Report from Dagstuhl Seminar 12411

Coalgebraic Logics

Edited by
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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 12411 "Coalgebraic Logics". The seminar deals with recent developments in the area of coalgebraic logic, a branch of logics which combines modal logics with coalgebraic semantics. Modal logic finds its uses when reasoning about behavioural and temporal properties of computation and communication, coalgebras have evolved into a general theory of systems. Consequently, it is natural to combine both areas for a mathematical description of system specification. Coalgebraic logics are closely related to the broader categories semantics/formal methods and verification/logic.

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1 Executive Summary

Alexander Kurz (University of Leicester, GB)
Ernst-Erich Doberkat (TU Dortmund, DE)

Background

Modal Logic is a field with roots in philosophical logic and mathematics. As applied to Computer Science it has become central in order to reason about the behavioural and temporal properties of computing and communicating systems, as well as to model properties of agents such as knowledge, obligations, and permissions. Two of the reasons for the success of Modal Logic are the following. First, many modal logics are—despite their remarkable expressive power—decidable and, therefore, amenable to automated reasoning and verification. Second, Kripke's relational semantics of modal logic turned out to be amazingly flexible, both in terms of providing techniques to prove properties of modal logics and in terms of allowing the different applications of Modal Logic to Artificial Intelligence, Software Agents, etc.

Coalgebra is a more recent area. Following on from Aczel's seminal work on non-well founded set theory , coalgebra has been developed into a general theory of systems. The basic idea is that coalgebras are given with respect to a parameter F. Technically, the parameter F is a *functor* on a *category* $\mathcal{C}$.

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Different choices of F yield, for example, the Kripke frames and models of modal logic, the labelled transition systems of process algebra, the deterministic automata of formal language theory, or the Markov chains used in statistics. Rutten showed that, in analogy with Universal Algebra, a theory of systems, called Universal Coalgebra, can be built uniformly in the parameter F, simultaneously covering the above and other examples. Crucial notions such as behavioural equivalence (observational equivalence, bisimilarity), final semantics and coinduction find their natural place here.

Coalgebraic Logic combines Coalgebra and Modal Logic to study *logics of systems* uniformly in the parameter F. Given the plethora of different transition systems and their ad hoc logics, such a uniform theory is clearly desirable. *Uniformity* means that results on, for example, completeness, expressivity, finite model property and complexity of satisfiability can be established at once for all functors (possibly satisfying some, usually mild, conditions). Additionally, there is also a concern for *modularity*: Typically, a parameter F is composed of basic features (such as input, output, non-determinism, probability). Modularity then means that the syntax/proof systems/algorithms for the logic of F are obtained compositionally from the syntax/proof systems/algorithms for the logics of the basic features.

What has been achieved: The power of uniformity and modularity Following on from Moss' seminal paper, Coalgebraic Logic is now growing into a successful area. Conferences in this area now treat topics such as completeness, expressivity, compositionality, complexity, rule formats for process calculi, containing several hitherto unknown results on these classic topics.

The uniformity achieved in the above cited work is based on varying the type F for a given base category $\mathcal{C}$, usually the category of sets. But it is also of interest to vary $\mathcal{C}$.

Here probabilistic approaches deserve to be mentioned. In a number of papers Markov transition systems could be shown to interpret modal logics under different assumptions on the probabilistic structure. It was shown that general measurable spaces provide too general a structure, but that analytic spaces with Borel transition laws offer just the right blend of generality and measure theoretic accessibility. In this context, it was shown that logical equivalence, bisimilarity, and behavioral equivalence are equivalent concepts . Recent work shows that this can be extended to distributional aspects as well: instead of comparing states proper, one has a look at distributions over the states of a Kripke model. This approach was recently generalized from general modal logics to coalgebraic logics ; these logics are interpreted through coalgebras in which the subprobability functor and the functor suggested by the phenomenon to be modelled form various syntactic alliances. This generalization brings stochastic coalgebraic logic into the mainstream of coalgebraic logics: the problems considered are similar, and one sees a convergence of methods.

Nevertheless it is to be mentioned that the probabilistic approach brings its own idiosyncratic touch due to measure theoretic problems. This entails among others that one sometimes has to work in a very specific topological context, for otherwise solutions are not available. On the other hand, leaving a topological context and working in general measurable spaces poses the question of the limits to the coalgebraic approach: What can be achieved in general measurable spaces, or in measurable spaces in which some of the properties are available (like Blackwell spaces, which are countably generated without being topological)?

Quantitative aspects are also considered when it comes to approximate Markov transition processes defined on uncountably infinite state spaces through finite processes . This is a classical problem that arises mostly in practical applications of Markov transition systems; it has to be investigated from a logical vantage point as well.

