Report from Dagstuhl Seminar 12391

Algorithms and Complexity for Continuous Problems

Edited by

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Abstract

From 23.09.12 to 28.09.12, the Dagstuhl Seminar 12391 Algorithms and Complexity for Continuous Problems was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar can be found in this report. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

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1 Executive Summary

Alexander Keller
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This was already the 11th Dagstuhl Seminar on Algorithms and Complexity for Continuous Problems over a period of 21 years. It brought together researchers from different communities working on computational aspects of continuous problems, including computer scientists, numerical analysts, applied and pure mathematicians. Although the seminar title has remained the same many of the topics and participants change with each seminar and each seminar in this series is of a very interdisciplinary nature.
Continuous computational problems arise in diverse areas of science and engineering. Examples include path and multivariate integration, approximation, optimization, as well as operator equations. Typically, only partial and/or noisy information is available, and the aim is to solve the problem within a given error tolerance using the minimal amount of computational resources. For example, in high-dimensional integration one wants to compute

an ε-approximation to the integral with the minimal number of function evaluations. Here it is crucial to identify first the relevant variables of the function. Understanding the complexity of such problems and construction of efficient algorithms is both important and challenging. The current seminar attracted 51 participants from more than 10 different countries all over the world. About 30% of them were young researchers including PhD students. There were 40 presentations covering in particular the following topics:

- Biomedical learning problems
- Random media
- Computational finance
- Noisy data
- Tractability
- Quantum computation
- Computational stochastic processes
- High-dimensional problems

The work of the attendants was supported by a variety of funding agencies. This includes the Deutsche Forschungsgemeinschaft, the Austrian Science Fund, the National Science Foundation (USA), and the Australian Research Council. Many of the attendants from Germany were supported within the DFG priority program SPP 1324 on "Extraction of Quantifiable Information from Complex Systems", which is strongly connected to the topics of the seminar.

As always, the excellent working conditions and friendly atmosphere provided by the Dagstuhl team have led to a rich exchange of ideas as well as a number of new collaborations. Selected papers related to this seminar will be published in a special issue of the Journal of Complexity.

2 Table of Contents

3 Overview of Talks

3.1 Quadrature of discontinuous functionals of the Heston Price using Malliavin calculus

Martin Altmayer (Universität Mannheim, DE)

Joint work of Altmayer, Martin; Neuenkirch, Andreas

The Heston Model is a popular stochastic volatility model in mathematical finance. While there exist several numerical methods to compute functionals of the Heston price, the convergence order is typically low for discontinuous functionals. In this talk, we will study an approach based on the integration by parts formula from Malliavin calculus to overcome this problem: The original function is replaced by a function involving its antiderivative and by a Malliavin weight. Using the drift-implicit Euler scheme for the square root of the volatility, we will construct an estimator for which we can prove that it has L^2-convergence order $1/2$ even for discontinuous functionals. This leads to an efficient multilevel algorithm.

3.2 Global Optimization Using Adaptive Delaunay Meshes

James M. Calvin (NJIT – Newark, US)

We describe a global optimization algorithm for twice-continuously differentiable functions that uses only adaptively chosen function evaluations. New evaluations are selected based on the function values at the vertices of the simplexes of the Delaunay triangulation of the previous evaluation points. In the case where a quality bound is available for the Delaunay triangulation, an asymptotic error bound is given.

3.3 The continuous shearlet transform in arbitrary space dimensions: general setting, shearlet coorbit spaces, traces and embeddings

Stephan Dahlke (Philipps-Universität Marburg, DE)

Joint work of Dahlke, Stephan; Steidl, Gabriele; Teschke, Gerd

In the context of directional signal analysis several approaches have been suggested such as ridgelets, curvelets, contourlets, shearlets and many others. Among all these approaches, the shearlet transform is outstanding because it is related to group theory, i.e., this transform can be derived from a square-integrable representation the so-called shearlet group. Therefore, in the context of the shearlet transform all the powerful tools of group representation theory can be exploited. Moreover, there is a very natural link to another useful concept, namely the coorbit space theory introduced by Feichtinger and Groechenig in a series of papers. By means of the coorbit space theory, it is possible to derive in a very natural way scales of smoothness spaces associated with the group representation. In this setting, the smoothness of functions is measured by the decay of the associated shearlet transform. Moreover, by a tricky discretization of the representation, it is possible to obtain Banach frames for these smoothness spaces.

Once these new shearlet smoothness spaces are established some natural questions arise.

- How do these spaces really look like?
- What are the relations to classical smoothness spaces such as Besov spaces?
- What are the associated trace spaces?

We show that for natural subclasses of shearlet coorbit spaces which correspond to 'shearlets on the cone', there exist embeddings into homogeneous Besov spaces and that for the same subclasses, the traces onto the coordinate axis can be identified with shearlet coorbit spaces.

