes as a function of a parameter of the input, assuming a polynomial time reduction/preprocessing algorithm.

A typical example is the famous Nemhauser-Trotter kernel for the Vertex Cover problem, showing that a "kernel" of at most $2k$ vertices can be obtained, with k the requested maximum size of a solution. A large number of results have been obtained in the past years, and the research in this area shows a rapid growth, not only in terms of number of papers appearing in top Theoretical Computer Science and Algorithms conferences and journals, but also in terms of techniques. Importantly, very recent developments were the introduction of new lower bound techniques, showing (under complexity theoretic assumptions) that certain problems must have kernels of at least certain sizes, meta-results that show that large classes of problems all have small (e.g., linear) kernels — these include a large collection of problems on planar graphs and matroid based techniques to obtain randomized kernels.

Kernelization is a vibrant and rapidly developing area. This meeting on kernelization consolidated the results achieved in the recent years, discussed future research directions, and exploreed further the applications potential of kernelization algorithms, and gave excellent opportunities for the participants to engage in joint research and discussions on open problems and future directions. This workshop was also special as we celebrated the 60th birthday of one of the founder of parameterized complexity, Prof. Michael R. Fellows. We organised a special day in which we remembered his contributions to parameterized complexity, science in general and mathematics for children.

The main highlights of the workshop were talks on the solution to two main open problems in the area of kernelization. We give a brief overview of these new developments below.

The AND Conjecture

The OR-SAT problem asks if, given m formulas each of size n, at least one of them is satisfiable. In 2008, Fortnow and Santhanam showed that if there is a reduction from OR-SAT to any language L with the property that the reduction reduces to instances of size polynomial in n (independent of m) then the polynomial-time hierarchy collapses. Such a reduction is called an OR-distillation, and this work motivated the notion of an OR-composition, which produces a boolean OR of parameterized instances of a given problem, without any restriction on the size. It was then established that an OR-composition and a polynomial kernel cannot co-exist, because these ingredients can be combined to lead to an OR-distillation. Thus, an OR-composition counts as evidence against the existence of a polynomial kernel, and it has turned into a very successful framework for establishing kernel lower bounds.

The question of whether there is similar evidence against the existence of an AND-distillation (defined analogously) has since been open. Such a result would imply that problems that have AND-compositions are also unlikely to admit polynomial kernels, and would therefore be a significant addition to the kernel lower bound toolkit. The question has been a central open problem for the kernelization community and was settled by Drucker in his work on classical and quantum instance compression. The route to the result is quite involved, and forges new connections between classical and parameterized complexity.

Tools from Matroid and Odd Cycle Traversal

The Odd Cycle Traversal problem asks if, given a graph G, there is a subset S of size at most k whose removal makes the graph bipartite. Equivalently, the question is if there is a subset S of size at most k that intersects every odd cycle in G. The problem was first shown to be FPT by Reed, Smith, and Vetta in 2004, and this was also the first illustration of the technique of iterative compression. However, the question of whether the problem admits a polynomial kernel was among the main open questions in the study of kernelization.

A breakthrough was recently made in work by Kratsch and Wahlström, providing the first (randomized) polynomial kernelization for the problem. It is a novel approach based on matroid theory, where all relevant information about a problem instance is encoded into a matroid with a representation of size polynomial in k.

Organization of the seminar and activities

The seminar consisted of twenty two talks, a session on open questions, and informal discussions among the participants. The organizers selected the talks in order to have comprehensive lectures giving overview of main topics and communications of new research results. Each day consisted of talks and free time for informal gatherings among participants. On the fourth day of the seminar we celebrated the 60th birthday of Mike Fellows, one of the founder of parameterized complexity. On this day we had several talks on the origin, history and the current developments in the field of parameterized complexity.

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3 Overview of Talks

3.1 Graph decompositions for algorithms and graph structure

Bruno Courcelle (Université Bordeaux, FR)

Several graph decompositions are important for algorithmic purposes, and not only tree-decompositions, rank-decompositions and those for clique-width. Many of them lead to "multi-kernelization" as they reduce a problem to several related problems for "prime" or "indecomposable" subgraphs.

I will review the algorithmic properties and uses of several known *canonical* decompositions: Tutte decomposition in 3-connected components, modular decomposition and split decomposition.