Structuring the Seminar

When we planned the seminar, we envisaged six broad topics. One of the outcomes of this seminar, as compared to the one of 2009, is that the different subcommunities in coalgebraic logic moved closer together, exchanging ideas, techniques, problems and also researchers. Consequently, it seems difficult, if not impossible, to divide up all the talks consistently among the distinct research topics. We will nevertheless try to describe some trends.

Probabilistic Transition Systems

The focus of Markov transition systems shifted from the consideration of specific problems (like interpreting a particular class of logics) to structural problems which are treated with the instruments provided by coalgebras. The talk presented Panangaden concentrated on the duality of Markov transition systems and various function spaces, most of them well known in functional analysis. The Radom-Nikodym Theorem provides a very sophisticated tool for switching between these representations. Doberkat's talk dealt with stochastic effectivity functions as an extension of Markov transition systems for the interpretation of more complicated logics like, e.g., Parikh's game logic. Urbat showed that both the Hausdorff and the Kantorovic functor, which are widely used to model probabilistic nondeterminism are finitary, improving some well known results; at the same time, this results raises some interesting topological questions.

Quite apart from structural problems, another approach has been presented by Srivastava; he gave a tutorial talk on deduction systems for probabilistic logics, based on the work by Goldblatt and by Zhou. The set theoretic problems which originate with bisimilarity were taken up by Terraf, who extended a well-known result from descriptive set theory on the structure of equivalence relations to bisimulations, hereby indicating some of the caveats one has to observe in classical set theory.

Coalgebras and automata theory

Whereas the final coalgebra describes all infinite behaviours, the theory of formal languages suggests that the regular or rational sets of behaviours should be of special interest. This is indeed the case and the talks of Milius, Myers, Sokolova and Winter presented some of the latest developments. More generally, this direction of generalising results from automata theory also saw talks of Hansen/Silva and of Venema.

Process algebra and operational semantics

Bonsangue presented a coalgebraic account of the 'bisimulation-up-to' proof technique and Staton had new results on finite power set functors. Another direction is concerned with applying coalgebraic techniques to other process equivalences than bisimulation. In particular, Hasuo and Cirstea studied trace equivalence, whereas Levy's tutorial on relation liftings was concerned with various notions of simulation.

Coalgebraic logic beyond sets

After the successes of set-based coalgebra, quite some effort goes now into extending results to more general settings. Jacobs presented a novel framework uniformly covering the classical, probabilistic and quantum case. Pavlovic introduced his ideas about a monoidal computer to bridge the gap between high-level specification and low-level computational models such as Turing machines. Talks by Bilkova, Dostal, and Velebil explored now to harness enriched category theory whereas Moshier is extending coalgebraic logics from the discrete to the setting of compact Hausdorff topological spaces, a topic that also surfaced in Hofmann's contribution. Petrisan studied final coalgebra in nominal sets.

Extensions of coalgebraic logics

Litak led a discussion session about the directions of generalising coalgebraic modal logic to formalisms with explicit quantifiers. Palmigiano reported latest results on extensions with fixpoint operators and Venema discussed some of the challenges and open problems in this area. Sano showed how to extend coalgebraic logic by an actuality operator and whereas Schröder explored the border of decidability for coalgebraic hybrid logic.

Applications

One of the outcomes of the seminar was the excitement generated by the wide range of applications which are now coming into the scope of coalgebraic techniques. Examples include Abramsky's results on infinite economic non-cooperative games, Trancon y Widemann's contributions to a reformulation of the foundations of ecology, and Kozen's ideas of making coalgebraic techniques available to the working programmer and to the working mathematician.

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3 Overview of Talks

3.1 Moss' coalgebraic logic in preorders and beyond

Marta Bilkova (Charles University – Prague, CZ)

Joint work of Bilkova, Marta; Velebil, Jiri

In this talk we built on results obtained in , namely existence of functorial relation lifting for functors preserving exact squares in the category of preorders, generalised in to the enriched case of V-categories, where V is a commutative quantale. For the preorder case we present Moss' coalgebraic language based on the logic of distributive lattices equiped with cover modalities nabla and delta, its semantics, and a sound axiomatics (it is a work in progress and completeness yet remains to be shown). As expected, axiomatics consists of certain distributive laws which are straightforward analogues of those known from the Set case, see . Formally, the laws look the same as in the Set case, yet the techniques used have to be more subtle and they reveal the hidden symmetries. Moreover, the proofs apply also to the case of the V-categories. For the case of V-categories we propose the propositional part of the logic to consist of connectives based on (weighted) limits and colimits. In the case of preorders they collapse to meets and joins.