3.4 Complexity of Banach space valued and parametric integration

Thomas Daun (TU Kaiserslautern, DE)

Joint work of Daun, Thomas; Stefan Heinrich

We study the complexity of Banach space valued integration. The input data are assumed to be r-smooth. We consider both definite and indefinite integration and analyse the deterministic and the randomized setting. We develop algorithms, estimate their error, and prove lower bounds. In the randomized setting the optimal convergence rate turns out to be related to the geometry of the underlying Banach space.

Then we study the corresponding problems for parameter dependent scalar integration. For this purpose we use the Banach space results and develop a multilevel scheme which connects Banach space and parametric case.

3.5 Quantization by empirical measures

Steffen Dereich (Universität Münster, DE)

Main reference S. Dereich, M. Scheutzow, R. Schottstedt, "Constructive quantization: approximation by empirical measures," Ann. Inst. Henri Poincaré (B)
URL http://arxiv.org/abs/1108.5346

We study the approximation of a probability measure μ on $\mathbb{R}^d$ by its empirical measure $\hat{\mu}_N$ interpreted as a random quantization. As error criterion we consider an averaged p-th moment Wasserstein metric. In the case where $2p < d$, we establish refined upper and lower bounds for the error, a high-resolution formula. Moreover, we provide a universal estimate based on moments, a so-called Pierce type estimate. In particular, we show that quantization by empirical measures is of optimal order under weak assumptions.

3.6 On the convergence analysis of Rothe's method with spatial adaptivity

Nicolas Döhring (TU Kaiserslautern, DE)

Joint work of Cioica, Petru A.; Dahlke, Stephan; Döhring, Nicolas; Friedrich, Ulrich; Kinzel, Stefan; Lindner, Felix; Raasch, Thorsten; Ritter, Klaus; Schilling, René L.
URL http://www.dfg-spp1324.de/download/preprints/preprint124.pdf

We study the approximation of the solution u of the heat equation

$$
\begin{aligned}
u'(t) &= \Delta u(t) + f(u(t)) \quad \text{on } \mathcal{O},\ t \in (0,T],\\
u(0) &= u_0,\\
u|_{\partial\mathcal{O}} &= 0,
\end{aligned}
$$

on a bounded Lipschitz domain $\mathcal{O} \subset \mathbb{R}^d$.

First, we discretize uniformly in time using a linear implicit Euler scheme. Then, for the spatial approximation, we use an adaptive wavelet algorithm. This approach of discretizing first in time and then in space is called Rothe's method or horizontal method of lines. We illustrate convergence rates and degrees of freedom needed to obtain this rate and show how to tune the spatial approximation errors in each step.

We show that this theory can be generalized to an abstract setting, covering deterministic evolution equations

$$u'(t) = F(t, u(t)),\ t \in (0,T],\ u(0) = u_0,$$

on a separable Hilbert space V with more general S-stage schemes and semi-linear stochastic PDEs

$$du(t) = (Au(t) + f(u(t)))dt + B(u(t))dW(t),\ u(0) = u_0,$$

on a suitable function space U.

This research is supported by the DFG priority program 1324.

3.7 Sparse Stabilization and Optimal Control of the Cucker-Smole model

Massimo Fornasier (TU München, DE)

In this talk we present the stabilization and optimal control of the Cucker and Smale nonlinear dynamical system modelling consensus emergence of interacting agents. The concept of optimality of such sparse stabilization and control procedure will be discussed. This will allow us to open the question of whether the proposed strategy is really optimal in terms of its complexity.

3.8 First exit times of continuous Itô processes

Stefan Geiss (Universität Innsbruck, AT)

Joint work of Bouchard, B.; Geiss, S.; Gobet, E.

We consider in a general setting exit times from certain domains for continuous Itô-processes and their approximation by discretized processes. Applications within the simulation of BSDEs on certain domains were the starting point of the investigations.

3.9 Infinite-dimensional integration: Optimal randomized multilevel algorithms in the ANOVA setting and related new results

Michael Gnewuch (Universität Kiel, DE)

Joint work of Gnewuch, Michael; Baldeaux, Jan; Dick, Josef
Main reference J. Baldeaux, M. Gnewuch. arXiv:1209.0882v1 [math.NA]; J. Dick, M. Gnewuch. arXiv:1210.4223 [math.NA]; M. Gnewuch. arXiv:1209.1808 [math.NA]
URL http://arxiv.org/archive/math

We present new upper and lower error bounds for the infinite-dimensional numerical integration problem on weighted Hilbert spaces with norms induced by an underlying function decomposition of ANOVA or anchored type. The weights model the relative importance of different groups of variables. We have results for randomized and deterministic algorithms, and our error bounds are in both settings sharp. The upper error bounds are based on randomized or deterministic multilevel algorithms [10, 7] and/or on deterministic changing dimension algorithms [13, 15].

Let us describe our findings in more detail:

In the paper [6] we provide lower error bounds for general deterministic linear algorithms and matching upper error bounds with the help of suitable multilevel algorithms and changing dimension algorithms.