I will introduce a new one for strongly connected graphs, linked to Tutte decomposition that I call the **atomic decomposition**. The initial motivation is the study of Gauss words (curves in the plane) but there are other applications in view. It is related but different to a noncanonical decomposition of the same graphs by Knuth (1974)

3.2 (Non)constructive advances

Hans L. Bodlaender (Utrecht University, NL)

The talk surveys early results of Fellows and Langston and memorates Mike Fellows contributions to the field.

3.3 Tight Compression Bounds for Problems in Graphs with Small Degeneracy

Marek Cygan (University of Warsaw, PL)

We study kernelization in d-degenerate graphs. It is known that a few problems admit $k^{O(d)}$ kernels in d-degenerate graphs, including Induced Matching, Independent Dominating Set, Capacitated Vertex Cover, Connected Vertex Cover. Moreover a $k^{O(d^2)}$ kernel is known for Dominating Set. Simple reductions show that for Capacitated Vertex Cover and Connected Vertex Cover $k^{\Omega(d)}$ lower bounds exist. We show $k^{\Omega(d)}$ lower bounds for Induced Matching and Independent Dominating Set.

Furthermore, most interestingly, we also prove $k^{\Omega(d^2)}$lower bound for Dominating Set, which matches the known upper bound by Philip et al. [TALG] for this problem as well.

3.4 New Evidence for the AND- and OR-Conjectures

Andrew Drucker (MIT – Cambridge, US)

In the OR(SAT) problem, one is given a collection of Boolean formulas, each of length at most k, and wants to know whether at least one is satisfiable. Similarly, in the AND(SAT) problem, one wants to know whether all the formulas are individually satisfiable.

These problems are not known to have polynomial kernels. Work beginning with [Harnik and Naor '06; Bodlaender, Downey, Fellows, and Hermelin '08] has established that, if OR(SAT) is not polynomially kernelizable, then many other natural problems fail to have polynomial kernels. Bodlaender et al. also showed that the "kernelization-hardness" of AND(SAT) would imply a number of other hardness results. Thus, these two hypotheses, the "OR-" and "AND- conjectures," have a great deal of explanatory power. But should we believe them? In support of the OR-conjecture, [Fortnow and Santhanam '08] showed that OR(SAT) does not have polynomial kernels unless NP is in coNP/poly.

In this work we provide equally strong evidence for the AND-conjecture: if AND(SAT) has poly kernels then NP is in coNP/poly, and even in SZK/poly. We also extend the hardness evidence for OR(SAT) in several ways; for instance, we give the first strong evidence against probabilistic kernelizations for OR(SAT) with two-sided bounded error. To prove our results, we exploit the information bottleneck of a kernelization reduction, using a new, general method to "disguise" information being fed into a compressive mapping.

3.5 Train marshaling is fixed parameter tractable

Rudolf Fleischer (German University of Technology – Oman, OM)

The train marshalling problem is about reordering the cars of a train using as few auxiliary rails as possible. The problem is known to be NP-complete. We show that it is fixed parameter tractable (FPT) with the number of auxiliary rails as parameter.

3.6 Parameterized Complexity of the Workflow Satisfiability Problem

Gregory Z. Gutin (RHUL – London, GB)

Joint work of Jason Crampton, Gregory Z. Gutin and Anders Yeo.

The Workflow Satisfiability Problem (WSP) defined below arises in Access Control in Information Security.

In WSP, we are given a set S of steps and a set U of users and asked to decide whether there is a function $\pi : S \to U$ that satisfies some constraints. Firstly, each step can be assigned (mapped to) some subset of U. Secondly, there are some relations ρ on U (ie., $\rho \subseteq U \times U$) such that all constraints of the type $(\rho, S', S")$, where S', S'' are subsets of S, must be satisfied meaning that there exist $s' \in S'$ and $s'' \in S''$ such that $(\pi(s'), \pi(s'')) \in \rho$. Examples of ρ include $=$ and $\neq$.

Wang and Li (ACM Trans. Inf. Syst. Secur., 2010) proved that WSP is NP-hard. They also observed that $k = |S|$ is relatively small (with respect to $n = |U|$) and proved that k-WSP is W[1]-hard. They obtained a fixed-parameter algorithm for special cases of k-WSP when only relations $=$ and $\neq$ are allowed.