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3.2 Coalgebraic Bisimulation-up-to

Marcello M. Bonsangue (Leiden University, NL)

Joint work of Rot, Jurriaan; Bonsangue, Marcello; Rutten, Jan
Main reference J. Rot, M. Bonsangue, J. Rutten, "Coalgebraic bisimulation-up-to," in Proc. of SOFSEM, 2013. To appear.
URL http://www.liacs.nl/~jrot/sofsem.pdf

In this talk I will present a systematic study of bisimulation-up-to techniques for coalgebras. These techniques enhance the bisimulation proof method for a large class of state based systems, including labelled transition systems but also stream systems and weighted automata. Our approach allows for compositional reasoning about the soundness of enhancements. Applications include the soundness of bisimulation up to bisimilarity, up to equivalence and up to congruence. All in all, this gives a powerful and modular framework for simplifed coinductive proofs of equivalence.

3.3 Coalgebraic logic over concrete categories

Liang-Ting Chen (University of Birmingham, GB)

Joint work of Chen, Liang-Ting; Jung, Achim

Coalgebraic logic for **Sets** coalgebras given by predicate lifting has been applied to different areas in computer science, e.g. modal logic, automata theory, and program verification. However, there are currently few studies beyond **Sets**. Exceptions are, for example, coalgebraic logic over the category of posets for positive modal logic (by Kapulkin, Balan, Kurz, Velebil) and coalgebra logic over the category of measurable spaces for stochastic coalgebraic logic. There are more general approaches based on dual adjunctions, but it is not clear how to describe modalities explicitly.

In this talk, we relate few different notions. A dual adjunction over concrete categories with a mild condition provides generalised predicates as morphisms to the dualising object, so we can give straightforward definitions and prove the adequacy of coalgebraic logic easily. Objects which contain predicate liftings are identified, and we derive a logic of all predicate liftings as a corollary from algebraic theory. As for expressivity, we argue that propositional geometric logic might be an interesting local logic to use for coalgebraic logic.

3.4 Interaction and observation: dialgebras in program semantics

Vincenzo Ciancia (CNR – Pisa, IT)

Main reference V. Ciancia, "Interaction and observation: categorical semantics of reactive systems trough dialgebras," arXiv:1209.5903v1 [cs.LO].
URL http://arxiv.org/abs/1209.5903v1

Interactive systems are collections of entities that may interact with each other, and produce a resulting entity and an observable effect. Their semantics is typically described by so-called reaction rules.

In this work, we try to address the question "what are reaction rules" in a categorical way. As an answer we propose dialgebras, generalising both algebras and coalgebras.

The focus is on providing a semantic model, alternative to coalgebras, where interaction is built-in, instead of relying on a (possibly difficult) understanding of the side effects of a component in isolation.

Dialgebras are arrows of the form FX -> GX for suitable endofunctors F and G. The functor G gives rise to observable effects, just like in coalgebras. The functor F takes into account interaction between different elements, like in algebras. Behavioural equivalence is defined as kernel equivalence. Due to the interplay of F and G, such equivalence gives non-trivial semantics to reaction rules, when letting G be some variant of the power set functor and F be a product.

Dialgebras lack a final object for useful cases of F and G. This requires a change of point of view, and the adoption of quotient categories as grounds for reasoning. As a consequence, comparing different categories of dialgebras becomes a "local" task which is better carried out in sub-categories of quotients of some given interesting objects. Minimization and simplification of dialgebras are similarly affected.

We will discuss and motivate these aspects, and look at some examples.

3.5 On Logics for Maximal Traces

Corina Cirstea (University of Southampton, GB)

The coalgebraic theory of finite traces is well understood , and some preliminary results exist on logics that characterise finite trace equivalence .

Initial steps towards a coalgebraic account of maximal, possibly infinite traces have also been made . Here we revisit the definition of maximal traces in loc. cit. by taking the view that only finite (but arbitrarily long) prefixes of infinite traces are observable, and use a dual adjunction approach similar to that of to derive logics that characterise maximal traces.

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3.6 Stochastic Game Frames

Ernst-Erich Doberkat (TU Dortmund, DE)

Parikh's Game Logic is interpreted through game frames, i.e., collections of stochastic effectivity functions. Stochastic game frames are introduced and compared to Kripke models. Goldblatt's Theorem on deduction systems helps to clarify the relationship between both. We show how to construct from such a game frame an interpretation of Game Logic. Some analogies to Kozen's proposal for interpreting PDL are drawn, they are helpful for compensating the lack of suitable algebraic structures in the space of stochastic effectivity functions.

This is work in progress.

3.7 Many-valued relation liftings and coalgebraic logics

Matej Dostal (Czech Technical University, CZ)

Joint work of Dostal, Matej; Bilkova, Marta; Velebil, Jiri
Main reference M. Dostal, "Many-valued coalgebraic logic," Diploma thesis, to be published 2013.