More precisely, the spaces of integrands are weighted reproducing kernel Hilbert spaces with norms induced by an anchored function space decomposition. The error criterion is the deterministic worst case error. We study two cost models [5, 13] for function evaluation which depend on the number of active variables of the chosen integration points, and two classes of weights, namely product and order-dependent (POD) weights [12] and the newly defined weights with finite active dimension. We show for both classes of weights that multilevel algorithms achieve the optimal convergence rate in one cost model while changing dimension algorithms achieve the optimal rate in the other model. In particular, we improve on results presented in [14, 8].

As an example, we discuss the infinite-dimensional anchored Sobolev space with smoothness parameter α and provide new optimal quasi-Monte Carlo multilevel algorithms and quasi-Monte Carlo changing dimension algorithms based on higher-order polynomial lattice rules [2, 3].

In [9] we consider the same spaces of integrands, but instead of deterministic algorithms and the deterministic worst-case error, we study randomized algorithms and the randomized (worst-case) error. Again, we investigate the two cost models mentioned above. We prove the first non-trivial lower error bounds for randomized algorithms in these cost models and demonstrate their quality in the case of product weights [16]. In particular, we show that the randomized changing dimension algorithms provided in [15] achieve optimal convergence rates.

In the paper [4] we discuss the randomized ANOVA setting, which is technically more demanding. Here we focus on the cost model proposed in [5]. Our analysis refines and extends the analysis provided in [11] substantially and leads to matching upper and lower (i.e., optimal) error bounds.

As an illustrative example, we discuss the infinite-dimensional unanchored Sobolev space as space of integrands and employ randomized quasi-Monte Carlo (QMC) multilevel algorithms based on scrambled polynomial lattice rules [1].

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3.10 Infinite-dimensional integration with respect to the countable product of standard normal distributions

Mario Hefter (TU Kaiserslautern, DE)

We consider a quadrature problem on the sequence space $\mathbb{R}^{\mathbb{N}}$, where the underlying measure $\mu = N(0,1)^{\mathbb{N}}$ is given by the countable product of standard normal distributions and the integrands $f : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ belong to a unit ball of a reproducing kernel Hilbert space $H_{\gamma,\sigma}$. The space is defined by

$$H_{\gamma,\sigma} = \bigotimes_{i \in \mathbb{N}} \left(W^{1,2}_{\sigma}, \|f\|_i^2 = f(0)^2 + \frac{1}{\gamma_i} \|f'\|^2_{L^2_\sigma} \right)$$

for a sequence $\gamma = (\gamma_i)_{i \in \mathbb{N}}$ of positive weights and a positive variance parameter $\sigma > 0$. Here $W^{1,2}_{\sigma}$ denotes the Sobolev space of once differentiable functions, where the function itself and the derivative have bounded L^2 norm with respect to the centered normal distribution $N(0, \sigma^2)$ with variance σ^2.

We consider deterministic algorithms in the worst-case-setting, where the cost of evaluating a function at a point is the index of the highest nonzero component. Upper and lower bounds for the complexity are derived. These bounds are sharp if the sequence γ tends to zero sufficiently fast or slow and σ tends to infinity.

Motivated by an option pricing problem for a path dependent payoff function in the Black-Scholes model, we consider an integrand $f : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ and determine γ and σ for which $f \in H_{\gamma,\sigma}$ holds. For weights γ with $\gamma_i \asymp \frac{1}{i^\beta}$ it turns out that $f \in H_{\gamma,\sigma}$ holds if and only if no good algorithm is available on $H_{\gamma,\sigma}$.

3.11 Complexity of Banach space valued and parametric initial value problems

Stefan Heinrich (TU Kaiserslautern, DE)

Joint work of Daun, Thomas; Heinrich, Stefan

We study the complexity of initial value problems for Banach space valued ordinary differential equations both in the deterministic and in the randomized setting. The right-hand side is assumed to be r-smooth. In the randomized setting the obtained complexity estimates are related to the type of the underlying Banach space. The results extend previous ones for the finite dimensional case.

Then we apply these results to initial value problems for parameter dependent ordinary differential equations. We develop a multilevel Monte Carlo algorithm, investigate its convergence, prove matching lower bounds, and thus, settle the complexity of this problem.

3.12 The Curse of Dimensionality for Numerical Integration of Smooth Functions

Aicke Hinrichs (Universität Jena, DE)

Joint work of Hinrichs, Aicke; Ullrich, Mario; Novak, Erich; Wozniakowski, Henryk

We prove the curse of dimensionality for multivariate integration for a number of classes of smooth functions. In particular, for the class of r times continuously differentiable d-variate functions whose values are at most one the curse holds iff the bound on all derivatives up to order r does not go to zero faster than $d^{-1/2}$. We also consider the case of infinitely many differentiable functions and prove the curse if the bounds on the successive derivatives are appropriately large. The proof technique is based on a volume estimate of a neighborhood of the convex hull of n points which decays exponentially quickly if n is small relative to$d$.