Using a result of Bjorklund, Husfeldt and Koivisto (SIAM J. Comput., 2009) we obtain a new fixed-parameter algorithm that significantly improves the runtime of Wang and Li and widen the special case for which k-WSP is fpt (including there organizations with hierarchical structures). In particular, we improve a result of Fellows, Friedrich, Hermelin, Narodytska, and Rosamond (IJCAI 2011). We also investigate the existence of polynomial-size kernels and obtain both positive and negative results using, in particular, a result of Dom, Lokshtanov and Saurabh (ICALP 2009).

3.7 Faster than Courcelle's Theorem on Shrubs

Petr Hlineny (Masaryk University, CZ)

URL http://arxiv.org/abs/1204.5194

Famous Courcelle's theorem claims FPT solvability of any MSO2-definable property in linear FPT time on the graphs of bounded tree-width (alternatively, of MSO1 on clique-width by Courcelle-Makowsky-Rotics). A drawback of this powerful algorithmic metatheorem is that its runtime has a nonelementary dependence on the quantifier alternation depth of the defining formula. This is indeed unavoidable in full generality (even on trees) as shown by Frick and Grohe.

We show a new kernelization approach to this problem, giving an MSO model checking algorithm on trees of bounded height in FPT with elementary dependence on the formula; actually, we "trade" a nonelementary runtime dependence on the formula for a nonelementary dependence of our kernel on the tree height. This implies a faster (than Courcelle's) new algorithm for all MSO2-definable properties on the graphs of bounded tree-depth, and similarly a faster algorithm for all MSO1-definable properties on the classes of bounded shrub-depth.

3.8 Preprocessing Subgraph and Minor Problems: When Does a Small Vertex Cover Help?

Bart Jansen (Utrecht University, NL)

We prove a number of results around kernelization of problems parameterized by the vertex cover of a graph. We provide two simple general conditions characterizing problems admitting kernels of polynomial size. Our characterizations not only give generic explanations for the existence of many known polynomial kernels for problems like Odd Cycle Transversal, Chordal Deletion, Planarization, η-Transversal, Long Path, Long Cycle, or H-packing, they also imply new polynomial kernels for problems like $\mathcal{F}$-Minor-Free Deletion, which is to delete at most k vertices to obtain a graph with no minor from a fixed finite set $\mathcal{F}$.

While our characterization captures many interesting problems, the kernelization complexity landscape of problems parameterized by vertex cover is much more involved. We demonstrate this by several results about induced subgraph and minor containment, which we find surprising. While it was known that testing for an induced complete subgraph has no polynomial kernel unless NP is in coNP/poly, we show that the problem of testing if a graph contains a given complete graph on t vertices as a minor admits a polynomial kernel. On the other hand, it was known that testing for a path on t vertices as a minor admits a polynomial kernel, but we show that testing for containment of an induced path on t vertices is unlikely to admit a polynomial kernel.

3.9 Max-Cut Parameterized Above the Edwards-Erdos Bound

Mark Jones (RHUL – London, GB)

We study the problem Max Cut: Given a graph, find a bipartite subgraph with the most edges. The Edwards-Erdos bound states that for any connected graph with n vertices, m edges, there is a bipartite subgraph with at least $m/2 + (n-1)/4$ edges.

We study Max Cut parameterized above this bound: Given a connected graph with n vertices, m edges, decide whether there is a bipartite subgraph with at least $m/2+(n-1)/4+k$ edges. We show that the problem is fixed-parameter tractable with running time $2^{(3k)}n^{O(1)}$, and has a kernel of size $O(k^5)$.

3.10 Data Reduction for Finding Diameter-Two Subgraphs

Christian Komusiewicz (TU Berlin, DE)

Given an undirected graph $G = (V, E)$ and an integer $l > 1$, the NP-hard 2-club problem asks for a vertex set $S \subseteq V$ of size at least l such that $G[S]$ has diameter at most 2.

We study the 2-club problem with respect to many-to-one- and Turing-kernelizability for a variety of parameters such as bandwidth of G, vertex cover size of G, the dual parameter $|V| - l$, and the feedback edge set number of G.

