Report from Dagstuhl Seminar 13311

Duality in Computer Science

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Abstract

Duality allows one to move between the two worlds: the world of certain algebras of properties and a spacial world of individuals, thereby leading to a change of perspective that may, and often does, lead to new insights. Dualities have given rise to active research in a number of areas of theoretical computer science. Dagstuhl Seminar 13311 "Duality in Computer Science" was held to stimulate research in this area. This report collects the ideas that were presented and discussed during the course of the seminar.

Seminar 29. July to 02. August, 2013 – www.dagstuhl.de/13311

1998 ACM Subject Classification F.3.2 Semantics of Programming Languages, F.4.1 Mathematical Logic, F.4.3 Formal Languages, D.2.4 Software/Program Verification

Keywords and phrases Stone-Priestley duality, Point free topology, Infinite computations, Exact real number computation, Computability in analysis, Hierarchies, Reducibility, Topological complexity, Domain theory, Semantics, Recognizability, Profinite topology

Digital Object Identifier 10.4230/DagRep.3.7.54

Edited in cooperation with Hannes Diener

1 Executive Summary

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This seminar concentrated on applications of duality in computation, semantics, and formal languages.

Duality and computation

Consider the area of exact real number computation. Real numbers are abstract infinite objects. Computing machines, on the other hand, can only transform finite objects. However, each real number is uniquely determined by the collection of rational open intervals that contain it, or a certain sub-collection thereof. Rational intervals can be finitely described as a pair of rationals. So, in order to compute with real numbers one has to compute with certain properties, i.e., one no longer works in the space of the reals but in the algebra generated by these properties. In doing so, the open intervals are considered as first-class objects and the concept of point is taken as a derived one. This is exactly the approach of pointless topology which tries to develop analytical concepts in a pointfree way, hereby using constructive logic.

Duality and semantics

In logic, dualities have been used for relating syntactic and semantic approaches. Stone's original result is in fact of this type as it shows that clopen subsets of Stone spaces provide complete semantics for classical propositional logic. This base case has been generalized in various directions. There is a general scheme underlying this work: given a logic, construct its Lindenbaum algebra which in these cases is a Boolean algebra with unary operators. Jonsson-Tarski duality relates such algebras to binary relational structures which in the modal case are just Kripke frames. In this setting, a wide spectrum of duality tools are available, e.g. for building finite models, for obtaining interpolation results, for deciding logical equivalence and other issues. For infinitary logics, Stone-type dualities have also played an important role starting with Scott's groundbreaking first model of the lambda-calculus which is a Stone space. Subsequently Abramsky, Zhang and Vickers developed a propositional program logic, the logic of finite observations. More recently work of Jung, Moshier, and others has evolved this link between infinitary and finitary logics in the setting of logics for computation much further.

Duality and formal languages

The connection between profinite words and Stone spaces was already discovered by Almeida, but Pippenger was the first to formulate it in terms of Stone duality. Gehrke, Pin and Grigorieff lately systematized and extensively developed this discovery which led to new research efforts in formal language theory. A final goal is a general theory of recognition.

The seminar brought together researchers from mathematics, logic and theoretical computer science that share an interest in the fields of computing with infinite data, semantics and formal languages, and/or the application of duality results. The researchers came from 12, mostly European, countries, but also from Argentina, Japan, Russia, South Africa, and the United States.

Some of the specific questions that were investigated in talks and discussions:

- Explore the use of the link between finitary and infinitary Stone dualities in other settings than semantics;
- Explore the link between complexity theory and semantics provided by the connection via duality theory;
- Identify the relationship between game semantics and dual spaces;
- Explore the link between the profinite semi-groups used in formal language theory and logics given by state-based transition systems or categorical models)

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3 Overview of Talks

3.1 Factoriality and the Pin-Reutenauer procedure

Jorge Almeida (University of Porto, PT)

The problem of separation of two rational languages by a language of a special type becomes a classical topological separation problem in the associated profinite completion of the free semigroup on the underlying alphabet. From an algorithmic viewpoint, it is better to work with an intermediate free algebra provided it is sufficiently rich to capture non-emptiness of the intersection of the closures of two rational languages. The natural signatures to consider for such algebraic structures are "implicit" in the sense that semigroup homomorphisms between finite semigroups remain homomorphisms in the enriched signature.