3.13 Approximation of infinitely many times differentiable functions in weighted Korobov spaces

Peter Kritzer (University of Linz, AT)

Joint work of Dick, Josef; Kritzer, Peter; Pillichshammer, Friedrich; Wozniakowski, Henryk
Main reference J. Dick, P. Kritzer, F. Pillichshammer, H. Wozniakowski. Approximation of infinitely many times differentiable functions in Korobov spaces. Preprint, 2012.

We discuss L_2 approximation of functions from a weighted Korobov space of periodic infinitely many times differentiable functions for which the Fourier coefficients decay exponentially fast. We would like to check conditions on the weights such that the approximation error converges exponentially fast. Furthermore, we discuss the concepts of weak, polynomial, and strong polynomial tractability, how they are related to each other, and which properties of the weights are necessary and/or sufficient for these concepts to hold.

Research supported by the Austrian Science Fund (FWF), Project P23389- N18.

3.14 Tractability of approximation in Gaussian reproducing kernel Hilbert spaces

Thomas Kühn (University of Leipzig, DE)

Let H_d be the RKHS generated by the Gaussian kernel $K(x,y) = \exp\left(-\sum_{j=1}^{d} \sigma_j^2 (x_j - y_j)^2\right)$, where (σ_j) is a bounded seq. $(\sigma_j > 0)$ and $(x_j), (y_j) \in [0,1]^d$. We show that the approximation problem $I_d\colon H_d \to C([0,1]^d)$ is weakly tractable, if $\lim \sigma_j = 0$ and quasi-polynomially tractable, if $\sum_{j=1}^{\infty} \sigma_j^2 = \infty$.

3.15 Orthogonal transforms for QMC

Gunther Leobacher (University of Linz, AT)

Joint work of Leobacher, Gunther; Irrgeher, Christian

It has been found in the late 1990's that certain transformations of the integrand can help to increase the efficiency of quasi-Monte Carlo methods. Prominent examples from quantitative finance are provided be the Brownian bridge construction [6] and the Principal Component Analysis construction [1] for sample paths of Brownian motion.

It was later observed in [7] that

1. those transformations do not increase efficiency for arbitrary problems, rather they can slow things down for some problems;
2. those transforms can be understood as orthogonal transforms of the standard normal input vector.

In [3] the authors had the idea of constructing orthogonal transforms tailored to a given (finance) problem. Their idea has been built upon, among others, by [4, 5, 2, 8].

Up to now, most work concentrates on making the problem "as one-dimensional as possible" by choosing some orthogonal transform that puts as much variance as possible onto the first input variable.

We generalize this idea and we want to know under which conditions an orthogonal transform can be found that makes QMC more efficient, and how such a transform can be constructed.

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3.16 The real number PCP theorem

Klaus Meer (BTU Cottbus, DE)

Joint work of Baartse, Martijn; Meer, Klaus

In this talk we deal with probabilistically checkable proofs in the realm of real and complex number computations as introduced by Blum, Shub, and Smale. For the corresponding complexity classes $NP_{\mathbb{R}}$ and $NP_{\mathbb{C}}$, respectively, we give a characterization via PCP classes along the classical PCP theorem for the Turing machine model.

3.17 Computing Quadrature Formulas for Marginal Distributions of SDEs I

Thomas Mueller-Gronbach (Universität Passau, DE)

Joint work of Mueller-Gronbach, Thomas; Ritter, Klaus; Yaroslavtseva, Larisa

We consider the problem of approximating the marginal distribution of the solution of a stochastic differential equation (SDE) by probability measures with finite support, i.e., by quadrature formulas with positive weights summing up to one. We study deterministic algorithms in a worst case analysis with respect to classes of SDEs, which are defined in terms of smoothness constraints for the coefficients of the equation. The worst case error of an algorithm is defined in terms of a metric on the space of probability measures on the state space of the solution. We present and discuss sharp asymptotic bounds on the respective N-th minimal errors.

3.18 Learning in variable RKHSs with Application to the Blood Glucose Reading

Valeriya Naumova (RICAM – Linz, AT)

Joint work of Naumova, Valeriya; Sergei, Pereverzyev; Sivananthan, Sampath
Main reference Naumova, Valeriya; Sergei, Pereverzyev; Sivananthan, Sampath, Extrapolation in variable RKHSs with application to the blood glucose reading, Inverse Problems 27 (2011), 075010.
URL http://dx.doi.org/10.1088/0266-5611/27/7/075010

Recent progress in diabetes technology is related to the so-called continuous glucose monitoring (CGM) systems estimating the blood glucose (BG) from the electric current measured in the interstitial fluid.

In accordance with the manufacture instruction a CGM should be re-calibrated several times per day, which means that each time a drop of the blood needs to be taken to measure the actual glucose and to correct the system. Even with this procedure the system is not always accurate. Therefore, the aim is to increase the CGM accuracy.

Mathematically the problem can be formulated as follows: we are given a data set, where each element is a value of unknown function paired with a point at which this value is attained. The function values may be blurred by noise, and the problem is to approximate the unknown function for the whole range of relevant values of the argument. This problem can be seen as an extrapolation, since it is not guaranteed that given data points span the required range.