3.11 Kernel lower bounds using co-nondeterminism: Finding induced hereditary subgraphs

Stefan Kratsch (Utrecht University, NL)

This work further explores the applications of co-nondeterminism for showing kernelization lower bounds. The only known example excludes polynomial kernelizations for the RAMSEY(k) problem of finding an independent set or a clique of at least k vertices in a given graph (Kratsch, SODA 2012). We study the more general problem of finding induced subgraphs on k vertices fulfilling some hereditary property Π, called Π-INDUCED SUBGRAPH(k). The problem is NP-hard for all non-trivial choices of Π by a classic result of Lewis and Yannakakis (JCSS 1980). The parameterized complexity of this problem was classified by Khot and Raman (TCS 2002) depending on the choice of Π. The interesting cases for kernelization are for Π containing all independent sets and all cliques, since the problem is trivial or W[1]-hard otherwise.

Our results are twofold. Regarding Π-INDUCED SUBGRAPH(k), we show that for a large choice of natural graph properties Π, including chordal, perfect, cluster, and cograph, there is no polynomial kernel with respect to k. This is established by two theorems: one using a co-nondeterministic variant of cross-composition and one by a polynomial parameter transformation from RAMSEY(k).

Additionally, we show how to use improvement versions of NP-hard problems as source problems for lower bounds, without requiring their NP-hardness. E.g., for Π-INDUCED SUBGRAPH(k) our compositions may assume existing solutions of size $k-1$. We believe this to be useful for further lower bound proofs, since improvement versions simplify the construction of a disjunction (OR) of instances required in compositions. This adds a second way of using co-nondeterminism for lower bounds

3.12 Planar $\mathcal{F}$-Deletion: Kernelization, Approximation and FPT Algorithms (I)

Daniel Lokshtanov (University of California – San Diego, US)

In the $\mathcal{F}$-Deletion problem you are given a graph G and integer k and asked whether there is a set S on at most k vertices such that G does not contain any minors from $\mathcal{F}$, where $\mathcal{F}$ is a finite list of graphs. We show that if $\mathcal{F}$ contains at least one planar graph, then the $\mathcal{F}$-Deletion problem admits polynomial kernels, constant factor approximation algorithms. If additionally all graphs in $\mathcal{F}$ are connected the $\mathcal{F}$-Deletion problem admits $c^k \cdot n$ time FPT algorithms. On the way we develop some new and interesting tools. Our results are stringed together by a common theme of polynomial time preprocessing

3.13 FPT suspects and tough customers: Open problems of Downey and Fellows

Dániel Marx (MTA – Budapest, HU)

We give an update on the status of open problems from the book "Parameterized Complexity" by Downey and Fellows.

3.14 Planar-$\mathcal{F}$ Deletion: Approximation, Kernelization and Optimal FPT Algorithms (II)

Neeldhara Misra (The Institute of Mathematical Sciences – Chennai, IN)

The notion of protrusions – constant treewidth subgraphs that can be separated from the instance by constant-sized separators – has been very useful in the context of kernelization algorithms on sparse graphs. When the optimization problem in question has certain properties, protrusions lend themselves to vastly general reduction rules, leading to a number of interesting meta theorems on sparse graphs. Unfortunately, however, the technique is not easily amenable to work the same way on general graphs.

In particular, for the Planar $\mathcal{F}$-deletion problem on general graphs, it turns out that even for apparently simply cases, non-trivial degree reduction rules crafted "by hand" have to come into play before protrusion-based reductions can be applied. It is not clear that this approach is amenable to generalization for more complex cases.

We therefore revisit the notion of a protrusion and introduce a more flexible variant, namely a near-protrusion. Informally, a near-protrusion is a subgraph which can become a protrusion in the future, after removing some vertices of some optimal solution. The usefulness of near-protrusions is that they allow us to find an irrelevant edge, i.e., an edge which removal does not change the problem.

We give a brief overview of the ideas involved in making protrusion-based reductions work in more general situations.