The coalgebraic approach to modal logic enables us to talk about logics for various kinds of Kripke-like structures. We concern ourselves with the coalgebraic logic based on the cover modality. It seems natural to wonder how far we can get when trying to generalise this approach to a many-valued setting. We present a notion of many-valued relation liftings for finitary functors as a tool for talking about many-valued coalgebraic logics and present some examples of logics that arise this way.

This work has been supported by grant no. SGS12/060/OHK3/1T/13 of SGS CVUT.

3.8 Functor- and Logic Patterns

H. Peter Gumm (Universität Marburg, DE)

Preservation properties of a Set-Endofunctor F have been recognized as essential for the structure of the category SetF of F-coalgebras. Over 30 years ago, V.Trnková in Prague has studied similar problems in the general context of categories with a factorization system (E;M). We recollect some relevant fragment of her findings, and we add some new results and open questions concerning functors weakly preserving kernel pairs.

Finitary functors can be described by sets of finitary patterns. Sets of 0-1 patterns determine logical modalities and pattern rules correspond to frame axioms in coalgebraic modal logic. We shall present examples of such correspondences.

3.9 Coalgebraic Trace Semantics for Higher-Order Computation, Especially The Quantum One

Ichiro Hasuo (University of Tokyo, JP)

Joint work of Hasuo, Ichiro; Hoshino, Naohiko
Main reference I. Hasuo, N. Hoshino, "Semantics of Higher-Order Quantum Computation via Geometry of Interaction," LICS 2011: 237–246.
URL http://dx.doi.org/10.1109/LICS.2011.26

Unlike the standard setting of the category of sets and functions where coinduction captures bisimilarity, coinduction in Kleisli categories (for certain monads with cpo-enriched structures) captures (finitary) trace semantics. Underlying this observation is the coincidence of an initial algebra and a final coalgebra, a phenomenon typical of cpo-enriched settings.

In [Jacobs, CMCS'10] it is observed that this coalgebraic "trace" semantics (a usage in concurrency theory) is closely related to "trace" in traced monoidal categories—in fact, most known "particle-style" examples of traced monoidal categories arise from coalgebraic trace semantics, i.e. coinduction in Kleisli categories. Traced monoidal categories, in turn, have been used as a categorical foundation for Girard's geometry of interaction (a denotational semantics "doctrine" that is close to game semantics), by Abramsky, Haghverdi and P. Scott. Therefore, here we are seeing an exciting opportunity of combining theory of coalgebra and that of denotational semantics for functional programming. I'll exhibit one example of a quantum lambda calculus.

The talk will be based on the joint work with Naohiko Hoshino (RIMS, Kyoto Univ.), presented at LICS 2011.

3.10 Variations on a theme of Vietoris

Dirk Hofmann (University of Aveiro, PT)

The motivation for this talk stems from several duality results extending the classical Priestley and Stone dualities to categories of algebras with operators (see , for instance), all of these make essential use of the Vietoris construction . Here we find it worthwhile to note that the upper Vietoris space is part of a lax idempotent monad on the category Top of topological spaces which can be restricted to the full subcategory of locally compact spaces and the (non-full) subcategory of stably compact spaces and spectral maps, as well as to compact Hausdorff spaces. In this talk we shall

- follow an idea of Halmos and consider larger categories "of algebras with hemimorphisms" where the above-mentioned operators appear as morphisms, and recall how functoriality of the Kleisli construction (see) can be used to deduce in a uniform manner that these categories are dually equivalent to full subcategories of the Kleisli category of (variants of) the Vietoris monad $\mathbb{V}$;
- employ a formal analogy between order sets and topological spaces to describe these Vietoris monads as the "covariant presheaf monad" which then allows the introduction of metric (and other) generalisations;
- describe the Kleisli category of these monads;
- use general results of to conclude that these categories "of algebras with hemimorphisms" are also dually equivalent to categories of certain algebras of $\mathbb{V}$.

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3.11 New Directions in Categorical Logic, for Classical, Probabilistic and Quantum Logic

Bart Jacobs (Radboud University Nijmegen, NL)

Main reference B. Jacobs, "New Directions in Categorical Logic, for Classical, Probabilistic and Quantum Logic," arXiv:1205.3940v2 [math.LO].
URL http://arxiv.org/abs/1205.3940v2

Traditionally in categorical logic predicates on an object/type X are represented as subobjects of X. Here we break with that tradition and use maps of the form p : X –> X + X with [id, id] o p = id as predicates. This new view gives a more dynamic, measurement-oriented view on predicates, that works well especially in a quantitative setting. In classical logic (in the category of sets) these new predicates coincide with the traditional ones (subsets, or characteristic maps X –> 0,1); in probabilistic logic (in the category of sets and stochastic matrices), the new predicates correspond to fuzzy predicates X –> [0,1]; and in quantum logic (in Hilbert spaces) they correspond to effects (positive endomaps below the identity), which may be understood as fuzzy predicates on a changed basis. It is shown that, under certain conditions about coproducts +, predicates p : X –> X + X form effect algebras and carry a scalar multiplication (with probabilities). Suitable substitution functors give rise to indexed/fibred categories. In the quantum case the famous Born rule – describing the probability of observation outcomes – follows directly from the form of these substitution functors: probability calculation becomes substitution in predicate logic. Moreover, the characteristic maps associated with predicates provide tests in a dynamic logic, and turn out to capture measurement in a form that uniformly covers the classical, probabilistic and quantum case. The probabilities incorporated in predicates (as eigenvalues) serves as weights for the possible measurement outcomes.