In the context of diabetes technology, a given function value is the glucose concentration in a blood sample, and it is paired with the value of subcutaneous electric current measured at the moment when the sample is taken.

It is well-known that extrapolation is ill-posed and requires special regularization that trades off between data fitting and a complexity of a data fitter. The latter one is often measured by the norm in some reproducing kernel Hilbert space (RKHS), such as a Sobolev space, for example.

Classical regularization theory restricts itself to the case when a RKHS is assumed to be a priori known. At the same time, for a wide variety of applications a choice of RKHS is not given a priori, but should be driven by data. There are very few papers discussing this issue. Furthermore, to the best of our knowledge, no study has been reported in which the choice of RKHS is oriented towards extrapolation.

In this talk we present a new scheme of a kernel adaptive regularization learning algorithm, where the kernel and the regularization parameter are adaptively chosen within the regularization procedure. Experiments with clinical data show that the proposed choice allows essential reduction of erroneous BG-estimations as compared to commercially available CGM-devices. Moreover, the proposed approach can be used for BG-prediction and is a part of the Patent Application EP 11163219.6 filed recently by Austrian Academy of Sciences and Novo Nordisk A/S (Denmark).

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3.19 Weighted Hilbert spaces in the porous flow problem and associated numerical challenges

James Nichols (UNSW – Sydney, AU)

Joint work of Nichols, James; Kuo, Frances; Sloan, Ian; Schwab, Christoph; Scheichl, Robert; Graham, Ivan

We present recent results in the theory of applying QMC rules to integrating PDEs with random coefficients, as for example arises in the Darcy flow problem in porous media. Proving good convergence of rank-1 lattice rules requires new results for generalised spaces. We present those results, and discuss decisions motivated by the challenging numerics of this problem.

3.20 Infinite-dimensional quadratures

Dirk Nuyens (KU Leuven, BE)

Driven by the application of calculating box integrals (expected distances between points) over a Cantor set we investigate variants of the multilevel Monte Carlo algorithm on a journey for infinite-dimensional quadratures. We propose the multidigit multilevel Monte Carlo algorithm.

3.21 Approximation of linear functionals in reproducing kernel Hilbert spaces

Jens Oettershagen (Universität Bonn, DE)

Main reference F. M. Larkin, Optimal Approximation in Hilbert Spaces with Reproducing Kernel Functions, Math. Comp. 24(112), 1970

We approximate linear functionals in reproducing kernel Hilbert spaces by linear combinations of point-evaluation functionals. We generalize a result of Larkin to the multivariate setting and give necessary conditions for a set of n d-dimensional points being optimal in the sense that the worst-case error is minimal among all linear algorithms, which rely on at most function evaluations.

3.22 On the role of tractability conditions for multivariate problems

Anargyros Papageorgiou (Columbia University, US)

Joint work of Papageorgiou, Anargyros; Petras, Iasonas

We introduce a new tractability condition and the corresponding notion of κ-weak tractability. We study linear tensor product problems and show necessary and sufficient conditions for κ-weak tractability.

3.23 Repeated Phase Estimation: Approximating the ground state energy of the Schrödinger equation

Iasonas Petras (Columbia University, US)

Joint work of Papageorgiou, Anargyros; Petras, Iasonas

We demonstrate a quantum algorithm which estimates the ground state energy for convex potentials uniformly bounded by $C > 1$ with relative error $\mathcal{O}(\varepsilon)$, using

$$\mathcal{O}\left(\varepsilon^{-(3+\frac{1}{2k})}C^{4+\frac{S+k}{2k}}d^{4+\frac{3+\eta}{2k}}\right)$$

bit queries and

$$\Theta\left(\log\varepsilon^{-3}\right)+\Theta\left(\log(C^2d^2)\right)+\Theta\left(d\log^2\varepsilon^{-1}\right)$$

qubits, where $2kH$ is the order of the Suzuki splitting method used to simulate the exponentials and η any positive constant.

3.24 Multivariate Integration of Infinitely Many Times Differentiable Functions in Weighted Korobov Spaces

Friedrich Pillichshammer (University of Linz, AT)

Joint work of Kritzer, Peter; Pillichshammer, Friedrich; Wozniakwski, Henryk
Main reference P. Kritzer, F. Pillichshammer, H. Wozniakowski: Multivariate integration of infinitely many times differentiable functions in weighted Korobov spaces. Math. Comp, to appear
URL http://www.finanz.jku.at/index.php?id=104

We study multivariate integration in the worst-case setting for a weighted Korobov space of periodic infinitely many times differentiable functions for which the Fourier coefficients decay exponentially fast and present conditions on the weights such that we have exponential convergence with weak, polynomial and strong polynomial tractability.