3.15 Planar-$\mathcal{F}$ deletion in parameterized single exponential time

Christophe Paul (CNRS, Université Montpellier II, FR)

Let $\mathcal{F}$ be a finite family of graphs containing at least one planar graph. In the parameterized PLANAR-$\mathcal{F}$ DELETION problem, we are given an n-vertex graph G and a non-negative integer k (the parameter), and the question is whether G has a set X of vertices of size at most k such that $G - X$ is H-minor-free for every H in $\mathcal{F}$. This problem encompasses a number of well-studied parameterized problems such as Vertex Cover, Feedback Vertex Set, or Treewidth-t Vertex Deletion for every value of $t \geq 0$. We present a algorithm for the parameterized PLANAR-$\mathcal{F}$ DELETION problem running in parameterized single-exponential time. Our approach significantly deviates from previous work as we do not use any reduction rule, but instead we apply a series of branching steps. This allows us to deal, in particular, with the case where the graphs in $\mathcal{F}$ are not necessarily connected, which was not known to admit a single-exponential algorithm

3.16 Graph separation: New incompressibility results

Marcin Pilipczuk (University of Warsaw, PL)

In the talk we plan to present the recent developments on the kernelization hardness of graph separation problems. We show that, unless NP is contained in coNP/poly, the following parameterized problems do not admit a polynomial kernel:

- Directed Edge/Vertex Multiway Cut, parameterized by the size of the cutset, even in the case of two terminals,
- Edge/Vertex Multicut, parameterized by the size of the cutset,
- and k-Way Cut, parameterized by the size of the cutset.

Our results complement very recent developments in designing parameterized algorithms for cut problems by Marx and Razgon [STOC'11], Bousquet et al. [STOC'11], Kawarabayashi and Thorup [FOCS'11] and Chitnis et al. [SODA'12].

The presented results are included in the ICALP'12 paper "Clique cover and graph separation: New incompressibility results" (joint work with Marek Cygan, Stefan Kratsch, Michal Pilipczuk and Magnus Wahlstrom).

3.17 Tight bounds for Edge Clique Cover

Michal Pilipczuk (University of Bergen, NO)

In the EDGE CLIQUE COVER problem, given a graph G and an integer k, we ask whether the edges of G can be covered with k complete subgraphs of G or, equivalently, whether G admits an intersection model on k-element universe. Gramm et al. [JEA 2008] have shown a set of simple rules that reduce the number of vertices of G to 2^k, and no algorithm is known with significantly better running time bound than a brute-force search on this reduced instance. In this work we show that the approach of Gramm et al. is essentially optimal: we present a polynomial time algorithm that reduces an arbitrary 3-CNF-SAT formula with n variables and m clauses to an equivalent EDGE CLIQUE COVER instance (G, k) with $k = O(\log n)$ and $|V(G)| = O(n + m)$. This implies that EDGE CLIQUE COVER does not admit an FPT algorithm that has better than doubly-exponential running time dependency on k, unless ETH fails. Moreover, we exclude subexponential kernels for the problem under ETH and under NP not contained in coNP/poly. This refines previous work together with Stefan Kratsch and Magnus Wahlstroem [ICALP 2012], in which we proved that polynomial kernelization would contradict the second complexity assumption.

3.18 Linear Kernels on Graphs Excluding a Topological Minor

Somnath Sikdar (RWTH Aachen, DE)

In this talk, we will sketch a proof of the following result: a parameterized graph problem that has finite integer index and satisfies a property that we call "treewidth-bounding" admits a linear kernel on the class of H-topological-minor free graphs, where H is an arbitrary but fixed graph. This builds on earlier work on the existence of linear kernels by Bodlaender et al. on graphs of bounded genus and by Fomin et al. on H-minor-free graphs. This result implies that several problems, including Chordal Vertex Deletion, Feedback Vertex Set and Edge Dominating Set, admit linear kernels on H-topological-minor-free graphs.

3.19 A Polynomial kernel for Proper Interval Vertex Deletion

Yngve Villanger (University of Bergen, NO)

It is known that the problem of deleting at most k vertices to obtain a proper interval graph (Proper Interval Vertex Deletion) is fixed parameter tractable. However, whether the problem admits a polynomial kernel or not was open. Here, we answers this question in affirmative by obtaining a polynomial kernel for Proper Interval Vertex Deletion. This resolves an open question of van Bevern, Komusiewicz, Moser, and Niedermeier

3.20 Uses of Matroids in Kernelization

Magnus Wahlström (MPI für Informatik – Saarbrücken, DE)

In some recent results (Kratsch and Wahlström, SODA 2012; Kratsch and Wahlström, preprint, 2012), tools from matroid theory have shown themselves to have powerful applications in polynomial kernelization; in particular, a tool known as representative sets (Marx, 2006; Lovász, 1980) has proved itself very useful.