We consider implicit signatures on finite semigroups determined by sets of pseudonatural numbers. We prove that, under relatively simple hypotheses on a pseudovariety V of semigroups, a finitely generated free algebra for the largest such signature is closed under taking factors within the free pro-V semigroup on the same set of generators. We also show that the natural analogue of the Pin-Reutenauer descriptive procedure for the closure of a rational language in the free group with respect to the profinite topology holds for the pseudovariety of all finite semigroups. As an application, we show that the Pin-Reutenauer procedure holds for a pseudovariety of semigroups with respect to the signature consisting of multiplication and pseudo-inversion if and only if the pseudovariety is full with respect to this signature.

3.2 The Sierpinski space as a higher inductive type

Andrej Bauer (University of Ljubljana, SI)

The Sierpinski space, which has two points one of which is open, is the dualizing object for duality between spaces and frames. The distributive lattice structure of Sierpinski space can be axiomatized in lambda calculus, which leads to a connection between topology on one side and logic and computation on the other. This fruitful connection has been used in the past to give logical characterizations of various topological properties, as well as to explain how we can directly compute with topological spaces.

We can ask whether the Sierpinski space can be constructed in type theory. Traditionally this is done with the use of the termination monad or a suitable co-inductive type. But in homotopy type theory we can construct the Sierpinski space as a higher inductive type, which leads to some new possibilities.

3.3 On normal numbers (including new results)

Veronica Becher (University of Buenos Aires, AR)

Joint work of Becher, Veronica; Bugeaud, Yann; Carton, Olivier; Heiber, Pablo; Slaman, Theodore
Main reference V. Becher, P. A. Heiber, T. A. Slaman, "A polynomial-time algorithm for computing absolutely normal numbers," Information and Computation, Vol. 232, Nov. 2013, pp. 1–9, 2013.
URL http://dx.doi.org/10.1016/j.ic.2013.08.013
URL http://www.dc.uba.ar/people/profesores/becher/normal_dagstuhl.pdf

This is a summary of recent result on normal numbers obtained by combining algorithmic, combinatorial and number-theoretic tools.

- Normality to different bases. Joint work by Becher and Slaman, 2013. We show that the discrepancy functions for different bases for which a real number can be normal are pairwise independent. As an application we answer two open questions. One is that the set of real numbers which are normal to at least one base is properly at the fourth level of the Borel hierarchy. The other is to show that that if R and S are a partition of bases closed under multiplicative dependence, then there are real numbers that are normal to each base in R and not simply normal to any base in S.
- On Simple normality Joint work by Becher, Bugeaud and Slaman, 2013. We give the conditions on a subset of positive integers such that there exists a real number that is simply normal to all the elements in the set and not simply normal to none of the element in it complement.
- A polynomial time algorithm for computing normal numbers Joint work by Becher, Heiber and Slaman, 2013. Using combinatorial and basic measure theoretic arguments we obtain al algorithm that computes an absolutely normal number with just above quadratic time complexity. We actually computed an instance.
- Normality and Automata. Joint work by Becher, Carton, Heiber, 2013 We extend the theorem that establishes that a real number is normal to a given base if, and only if, its expansion in that base is incompressible by lossless finite-state compressors, a particular class of finite-state automata. We show that the incompressibility results also hold for non-deterministic automata even if they are augmented by counters.

3.4 A coinductive approach to computable analysis

Ulrich Berger (Swansea University, UK)

We describe continuous real functions by a quantifier free coinductive predicate and show that from a proof of a function f having this property one can extract a memoized implementation of f that operates on real numbers in signed digit representation.

We will show that this is an example of a general approach to computation where one replaces operations on (constructive representations of) mathematical objects by constructive proofs of properties of these objects.

Technically, this approach is based on a realizability interpretation of Church's Simple Theory of Types.