3.25 Optimal approximation of SDEs with time-irregular coefficients via randomized Euler algorithm

Pawel Przybylowicz (AGH Univ. of Science & Technology-Krakow, PL)

Joint work of Przybylowicz, Pawel; Morkisz, Pawel

We investigate pointwise approximation of the solution of a scalar stochastic differential equation in the case when a drift coefficient is a Caratheodory mapping and a diffusion coefficient is only piecewise Holder continuous. It is known that under imposed assumptions and in the worst case setting the classical Euler algorithm does not converge to the solution of the equation. We give a construction of the randomized Euler scheme and investigate its error and optimality in the worst case and asymptotic setting.

Part of this talk is based on joint work with Pawel Morkisz.

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3.26 Weighted Hilbert Spaces and Integration of Functions of Infinitely Many Variables

Klaus Ritter (TU Kaiserslautern, DE)

Joint work of Gnewuch, Michael (UNSW, Sydney; supported by the German Research Foundation DFG (GN 91/3-1) and the Australian Research Council ARC); Mayer, Sebastian (Universität Bonn); Ritter, Klaus (partially supported by the DFG within Priority Program 1324)

We study two issues that arise for integration problems for functions of infinite many variables, which were first studied in [5] and which have recently been studied intensively, see, e.g., [1, 2, 3, 4, 8, 9, 10, 11] as well as [12, 13, 14] and [7, 6] for closely related problems. The setting is based on

- a reproducing kernel for functions on a domain D,
- a family of non-negative weights γ_u, where u varies over all finite subsets of $\mathbb{N}$,
- a probability measure ρ on D.

For the construction of the function space we consider the tensor product kernels

$$k_u(\mathbf{x}, \mathbf{y}) = \prod_{j \in u} k(x_j, y_j)$$

with $\mathbf{x}, \mathbf{y} \in D^u$, as well as the weighted superposition

$$K = \sum_u \gamma_u k_u.$$

We show that, under mild assumptions, K is a reproducing kernel on a properly chosen domain $\mathfrak{X} \subseteq D^{\mathbb{N}}$, and the quasi-reproducing kernel Hilbert space associated to K is isomorphic to the reproducing kernel Hilbert space with kernel K in a natural way. Furthermore, $H(K)$ is the orthogonal sum of the spaces $H(\gamma_u k_u)$.

Thereafter, we relate two approaches to define an integral for functions on $H(K)$, namely via a canonical representer or with respect to the product measure $\rho^{\mathbb{N}}$ on $D^{\mathbb{N}}$. In particular, we provide sufficient conditions for the two approaches to lead to the same notion of integral.

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3.27 On the slice sampler

Daniel Rudolf (Universität Jena, DE)

Joint work of Rudolf, Daniel; Łatuszyński, Krzysztof

We consider a slice sampling procedure to sample a possibly non-normalized density function. For the slice sampler, say the simple slice sampler, it is often assumed that one can sample the uniform distribution on the slices of the density. In contrast we consider slice sampler where one has a Markov chain with the correct limit distribution on every slice. The goal is to show a lower bound of the spectral gap in terms of the spectral gap of the simple slice sampler and properties of the Markov chain on the slice.

3.28 Regularization of Ill-posed Linear Equations by the Non-stationary Augmented Lagrangian Method

Otmar Scherzer (Universität Wien, AT)

Joint work of Frick, Klaus; Scherzer, Otmar
Main reference Regularization of Ill-posed Linear Equations by the Non-stationary Augmented Lagrangian Method
URL http://dx.doi.org/10.1216/JIE-2010-22-2-217

In this work we make a convergence rates analysis of the non-stationary Augmented Lagrangian Method for the solution of linear inverse problems. The motivation for the analysis is the fact that the Tikhonov-Morozov method is a special instance of the Augmented Lagrangian Method. In turn, the latter is also equivalent to iterative Bregman distance regularization, which received much attention in the imaging literature recently.

We base the analysis of the Augmented Lagrangian Method on convex duality arguments. Thereby, we can reprove some of the convergence (rates) results for the Tikhonov-Morozov Method. In addition, by the novel analysis we can prove properties of the dual variables of the Augmented Lagrangian methods. Reinterpretation of the dual variables for the Tikhonov-Morozov method gives some new convergence rates results for the linear functionals of the regularized solutions. As a benchmark for achievable convergence rates of the Augmented Lagrangian Method in the general convex context we use the results on evaluation of unbounded operators of Groetsch, which is a special instance of the Tikhonov-Morozov method.

3.29 Novel tensor formats and non-linear Galerkin approximation

Reinhold Schneider (TU Berlin)

Hierarchical Tucker tensor format (Hackbusch) and Tensor Trains (TT) (Tyrtyshnikov) have been introduced recently offering stable and robust approximation by a low order cost. In case $V = \bigotimes_{i=1}^{d} V_i$ which is proportional to d and polynomial in the ranks. We demonstrate the behavior of these ranks, depending on bilinear approximation rates and corresponding trace class norms. For many problems, which could not be handled so far, this approach can circumvent from the curse of dimensionality. We became aware, in case $V = \bigotimes_{i=1}^{d} \mathbb{C}^2$, that these formats are equivalent to tree tensor networks states and matrix product states (MPS) introduced for the treatment of quantum spin systems. Under the assumption of moderate ranks, i.e. low entanglement, this approximation enables quantum computing without quantum computers. For numerical computations, we consider the solution of quadratic optimization problems constraint by the restriction to tensors of prescribed ranks r. For approximation by elements from this highly nonlinear subset, we developed a non-linear Galerkin framework. We analyse the (open) manifold of such tensors and its projection onto the tangent space. We further derive differential equations for the gradient flow and stationary equations based on Dirac-Frenkel variational principle.