In this talk, I will give an overview of the use of these tools, illustrating with applications to kernels for Almost 2-SAT and for graph cut problems.

3.21 Different parameterizations of the Test Cover problem

Anders Yeo (RHUL – London, GB)

In the Test Cover problem we are given a set $\{1, \ldots, n\}$ of items together with a collection, T, of distinct subsets of these items called tests. We assume that T is a test cover, i.e., for each pair of items there is a test in T containing exactly one of the items. The objective is to find a minimum size subcollection of T which is still a test cover.

This problem is NP-hard, so we consider the following parameterizations of the problem, where k is the parameter and m is the number of tests available.

1. Is there a solution with at most k tests?
2. Is there a solution with at most $n - k$ tests?
3. . Is there a solution with at most $m - k$ tests, where m is the size of T?
4. Is there a solution with at most $(\log n) + k$ tests?

The above is of interest as n and m are upper bounds for the size of an optimal solution and $\log n$ is a lower bounds. We state the FPT-complexities of the above parameterizations and focus on (non-)polynomial kernel results. In particular we will illustrate why parameterization 1 has no polynomial kernel (unless NP is a subset of coNP/poly).

4 Open Problems

4.1 Above Guarantee Independent Set on Planar Graphs

Venkatesh Raman (The Institute of Mathematical Sciences – Chennai, IN)

It is well known that every planar graph admits an independent set on at least $n/4$ vertices, as an easy consequence of the Four Color Theorem. The above guarantee version of the question involve asking for an independent set of size at least $\frac{n}{(4+k)}$. The parameterized complexity of this question, parameterized by k, is open. As an aside, we note that the question is non-trivial even when $k = 1$.

4.2 Biclique

Mike Fellows (Charles Darwin University – Darwin, AU)

The parameterized complexity of finding $K_{k,k}$ as a subgraph when parmeterized by k is a long-standing and notorious open problem.

The multicolored variant (where the vertex set is partitioned into $2k$ parts and we would like to find a subgraph that involves exactly one vertex from each part) is known to be W[1]-hard (see Appendix, [10]). It is also known that counting bicliques is #W[1]-hard parameterized by k. If the input graph has no induced paths of length s, then the problem is fixed-parameter linear parameterized by k and s [1].

4.3 Chromatic Number of P_5-free graphs

Fedor Fomin (University of Bergen, NO)

The chromatic number of a P_4-free graph can be computed in polynomial time and it is NP-hard to compute the chromatic number of a P_5-free graph. However, the question of whether the chromatic number of a P_5-free graph is at most k admits a XP algorithm [21]. It is open as to whether there the problem is FPT.

4.4 Clique for Line Segments

Dániel Marx (MTA – Budapest, HU)

The problem of finding a maximum sized clique on the intersection graph of line segments in the plane is known to be NP-hard [4]. However, the parameterized complexity remains open.

4.5 Cliquewidth

Daniel Lokshtanov (University of California – San Diego, US)

While problems parameterized by cliquewidth are studied extensively, computing cliquewidth remains elusive.

A well-known fact is that if the tree-width of a graph is t then its clique-width is bounded by $3 \cdot 2^{t-1}$ [7]. On the other hand, complete graphs have clique-width 2 and unbounded tree-width. However, for sparse graphs the treewidth and cliquewidth are linearly related.

Hlineny and Oum obtained an algorithm running in polynomial time and computing $(2^{k+1} - 1)$-expressions for a graph G of clique-width at most k [20].

An FPT algorithm for computing cliquewidth remains an open problem.

4.6 Contraction Decomposition Beyond H-minor Free Graphs

MohammadTaghi Hajiaghayi (University of Maryland – College Park, US)

A contraction decomposition is a partition of the edges of a graph into a desired number k of color classes such that contracting the edges in any one color class results in a graph of treewidth linear in k.

A series of results on contraction decompositions finally culminated in this paper [11], which showed that such decompositions exist for H-minor free graphs and can be computed in polynomial time. This leads to a general framework for approximation and fixed-parameter algorithms for problems closed under contractions in graphs excluding a fixed minor. For example, one of the implications is (another) fixed-parameter algorithm for k-cut in H-minor-free graphs, which was an open problem of Downey et al. even for planar graphs.

Can such decompositions be constructed for other classes of graphs?