3.12 Programming with Coinductive Types

Dexter Kozen (Cornell University – Ithaca, US)

Joint work of Kozen, Dexter; Jean-Baptiste Jeannin; Alexandra Silva

We present CoCaml, a functional programming language extending OCaml, which allows us to define functions on coinductive datatypes parameterized by an equation solver. We provide numerous examples that attest to the usefulness of the new programming constructs, including operations on infinite lists, infinitary lambda-terms and p-adic numbers.

3.13 Tutorial on relators

Paul Blain Levy (University of Birmingham, GB)

Relators (also known as relational extensions and lax relational extensions of a functor) play a key role in the coalgebraic study of bisimulation and simulation.

Just as a functor goes between categories, a relator goes between framed categories, so we beging by looking at these. We see examples including relations, corelations, and bimodules between preordered sets. We explore some properties of framed categories, in particular tabulations and cotabulations.

Then we look at the required properties of a relator: monotonicity, lax functoriality, and preservation of inverse images. We consider several variations of these requirements. In particular, we see that some relators preserve identities, some composition and some both.

Finally we give four "functor theorems" showing that a relator of a particular kind can be encoded as a functor, bringing together several notions and results from the literature.

3.14 A special discussion session on coalgebraic predicate formalisms

Tadeusz Litak (Universität Erlangen-Nürnberg, DE)

Joint work of Litak, Tadeusz; Pattinson, Dirk; Sano, Katsuhiko; Schröder, Lutz
Main reference T. Litak, D. Pattinson, K. Sano, L. Schröder, "Coalgebraic predicate logic," in Proc. of the 39th Int'l Colloquium on Automata, Languages and Programming (ICALP'12) – Volume II, pp. 299–311, Springer, LNCS, Vol. 7392, 2012.
URL http://dx.doi.org/10.1007/978-3-642-31585-5_29

In our ICALP 2012 paper, Dirk Pattinson, Katsuhiko Sano, Lutz Schröder and myself proposed the Coalgebraic Predicate Logic (CPL) – a one-sorted language for ordinary first-order models enriched with a coalgebraic structure. In the neighbourhood setting, a notational variant of this formalism has been first investigated by C. C. Chang in early 1970's as a "logic for social situations" or as a simplification of Montague's "pragmatics". Later on, its restriction to topological spaces has been rediscovered as a relatively weak language for topological model theory (Ziegler, Makowsky, Flum, Sgro ...) but remained largely unknown or forgotten elsewhere.

Our generalization to arbitrary Set-coalgebras has been based, as may be expected, on the notion of predicate lifting; a variant of the language based on Moss' nabla would also be conceiveable. There are also other, more expressive possible choices: one of them found in FoSSaCS 2010 paper of Schroeder and Pattinson. These more expressive variants require at least a sort for neighbourhoods, possibly also a sort for elements of the transition structure and more involved syntax in general. Furthermore, it is not clear at all whether some of our positive results, such as natural Henkin-style axiomatization or (in a follow-up paper) syntactic proofs of cut-elimination for restricted classes of functors and predicate liftings could be found for too powerful extensions of CPL. Nevertheless, let us not forget that, e.g. first-order formalisms employed in topological model theory are usually stronger than CPL (restricted to topological spaces as a subclass of coalgebras for the neighbourhood functor).

There are also more radically different "coalgebraic predicate languages". Under a sufficiently broad understanding of the notion, one can even include here Jacobs' recent work on predicate formalism for the Kleisli category of a monad. Its relationship to either the formalism of FoSSaCS 2010 or to the one of ICALP 2012 seems an open question to everybody involved; similarly with other formalisms used in categorical logic.

So with all this, what – if any – is the "right" coalgebraic predicate formalism? And more importantly, what sort of results and applications one would expect from such a beast? Does not the very idea of formalism not invariant under bisimulation or behavioural equivalence go against the main thrust of research in coalgebraic logic? In short: where is coalgebraic predicate logic going and should it go anywhere at all?

These are serious questions, worthy of a public debate. I am convinced that a genuine discussion session on the subject would be more beneficial to us and to the community than one more talk advertising the results of the ICALP 2012 paper and its follow-up.