3.30 QMC Quadratures for infinite-dimensional parametric PDE problems

Christoph Schwab (ETH Zürich, CH)

Joint work of Schwab, Christoph; Kuo, Frances; Sloan, Ian
Main reference Frances Kuo and Christoph Schwab and Ian Sloan: Quasi-Monte Carlo finite element methods for a class of elliptic partial differential equations with random coefficients ,SIAM. J. Numer. Anal. 2013 (accepted for publication)
URL http://www.sam.math.ethz.ch/reports/2011/52

We present recent results on well-posedness and regularity for several classes of infinite-dimensional, parametric ordinary and partial differential equation (PDE) problems. Both, linear and nonlinear PDE problems are covered.

Such problems arise, among others, in connection with diffusion, vibration and wave-propagation in random media, when a parametric representation of the law of the random solutions is sought in in terms of countably many parameters of the problems' random inputs, e.g. in terms of a Karhunen-Loeve expansion.

We present recent results on the regularity of these parametric representations of random fields, in terms of weighted reproducing kernel Hilbert spaces in infinite dimension.

We then show how recent results on Quasi Monte-Carlo quadratures on these weighted reproducing kernel Hilbert spaces in infinite dimension allow for the efficient numerical evaluation of mean fields and statistical moments of the random solutions.

The QMC results are joint work with Frances Kuo and Ian Sloan, of UNSW, Sydney, Australia.

The results are part of the research reports
http://www.sam.math.ethz.ch/reports/2012/18
http://www.sam.math.ethz.ch/reports/2012/25
http://www.sam.math.ethz.ch/reports/2011/52

3.31 Lattice Methods with Designed Weights for PDE with Random Coefficients

Ian Sloan (University of New South Wales, AU)

Joint work of Kuo, Frances (UNSW); Graham, Ivan (Bath); Nichols, James (UNSW); Scheichl, Rob (Bath); Schwab, Christoph (ETH); Sloan, Ian (UNSW)

In this talk I will present recent developments on applying quasi-Monte Carlo methods (specifically lattice methods) to the computation of high-dimensional expected values (treated as multivariate integrals) of functionals of the solution of a PDE with random coefficients. A guiding example is the flow of a liquid through a porous material, with the permeability modelled as a random field. In this work we apply the theory of lattice methods in weighted spaces, together with estimates derived from the PDE, to design special lattice methods with proved good convergence properties for the computed expected values.

3.32 Adaptive (piecewise) tensor product wavelet Galerkin method

Rob Stevenson (University of Amsterdam, NL)

In this talk we summarize the convergence theory of adaptive wavelet Galerkin methods for solving well-posed linear or nonlinear operator equations. We discuss the application of these methods with the use of (piecewise) tensor product bases. Finally, we focus on the adaptive solution of simultaneous space-time variational formulations of evolution problems.

3.33 A rapidly mixing Markov chain for the two-dimensional Ising model

Mario Ullrich (Universität Jena, DE)

Main reference M. Ullrich, Rapid mixing of Swendsen-Wang and single-bond dynamics in two dimensions
URL http://arxiv.org/abs/1202.6321

We prove that the Swendsen-Wang dynamics (SW) for the Ising model on the two-dimensional square lattice is rapidly mixing at all temperatures. For this, we present three comparison results. First we show rapid mixing at and above the critical temperature by comparison with the single-site heat- bath dynamics. Then we prove that rapid mixing of SW and the single-bond dynamics (SB) for the corresponding random-cluster model is equivalent. And finally, we relate the mixing properties of SB at high and low temperatures (using dual graphs).

3.34 Optimal Cubature in Sobolev-Besov Spaces with Dominating Mixed Smoothness

Tino Ullrich (Universität Bonn, DE)

Main reference Tino Ullrich, Optimal Cubature in Besov Spaces With Dominating Mixed Smoothness on the Unit Square, Preprint, Bonn 2012

We present new constructive and asymptotically optimal error bounds for numerical integration in bivariate periodic Besov spaces with dominating mixed smoothness $S^r_{p,q}B(T^2)$, where $1 \leq p, q \leq \infty$ and $r > 1/p$. Our first result uses Quasi-Monte Carlo integration on Fibonacci lattice rules and improves on the so far best known upper bound achieved by using cubature formula taking function values from a Sparse Grid. It is well known that there is no proper counterpart for Fibonacci lattice rules in higher dimensions. To this end, our second result is based on Hammersley (or Van der Corput) type point grids. Instead of exploiting a Hlawka-Zaremba type discrepancy duality, which is limited to small smoothness parameters $1/p < r \leq 1$, we extend Hinrichs' recent results to larger orders r, namely $1/p < r < 2$. This direct approach is strongly conjectured to have a proper counterpart for higher orders r and, in addition, for functions on the d-torus T^d . Last, but not least, we prove that any cubature rule based an a sparse grid in d dimensions has a significantly worse error order than the previously described methods. These results are a first step to approach the problem of optimal recovery of functions from a discrete set of function values in a completely new way.