3.5 The Unreasonable Effectiveness of Choice or Metamathematics in the Weihrauch Lattice

Vasco Brattka (Universität der Bundeswehr – München, DE)

URL http://arxiv.org/find/all/1/au:+Brattka_V/0/1/0/all/0/1

The concept of Weihrauch reducibility captures the idea of reducing one problem computationally (or continuously) to another problem. Formally, the reducibility yields a lattice with a rich algebraic structure that has some similarities to the resource oriented interpretation of linear logic. This lattice can be seen as a refinement of the Borel hierarchy for multi-valued functions and it can be used to classify the computational content of mathematical theorems. In this approach for all-exists theorems are just interpreted as multi-valued functions and positioned in the Weihrauch lattice. If Theorem A can be reduced to Theorem B in this lattice, then this means that the input and output data of these theorems can be computably (or continuously) transferred into each other such that a reduction of A to B is obtained. Over the previous 5 year a large number of theorems have been classified in this way by several authors and interesting separation techniques have been developed along the way. A particular role is played by so-called choice principles, which can be used to calibrate not only a large class of theorems, but also several natural classes of functions. The aim of this talk is to present a survey on the classification of theorems in the Weihrauch lattice with some outlook on recent results.

3.6 Complete quasi-metric spaces in computer science, logic, and algebra

Matthew de Brecht (NICT – Osaka, JP)

We will present our research on the descriptive set theory of quasi-Polish spaces, which are defined as the countably based topological spaces which admit a Smyth-complete quasi-metric. Quasi-Polish spaces are a natural generalization of Polish spaces to include important classes of non-metrizable spaces while retaining a rich descriptive set theory. Most countably based spaces in computer science, including omega-continuous domains, are quasi-Polish. From the locale theoretic perspective, quasi-Polish spaces correspond to the locales with a countable presentation in terms of generators and relations. All countably based spectral spaces are quasi-Polish, which suggests the relevance of quasi-Polish spaces to the field of commutative algebra.

3.7 Reverse Reverse Mathematics

Hannes Diener (University of Siegen, DE)

Topological—or more general Heyting-valued—models have been a useful tool to provide counterexamples to various principles in constructive mathematics. In particular, topological models can be used to separate principles that have been investigated in the program of constructive reverse mathematics. The goal of (constructive) reverse mathematics is not to ask what principles are sufficient to prove theorems (normal "forward" mathematics) but also what principles are necessary to prove them.

In this talk we will present some work (very much) in progress that could be labeled "reverse reverse mathematics": namely, we will ask the question what properties the space underlying a topological model needs to satisfy in order to be able to separate between certain principles.

3.8 Ramsey theory and Gelfand duality

Willem Fouché (University of South Africa, ZA)

We discuss non-archimedean groups which are extremely amenable and the role that Ramsey theory plays in identifying these extremely amenable groups. We indicate how Gelfand duality can be used to lead to a better understanding of the constructive content of the topological versions of structural Ramsey theory. For further background:

References

1. W. L. Fouché: "Algorithmic Randomness and Ramsey Properties of Countable Homogeneous Structures. L. Ong and R. de Queiroz (Eds.): WoLLIC 2012, LNCS 7456, pp. 246–256, 2012. Springer-Verlag Berlin Heidelberg.
2. W. L. Foucheé. "Martin-Löf randomness, invariant measures and countable homogeneous structures." Theory of Computing Systems, 52:65–79, 2013.

3.9 Boolean topological algebras as dual spaces

Mai Gehrke (University Paris-Diderot, FR)

Joint work of Gehrke, Mai; Grigorieff, Serge; Pin, Jean-Eric
Main reference M. Gehrke, "Stone Duality, Topological Algebra, and Recognition," HAL-00859717, version 2, 2013.
URL http://hal.archives-ouvertes.fr/hal-00859717

Based on Stone Priestley duality for additional operations on lattices, one can show that topological algebras based on Boolean spaces are the dual spaces of certain Boolean Algebras with additional Operations (BAO). In the special case of the profinite completion of an abstract algebra the dual BAO is always the BA of recognizable subset equipped with the residuals of the lifted operations of the original algebra. This result, as well as duality between sublattices and quotient spaces are the main duality theoretic results behing our joint work with Serge Grigorieff and Jean-Eric Pin generalising various elements of the theory of regular languages.