Our (that is, at least that of Lutz Schroeder and myself) personal belief is that the most fruitful line of research may lie in investigating further the connection with finite model theory and preservation results. We already have suitable variants of the Van Benthem-Rosen theorem for several flavors of coalgebraic predicate formalisms. A natural next step is to attack the only other existing major preservation result surviving in the finite model theory context: invariance of existential-positive formulas under homomorphism (see Rossman in LiCS 2005 and ACM 2008). The key to robustness of these two results seems to be lie in the fact that their proofs rely heavily on notions such as Gaifman graphs – and so does the coalgebraic Van Benthem-Rosen theorem (at least in the FoSSaCS 2010 paper). There seems to be a tantalizing slogan lurking in the background that "preservation results survive in the finite model theory context iff they survive in the coalgebraic context" and its full implications are yet to be understood. The bigger challenge here is developing finite model theory (or "metafinite model theory", as in the paper of Gradel and Gurevich) for non-relational structures.

Yde Venema and the Amsterdam group have already done some work on the generalization of another preservation and characterization result – the Janin-Walukiewicz theorem (which is itself a generalization of the van Benthem-Rosen to the second-order case). All this should lead to a more complete version of "abstract coalgebraic model theory". So far, for example, the only existing coalgebraic variants of the Lindstroem theorem focused on the modal propositional language. Clearly, it is not what the original Lindstroem theorem was about. But what is the right notion of E-F games in the coalgebraic case? We do in fact have E-F games for CPL (developed with Lutz Schroeder, yet unpublished) – are they likely to be of general interest? Are there some unexpected potential applications?

3.15 On the specification of operations on the rational behaviour of systems

Stefan Milius (TU Braunschweig, DE)

Joint work of Bonsangue, Marcello; Milius, Stefan; Myers, Rob; Rot, Jurriaan
Main reference M.M. Bonsangue, S. Milius, J. Rot, "On the specification of operations on the rational behaviour of systems," in Proc. of EXPRESS/SOS 2012. Electron. Proc. Theoret. Comput. Sci. 89 (2012), 3–18.
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Structural operational semantics can be studied at the general level of distributive laws of syntax over behaviour. This yields specification formats for well-behaved algebraic operations on final coalgebras, which are a domain for the behaviour of all systems of a given type functor. We introduce a format for specification of algebraic operations that restrict to the *rational fixpoint* of a functor, which captures the behaviour of *finite* systems. Our format can be seen as a generalization of Aceto's simple GSOS format from process algebra to the realm of distributive laws. We show that rational behaviour is closed under operations specified in our format. As applications we consider operations on regular languages, regular processes and finite weighted transition systems. We also obtain a generalization of Aceto's theorem stating that for a transition system specification in the simple GSOS format the associated LTS is regular.

3.16 Modal proximity lattices and modal compact Hausdorff spaces

M. Andrew Moshier (Chapman University – Orange, US)

Joint work of Moshier, M. Andrew; Nick Bezhanishvili

Thanks to a well-known completeness theorem, normal modal logic can be regarded as a specification language for non-deterministic transition systems modelled as compact, zero dimensional state spaces equipped with a coalgebra for the Vietoris functor. The zero dimensionality requirement, however, severely limits potential applications. After all, this rules out such state spaces as spheres, tori, and other garden variety compact Hausdorff spaces.

In this talk we consider how to generalize normal modal logic to account for general compact Hausdorff state spaces. The result is again a completeness theorem for the generalized normal logic. We then prove that Sahlqvist's Theorem still works in this generalization, provided we replace Sahlqvist formulae with a suitable notion of Sahlqvist inference rule.

3.17 Eilenberg's Theorem Coalgebraically

Rob Myers (TU Braunschweig, DE)

Eilenberg's theorem is one of the central theorems in algebraic automata theory. It describes an isomorphism between the lattice of varieties of finite monoids and the lattice of varieties of regular languages. Recent work by Gehrke, Grigorieff and Pin has shown this theorem arises locally as a duality. Our contribution is that this duality is inherently coalgebraic, to the extent that the entire theorem can be proved using canonical constructions. Since our approach is parametric in a functor we obtain many new theorems and unify much previous work. For example we not only cover the version involving ordered monoids and idempotent semirings, but we also obtain new examples involving associative algebras and modules.

3.18 The Generic Kleene Theorem

Rob Myers (TU Braunschweig, DE)

Kleene theorems are usually proved relative to particular operations satisfying particular equations. The message of this talk is that one can prove them parametric in an arbitrary finitary functor on an arbitrary finitary variety. We justify this by providing many examples such as boolean automata, subsequential transducers, context free grammars and linear automata over arbitrary semirings. We also discuss an ongoing application of this approach, namely the first definition of the minimal nondeterministic automaton accepting a regular language.