3.35 Learning Functions of Few Arbitrary Linear Parameters in High Dimensions

Jan Vybiral (TU Berlin, DE)

Joint work of Fornasier, Massimo; Schnass, Karin; Vybiral, Jan
Main reference Found. Comput. Math. 12 (2) (2012), 229-262
URL http://dx.doi.org/10.1007/s10208-012-9115-y

We study the uniform approximation of functions of many variables with the following inner structure. We assume, that $f(x) = g(Ax)$, where $x \in \mathbb{R}^d$, A is a $k \times d$ matrix and g is a (smooth) function on $\mathbb{R}^k$. Both g and A are unknown and their recovery is a part of the problem.

Under certain smoothness and variation assumptions on the function g, and an arbitrary choice of the matrix A, we present a sampling choice of the points drawn at random for each function approximation and algorithms for computing the approximating function. Due to the arbitrariness of A, the choice of the sampling points will be according to suitable random distributions and our results hold with overwhelming probability. Our approach uses tools taken from the compressed sensing framework, recent Chernoff bounds for sums of positive-semidefinite matrices, and classical stability bounds for invariant subspaces of singular value decompositions.

3.36 Average Case Tractability of Approximating ∞-Variate Functions

Grzegorz Wasilkowski (University of Kentucky, US)

We discuss function approximation in the average case setting for spaces of ∞-variate functions that have a weighted tensor product form and are endowed with a Gaussian measure that also has a weighted tensor product form. We assume that the cost of function evaluation depends on the number of active variables and we allow it to be from linear to exponential. We provide a necessary and sufficient condition for the problem to be polynomially tractable and derive the exact value of the tractability exponent. In particular, the approximation problem is polynomially tractable under modest conditions on weights even if the function evaluation cost is exponential in the number of active variables. The problem is weakly tractable even if this cost is doubly exponential.

3.37 Probabilistic star discrepancy bounds for double infinite random matrices

Markus Weimar (Universität Jena, DE)

In 2001 Heinrich, Novak, Wasilkowski and Wozniakowski proved that the inverse of the discrepancy depends linearly on the dimension by showing that a random point set P of N points in the s-dimensional unit cube satisfies the discrepancy bound $D_N^{*s}(P) < cs^{1/2}N^{-1/2}$ with positive probability. Later their results were generalized by Dick to the case of double infinite random matrices.

In this talk we give explicit, asymptotically optimal bounds for the star discrepancy of such random matrices, and give estimates for the corresponding probabilities. Using the same techniques we derive similar discrepancy bounds for randomly generated completely uniformly distributed (c.u.d.) sequences which find applications in Markov Chain Monte Carlo.

The talk is based on a recent paper which is joint work with C. Aistleitner [1].

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3.38 Tractabilty of multi-parametric Euler and Wiener integrated processes

Henryk Woźniakowski (Columbia University, US and University of Warsow, PL)

Joint work of Lifshitz M. (State University of Saint Petersburg); Papageorgiou, Anargyros (Columbia University); Henryk Woźniakowski

We provide necessary and sufficient conditions on weak, polynomial and strong polynomial tractability for multivariate approximation in the average case setting for Gaussian measures with Euler and Wiener integrated covariance kernels. These conditions are expressd in terms of the sequence $\{r_k\}$, where r_k measures the smoothness of functions with respect to the k-th variable.

3.39 Computing Quadrature Formulas for Marginal Distributions of SDEs II

Larisa Yaroslavtseva (Universität Passau, DE)

Joint work of Yaroslavtseva, Larisa; Mueller-Gronbach, Thomas

This talk is a continuation of the talk of Thomas Mueller-Gronbach. Here we consider the case of scalar SDEs with bounded coefficients that are 6 times continuously differentiable and have bounded derivatives. Furthermore, the diffusion coefficient is assumed to be bounded away from zero. For the definition of the error we employ the Wasserstein distance. We present a deterministic algorithm, which is based on sparse discrete approximations of Wagner-Platen steps with support points in small grids and is easy to implement. The method achieves the optimal order of convergence in terms of the computational cost, up to an arbitrarily small power of the cost.

3.40 Pointwise approximation for additive random fields

Marguerite Zani (Université Paris-Est Créteil, FR)

Joint work of Lifshitz, Mikhail A. (St. Petersburg); Zani, Marguerite

We consider standard information in the average case setting for additive random fields. We consider a multilevel algorithm using function evaluations and show that the L^2 approximation error is somehow comparable to the one in the linear case.

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