3.10 Duality for sheaves of distributive-lattice-ordered algebras over stably compact spaces

Sam Van Gool (Radboud University Nijmegen, NL)

A sheaf representation of a universal algebra A over a topological space Y can be viewed as a special subdirect product decomposition of A indexed by Y. Indeed, in case Y is a Boolean space, sheaf representations of A correspond exactly to weak Boolean product decompositions of A. Moreover, if A is a distributive lattice, then Boolean sheaf representations of A correspond to decompositions of the (Stone-Priestley) dual space of A into a Boolean sum, i.e., a disjoint sum indexed over the underlying Boolean space satisfying a certain patching property for the topology on the sum.

We study sheaf representations of distributive lattices over stably compact spaces, which are the natural T_0 spaces associated to compact ordered spaces. In particular, the class of stably compact spaces contains both the class of compact Hausdorff spaces and the class of spectral spaces. We introduce an appropriate condition on sheaves, that we call fitted, which allows us to prove the following:

Theorem. Fitted sheaf representations of a distributive lattice A over a stably compact space Y are in one-to-one correspondence with patching decompositions of the Stone-Priestley dual space of A over the space Y.

Here, a patching decomposition of a topological space X over a space Y is most conveniently described by a continuous map from X to Y which satisfies a certain patching property, reflecting the patching property of the sheaf. If the indexing space Y is Boolean, then such a patching decomposition precisely corresponds to a Boolean sum. However, since stably compact spaces may have a non-trivial specialization order, the correct notion of patching decomposition is no longer a disjoint sum of spaces, but rather corresponds to an ordered sum with an appropriate topological property.

In our recent joint work with V. Marra [1], we showed that sheaf representations of MV-algebras can be obtained via decompositions of the Stone-Priestley dual spaces of their distributive lattice reducts. Thus, the results described here show in particular that the methods employed in [1] are an instance of a more general phenomenon, and open the way for studying sheaf representations of other varieties of distributive-lattice-ordered algebras via decompositions of their dual spaces, which may be indexed over any stably compact space.

References

1 M. Gehrke, S. J. van Gool, and V. Marra, *Sheaf representations of MV-algebras and lattice-ordered abelian groups via duality*, (2013), submitted, preprint available at arXiv:1306.2839.

3.11 Approximation spaces (spaces of Choquet Games), à la domain approach to quasi-Polish spacesy

Serge Grigorieff, (University Paris-Diderot, FR)

Quasi-Polish spaces generalize both Polish spaces and countably based continuous domains. They can be seen as the analog of Polish spaces when Hausdorff T_2 separation axiom is replaced by Kolmogorov T_0 axiom. In a fundamental recent paper, Matthew de Brecht showed that a large part of the theory of Polish spaces admits a counterpart with quasi-Polish spaces.

We introduce another class of topological spaces, extending that of quasi-Polish spaces: approximation spaces. The definition is based on an approximation relation which formalizes a "strong containment relation" between basic open sets such that strongly decreasing chains have a non empty intersection.

An example of an approximation relation is obtained by lifting to basic open sets the way-below (or approximation) relation in a continuous dcpo.

The subclass of convergent approximation spaces is obtained by requiring that every strongly decreasing chain is a fundamental system of neighborhoods of some point.

We prove that quasi-Polish spaces are exactly the T_0 second-countable convergent approximation spaces. This gives an "à la domain" characterization of quasi-Polish spaces.

Approximation spaces are exactly the spaces in which player NonEmpty has a winning memoryless strategy in the Choquet topological game where

- at its i-th-move player Empty plays a nonempty open set U_{2i} and a point x_i in U_{2i}. For $i \geq 1$, the set U_{2i} must be included in U_{2i-1}.
- at its i-th-move player NonEmpty plays a nonempty open set U_{2i+1} which must be included in U_{2i} and contain the point x_i.
- NonEmpty wins if and only if the intersection of the U_j's is nonempty.

A strategy for NonEmpty is convergent if the U_j's is a basis of neighborhoods of some point in the intersection of the U_i's. A strategy for NonEmpty is memoryless if its i-th-th move depends only on the i-th move of Empty.