4.7 Directed Feedback Vertex Set

Saket Saurabh (The Institute of Mathematical Sciences – Chennai, IN), Stefan Kratsch (Utrecht University, NL), and Magnus Wahlström (MPI für Informatik – Saarbrücken, DE)

Directed Feedback Vertex Set is known to be FPT parameterized by solution size [5], however, the question of whether it admits a polynomial kernel remains open. Little progress has been made on this question, even on special graph classes.

4.8 Disjoint Paths

Saket Saurabh (The Institute of Mathematical Sciences – Chennai, IN)

The Disjoint Paths problem asks for vertex disjoint paths connecting k terminal pairs. It is known to be NP-complete even on planar graphs [25], and is FPT parameterized by k. However, the FPT result relies on graph minor theory. An explicit FPT algorithm that avoids this route – even an explicit FPT approximation – remains open.

4.9 Even Set

Mike Fellows (Charles Darwin University – Darwin, AU)

The Even Set problem takes a red/blue bipartite graph as input and asks for a non-empty set of at most k red vertices R such that each blue vertex has an even number of neighbors in R. The problem is known to be NP-complete [29] and the exact version of the problem, where $|R| = k$, is W[1]-hard [13]. The problem can be reformulated in a number ways in a variety of contexts, and its parameterized complexity is an important unsolved problem.

4.10 Group Feedback Edge/Vertex Set

Stefan Kratsch (Utrecht University, NL) and Magnus Wahlström (MPI für Informatik – Saarbrücken, DE)

Group Feedback Vertex Set (GFVS) is a broad generalization of the Odd Cycle Transversal problem (OCT). In this problem, the input is a graph G with edges labeled by elements from a group Γ, and the task is to delete vertices (or edges) to remove any cycles whose labels do not sum up to zero. OCT corresponds to the case where Γ =GF(2); see the literature for more precise definitions. The problem is known to be FPT, even in quite general variants; see [19, 9]. When Γ is fixed, the problem has a polynomial kernel [23], but the case of a non-fixed group is open and interesting. Depending on the choice of group representation and parameter, the strength of the problem seems to vary, but it is known to subsume both Multiway Cut, for an arbitrary large-enough group [23], and Subset Feedback Vertex Set, for a group with $2^{O(n)}$ elements given via oracle access [9]. A polynomial kernel for this most-general setting would be surprising, but even this is not excluded; failing this, though, the question is for what settings and parameter combinations the problem allows a polynomial kernel.
Open: GFVS(k+$|\boldsymbol{\Gamma}|$) and GFVS(k). (In explicit or oracle representation of the group)

4.11 Knapsack Parameterized by Items

Stefan Kratsch (Utrecht University, NL) and Magnus Wahlström (MPI für Informatik – Saarbrücken, DE)

We know that subset sum is W[1]-hard if parameterized by the size of the subset of numbers sought. On the other hand, it is also admits a randomized polynomial kernel parameterized by n. Does an analogous result hold (in the kernelization context) for knapsack parameterized by the number of items?

4.12 Lower Bounds for Turing Kernels

Mike Fellos (Charles Darwin University – Darwin, AU)

Turing kernels were introduced as a potential coping strategy for problems that are not expected to have polynomial kernels under standard complexity-theoretic assumptions [16]. Informally speaking, these are "many polynomial kernels" – that are independent of each other and can therefore be processed in parallel. The number of kernels is allowed to be a function of n. Formulating a lower bound framework for Turing kernels remains an open problem.

4.13 Multiway Cut

Stefan Kratsch (Utrecht University, NL) and Magnus Wahlström (MPI für Informatik – Saarbrücken, DE)

Arguably a polynomial kernelization for Multiway Cut parameterized by the size k of the requested cutset is one of the biggest open questions in kernelization of cut problems (along with DFVS). It is known that there are randomized polynomial kernels with $O(k^{s+1})$ vertices when the number of terminals is bounded by some constant s, and with $O(k^3)$ vertices when terminals are deletable (the latter is equivalent to having terminal-degrees equal to one) [23]. The main open question is what happens when s may be unbounded; note though, that known reduction rules give $s \leq 2k$ so parameterization by $k + s$ is just as hard. Furthermore, the edge deletion variant holds independent interest.
Open: Multiway Cut(k).