3.19 Monoidal computer and coalgebras

Dusko Pavlovic (RHUL – London, GB)

Main reference D. Pavlovic, "Monoidal computer I: Basic computability by string diagrams," arXiv:1208.5205v2 [cs.LO].
URL http://arxiv.org/abs/1208.5205v2

Church's Thesis is great, but low level programming of Turing Machines, and of lambda terms, makes the bridge between theoretical computer science and practice often longer than one would expect. E.g., the task of measuring the logical distance between algorithms (which I recently put forward in "Gaming security by obscurity" – http://arxiv.org/abs/1109.5542) quickly leads to unreasonably verbose low level programming. To overcome this, we need a high level language for complexity theory, algorithmic information, and cryptorgraphic constructions. The structure of monoidal computer is an effort to formalize the diagrammatic language that I have been using for this purpose informally. The first step is in the uploaded paper. Modeling protocols and Interactive Proof Systems in monoidal computer naturally leads to coalgebras, which will be elaborated in a sequel paper.

3.20 Nominal coalgebriac data types

Daniela Petrisan (University of Leicester, GB)

Pitts and Gabbay proved structural recursion and induction principles for syntax with binding and further introduced nominal algebraic data types. This talk is about nominal coalgebraic data types. In particular we will discuss final coalgebras for functors obtained from a binding signature. Applications include an alpha-corecursion principle for the infinitary lambda calculus and corecursive definitions of infinite normal forms. This talk is based on joint work with Alexander Kurz, Paula Severi and Fer-Jan de Vries.

3.21 Actuality in Coalgebraic Modal Logic

Katsuhiko Sano (JAIST – Nomi, JP)

This talk reports a current ongoing work on coalgebraic modal logic with the actuality operator, originally invented by Kamp and Kaplan. Semantically, the addition of the actuality operator to coalgebraic modal logic corresponds to an addition of an initial state to a given coalgebra (i.e., pointed coalgebra). First, we observe that this addition of the actuality operator still does not increase the expressive power with respect to truth at the initial state. This is done by a generalization of a recent study on the actuality operator in Kripke and neighborhood semantics by Hazen, Rin, and Wehmeier. Second, we demonstrate how to convert a sequent calculus of coalgebraic modal logic into the one with the actuality operator. For this aim, we employ the framework of sequent calculus by Pattinson and Schröeder (2011).

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3.22 Coalgebraic Logic and Self-Reference

Lutz Schroeder (Universität Erlangen-Nürnberg, DE)

Decidability of modal logics tends to break down quickly when features for self-reference are added, such as the down-arrow binder of hybrid logic, or its single-variable version popularized by Marx as the I-me construct. We have shown in earlier work that decidability (and in fact low complexity) of logics with I-me is regained if the number of modal operators between between each use of me and its enclosing I is bounded by two; these results are stable under adding graded modalities . Here, we report on ongoing work aimed at a coalgebraic generalization of the algorithmic principles involved, in particular a PSPACE upper bound for local reasoning.

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3.23 Brzozowski's algorithm (co)algebraically

Alexandra Silva (Radboud University Nijmegen, NL)

Joint work of Bonchi, Filippo; Bonsangue, Marcello; Hansen, Helle; Panangaden, Prakash; Rutten, Jan; Silva, Alexandra
Main reference F. Bonchi, M.M. Bonsangue, J.J.M.M. Rutten, A. Silva, "Brzozowski?s Algorithm (Co)Algebraically," in R.L. Constable, A. Silva, (eds), Logic and Program Semantics – Essays Dedicated to Dexter Kozen on the Occasion of His 60th Birthday, LNCS, Vol. 7230, pp. 12–23, 2012.
URL http://dx.doi.org/10.1007/978-3-642-29485-3_2

We give a new presentation of Brzozowski's algorithm to minimize finite automata, using elementary facts from universal algebra and coalgebra, and building on earlier work by Arbib and Manes on the duality between reachability and observability. This leads to a simple proof of its correctness and opens the door to further generalizations. Notably, we derive algorithms to obtain minimal, language equivalent automata from Moore, non-deterministic and weighted automata.

3.24 Congruences of Convex Algebras

Ana Sokolova (Universität Salzburg, AT)

Joint work of Sokolova, Ana; Woracek, Harald
Main reference A. Sokolova, H. Woracek, "Congruences of convex algebras," ASC report 39/2012, TU Vienna.
URL http://www.asc.tuwien.ac.at/preprint/2012/asc39x2012.pdf

We provide a full description of congruence relations of convex, positive convex, and totally convex algebras. As a consequence of this result we obtain that finitely generated convex (positive convex, totally convex) algebras are finitely presentable. Convex algebras, in particular positive convex algebras, are important in the area of probabilistic systems. They are the Eilenberg-Moore algebras of the subdistribution monad.