We prove that a T_0 second-countable space is quasi-Polish spaces if and only if player NonEmpty has a winning convergent strategy if and only if player NonEmpty has a winning memoryless convergent strategy.

3.12 Clarke's Generalized Gradient and Edalat's L-derivative

Peter Hertling (Universität der Bundeswehr – München, DE)

Clarke (1973, 1975, 1983) introduced a generalized gradient for Lipschitz continuous functions on Banach spaces and a notion of upper semicontinuity for set-valued functions. He showed that the function that maps a point to its generalized gradient is upper semicontinuous in this sense if the underlying Banach space is finite-dimensional. Edalat (2005) posed the question whether this is true as well if the Banach space is infinite-dimensional. We show that the answer to this question is in general negative for upper semicontinuity in

Clarke's sense and positive for another notion of upper semicontinuity considered by Edalat. Furthermore, Edalat introduced as so-called L-derivative for real-valued functions, showed that it is identical with Clarke's generalized gradient if the underlying Banach space is finite-dimensional, and asked whether this is true also if the Banach space is infinite-dimensional. We show that this is the case.

3.13 Continuous domain theory in logical form

Achim Jung (University of Birmingham, UK)

In 1987 Samson Abramsky presented "Domain Theory in Logical Form" in the Logic in Computer Science conference. His contribution to the conference proceedings was honoured with the Test-of-Time award 20 years later. In this talk I will begin by giving a brief overview of this seminal work and then focus on three technical issues that (among others) Samson had to overcome in order to make DTLF work. These issues are chosen because they have proved to be particularly difficult to generalise to the continuous case. I will explain what is known about the generalisations and what the key open questions are.

3.14 On the duality between state transformer and predicate transformer semantics

Klaus Keimel (TU Darmstadt, DE)

Domain Theory provides two kinds of semantics for programs. State transformer semantics describes the input–output behavior of programs. Predicate transformer semantics at the contrary assigns to every (desired) property of the output the weakest precondition on the input which guarantees the (desired) property for the output. In several situations the two semantics have been proved to be dually equivalent.

For deterministic programs this is a straightforward but extremely useful observation for the verification of programs. It has been extended to non-deterministic programs by M. B. Smyth. For programs involving probabilistic features this has has been noted by D. Kozen and in a domain theoretical setting by C. Jones. For situations combining probabilistic and ordinary non-determinism, K. Keimel and G. D. Plotkin as well as J. Goubault-Larrecq have also established this dual equivalence. The methods needed for the proofs became more and more involved.

The question arises whether such a dual equivalence between predicate and state transformer semantics may be expected quite generally. The talk will be devoted to this question. We will show that such a dual equivalence can arise in quite special situations only.

We will use the continuation monad over a domain of 'observations' and monads subordinate to the continuation monad. Notions from universal algebra like entropicity will play a role. We will see that the commutativity of the monads is a property which is essential for the desired dual equivalence.

3.15 Perspectives on non-commutative Stone dualities

Ganna Kudryavtseva (University of Ljubljana, SI)

I will outline several approaches to non-commutative generalizations of Stone duality that have been developed in recent years. The general idea can be very briefly explained as follows: we consider an algebra that generalizes a Boolean algebra (or a distributive lattice, or a frame or a coherent frame…) and enquire how the dual topological (localic) object of the commutative structure can be upgraded to dualize the whole algebra. For more details, please see the extended abstract of my talk.

3.16 Endpoints of quasi-metric spaces

Hans-Peter Albert Künzi (University of Cape Town, ZA)

Joint work of Agyingi Collins Amburo; Haihambo Paulus; Kuenzi, Hans-Peter Albert
Main reference C. A. Agyingi, P. Haihambo, H.-P. A. Künzi, "Endpoints in T_0-Quasimetric Spaces: Part II," Abstract and Applied Analysis, Volume 2013, Article ID 539573, 10 pp. , 2013.
URL http://dx.doi.org/10.1155/2013/539573

In his well-known paper dealing with the construction of the injective hull of a metric space Isbell introduced the concept of an endpoint of a compact metric space.