4.14 Multicut

Stefan Kratsch (Utrecht University, NL) and Magnus Wahlström (MPI für Informatik – Saarbrücken, DE)

The standard parameterization by the size of the requested cutset was recently showed not to admit a polynomial kernelization under the standard assumption [8], following the recent breakthrough results that show its fixed-parameter tractability [3, 27]. However, the used cross-composition from 3-Multiway Cut creates a large number of terminal pairs [8]. Hence, it is interesting to know whether parameterization by $k + s$ (here s is the number of terminal pairs) is helpful for getting a polynomial kernelization. Similarly to Multiway Cut, there is a randomized polynomial kernelization when the number of terminal pairs is bounded by some constant s. Note that deleteable terminals do not help, since terminals can be easily copied without creating undesired requests (unlike for Multiway Cut).
Open: Multicut($k + s$).

4.15 Multiway Cut and Multicut in directed graphs

Stefan Kratsch (Utrecht University, NL) and Magnus Wahlström (MPI für Informatik – Saarbrücken, DE)

It is known that the standard parameterizations of Directed Multiway Cut and Directed Multicut do not admit polynomial kernelizations even when there are only two terminals respectively one terminal pair [8]. Note that Directed Multiway Cut is FPT [6] and Directed Multicut is W[1]-hard [27]. For Directed Multicut the restriction to directed acyclic graphs (DAGs) remains W[1]-hard when parameterized by the cutset only but it is

FPT when parameterized by the cutset k plus the number s of terminal pairs [22]; this leaves open whether it admits a polynomial kernelization parameterized by $k+s$ or parameterized by k and with s fixed (the restriction to DAGs prevents the lower bound construction used for general directed graphs).
Open: Multicut-in-DAGs($k+s$) and s-Multicut-in-DAGs(k).

4.16 Parameterized Approximation for Dominating Set

Mike Fellows (Charles Darwin University – Darwin, AU)

The following question is open: Is there an FPT algorithm that, given a graph G and parameter k, either determines that G has no k-Dominating Set, or or produces a dominating set of size at most $g(k)$ (where $g(k)$ is some fixed function of k?

It is known that there is no such FPT algorithm for $g(k)$ of the form $(k+c)$ (where c is a fixed constant), unless FPT = W[2]. Also, it is known that there is no such FPT algorithm for *any* $g(k)$ for the Independent Dominating Set problem unless FPT = W[2] [14]. The Threshold Set problem is also known to be FPT inapproximable for any function g unless FPT = W[1] [26].

4.17 Polynomial Kernels for $\mathcal{F}$-deletion

Daniel Lokshtanov (University of California – San Diego, US)

The $\mathcal{F}$ deletion problem asks for a subset of vertices of size at most k whose removal makes a graph H-minor free for allowed $H \in \mathcal{F}$. It is known that the problem admits a polynomial kernel (parameterized by k) if $\mathcal{F}$ contains at least one planar graph [17], but the kernelization complexity is open for the case when $\mathcal{F}$ contains only non-planar graphs.

4.18 Polynomial Kernel for Imbalance

Saket Saurabh (The Institute of Mathematical Sciences – Chennai, IN)

The problem of checking if a graph admits a layout with imbalance at most k is known to be FPT parameterized by k [24]. The question of whether the problem admits a polynomial kernel is open.

4.19 Quadratic Integer Programming

Daniel Lokshtanov (University of California – San Diego, US)

While integer linear programs are known to have FPT algorithms parameterized by the number of variables, the parameterized complexity is unsettled for quadratic integer programs parameterized by the number of variables.

A FPT algorithm would imply that Optimal Linear Arrangement is FPT when parameterized by Vertex Cover [15].

4.20 Treewidth

Hans L. Bodlaender (Utrecht University, NL)

Treewidth is a fundamental structural parameter that has played a central role in algorithmic graph theory in general and in obtaining FPT algorithms in particular. The complexity of computing treewidth is of great interest, and here are some of the challenging open problems.

1. What is the complexity of treewidth on planar graphs? A (3/2)-approximation is known since branchwidth can be computed in polynomial time [28], but even NP-hardness remains open. It is also known that the treewidth of a planar graph is linear in the tree-length of the graph [12].
2. Is there a constant-factor approximation for treewidth? The answer is in the negative assuming the Small Set Expansion conjecture [2].
3. Is $2^{(O(k^3))}$ optimal for tree width in terms of the function of k?

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