3.25 Universal properties of finite powersets

Sam Staton (University of Cambridge, GB)

I discussed the two different universal properties of finite powerset functors.

The first universal property is finite powersets that they classify maps that are fibrewise finite, in the sense that to give a function $X \to P(Y)$ is to give a relation $R \subseteq X \times Y$ whose left leg is fibrewise finite. This universal property is similar to the characterization of the Vietoris functor on Stone spaces as the classifier of open maps. One can use this universal property to prove properties of powerset-like functors in an axiomatic way.

The second universal property is that the finite powerset is a free semilattice. This enables us to define maps $X \to P(Y)$ using Moggi's monadic metalanguage. We can get variations on the powerset by varying the algebraic theory. I demonstrated this with a new powerset functor on a presheaf category which is useful for the operational semantics of programs with free variables.

3.26 Bisimilarity is not Borel

Pedro Sanchez Terraf (Universidad Nacional de Córdoba, AR)

Main reference P. Sánchez Terraf, "Bisimilarity is not Borel," arXiv:1211.0967v1 [math.LO].
URL http://arxiv.org/abs/1211.0967v1

In this work in progress, we prove that the relation of bisimilarity between countable labelled transition systems is not Borel, by reducing bounded classes of countable wellorders continuously to it.

This has an impact on the theory of probabilistic and nondeterministic processes over uncountable spaces, since the proofs of logical characterizations of bisimilarity based on the unique structure theorem for analytic spaces require a countable logic whose formulas have measurable semantics. Our reduction shows that such a logic does not exist in the case of image-infinite process.

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3.27 Systematic Construction of Temporal Logics for Dynamical Systems via Coalgebra

Baltasar Trancon y Widemann (Universität Bayreuth, DE)

Joint work of Trancon y Widemann, Baltasar; Hauhs, Michael

Temporal logics are an obvious high-level descriptive companion formalism to dynamical systems which model behaviour as deterministic evolution of state over time. A wide variety of distinct temporal logics applicable to dynamical systems exists, and each candidate has its own pragmatic justification. Here, a systematic approach to the construction of temporal logics for dynamical systems is proposed: Firstly, it is noted that dynamical systems can be seen as coalgebras in various ways. Secondly, a straightforward standard construction of modal logics out of coalgebras, namely Moss's coalgebraic logic, is applied. Lastly, the resulting systems are characterized with respect to the temporal properties they express.

3.28 Two Finitary Functors

Henning Urbat (TU Braunschweig, DE)

The Hausdorff and the Kantorovich functor are widely used to model (probabilistic) nondeterminism from a coalgebraic perspective. We show that both functors are finitary, improving on previous work by van Breugel et. al. In fact, we derive our result from the general observation that all equationally defined free monads are finitary, provided that the underlying category is well-behaved with respect to filtered colimits.

3.29 Regularity and exactness of quasivarieties and varieties of ordered algebras

Jiri Velebil (Czech Technical University, CZ)

Joint work of Kurz, Alexander; Velebil, Jiri

Ordered algebras are algebras for a signature consisting of n-ary operations (n a cardinal) that are interpreted as monotone operations on a poset. Homomorphisms of such algebras are monotone maps preserving the specified operations.

We characterise quasivarieties and varieties of ordered algebras intrinsically. Our characterisation has the same form as in ordinary universal algebra over sets. Namely, we prove the following result:

Theorem: For a category A, the following are equivalent: (i) A is equivalent to a quasivariety (variety, resp.) of ordered algebras. (ii) A is a "regular" ("exact", resp.) category, and there exists an object P such that (a) P has "copowers". (b) P is a "presentable object". (c) P is a "regular projective", "regular generator".

The notions in the theorem above that are in quotes are to be interpreted as notions that are appropriate for category theory enriched over posets. For example, the notion of regularity and exactness of a category enriched in posets is the (suitably modified) notion due to Ross Street .

As an example, the enriched category of posets and monotone maps is an exact category in the above sense (as opposed to an ordinary category of posets: this ordinary category is not even regular in the sense of Michael Barr).

We exemplify the above notions on various examples and we give connections to the categorical notion of monadicity.

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3.30 Automatic sequences as context-free systems.

Joost Winter (CWI – Amsterdam, NL)

Main reference M.M. Bonsangue, J. Rutten, J. Winter, "Defining Context-Free Power Series Coalgebraically," in CMCS 2012: pp. 20–39.
URL http://homepages.cwi.nl/~winter/articles/cmcs12.pdf

We recall some basic results of the coalgebraic approach to the theory of formal languages. We show that all q-automatic sequences are in the class of $\mathbb{F}_q$ context-free streams, and provide a construction of the Thue-Morse sequence as a context-free stream.

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