In my talk I shall introduce similarly the notion of an endpoint in a joincompact T_0-quasi-metric space. It turns out that in a joincompact T_0-quasi-metric space there is a dual concept which I shall call a startpoint. Some classical results on endpoints in metric spaces can then be generalized to the quasi-metric setting.

In particular I shall specialize some of the results to the case of two-valued T_0-quasi-metrics, that is, essentially, to partial orders.

3.17 Computational Semantics for Intuitionistic Logic Using the Muchnik Lattice

Rutger Kuyper (Radboud University Nijmegen, NL)

Around 1960, Medvedev and Muchnik introduced two computationally motivated lattices in an attempt to capture the computational content of intuitionistic logic (as suggested by the BHK interpretation). Unfortunately, these two lattices (which are, in fact, Brouwer algebras, i.e. order duals of Heyting algebras) fall short: the satisfy the weak law of the excluded middle $\neg A \vee \neg\neg A$. Surprisingly, in 1988 Skvortsova showed that one can repair this deficiency by looking at a factor of the Medvedev lattice by a principal filter: there is a principal factor of the Medvedev lattice which exactly captures IPC. The analogous result for the Muchnik lattice was recently proven by Sorbi and Terwijn. Unfortunately, the elements generating these principal filters are unnatural, in the sense that they do not have clear computational interpretations. Therefore this does not completely give us computational semantics for IPC.

It turns out that, at least in the case of the Muchnik lattice, there are also natural factors which characterise IPC. I will explain how some well-known concepts from computability theory give us such factors.

3.18 Quasi-uniformities and Duality

Jimmie D. Lawson (Louisiana State University, US)

The duality of stably compact spaces and accompanying dualities of structures they carry have received considerable attention in recent work in the theoretical computer science community and beyond. We consider the duality of stably compact spaces as a special case of a natural duality of quasi-uniform spaces. We give an overview of some important aspects of the latter theory, make explicit connections with stably compact spaces through compactifications, and seek to identify cases where these compactifications can be characterized order theoretically as a special type of completion we call the D-bicompletion.

3.19 Category Theory vs. Universal Algebra in Duality Theory

Yoshihiro Maruyama, (University of Oxford, UK)

Duality theory has been developed mainly in two contexts: category theory and universal algebra. Based upon my recent work in both contexts, I aim at comparing the two ways of duality theory, thereby explicating what are genuine merits of each, and seeking new possibilities to integrate them. Especially, I shed light upon the distinction between finitary Stone dualities (e.g., in logic) and infinitary Stone dualities (e.g., in domain theory), with particular emphasis on links with dualities in algebraic geometry, and their dimension-theoretical consequences.

3.20 A microcosm principle in logic

Paul-Andre Mellies (University Paris-Diderot, FR)

The microcosm principle is a fundamental principle of higher-dimensional algebra, which states that every algebraic structure of dimension n needs an appropriate algebraic framework of dimension $n + 1$ in order to be formulated. Typically, the definition of a monoid in Set requires the category Set itself to be cartesian.

My main purpose in this introductory talk will be to describe a similar microcosm principle in logic. The principle expresses that every universe C of discourse (typically a Heyting algebra or a cartesian closed category) relies on the ability to define an opposite universe C^{op} by reversing the orientation of every logical implication in C. In the same way as a change of frame of reference in galilean mechanics, this involutive operation $C \mapsto^{\text{op}}$ performs a change of point of view between the two sides of the logical dispute. An important point is that every formula A seen in the universe C becomes its negation A^* when seen in the universe C^{op}.

In this way, every logical system (even intuitionistic) becomes equipped with an involutive negation $A \mapsto A^*$. By way of illustration, I will show how to decompose the implication $A \Rightarrow B$ of intuitionistic logic as a disjunction $A^* \vee B$, in exactly the same way as in classical logic. If time permits, I will briefly explain how this analysis is related to tensorial logic and to its purely logical reconstruction of game semantics.

Tentative references for the talk:

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3.21 Wadge-like reducibilities on arbitrary quasi-Polish spaces: a survey.

Luca Motto Ros (Universität Freiburg, DE)

Joint work of Motto Ros, Luca; Schlicht, Philipp; Selivanov, Victor
Main reference L. Motto Ros, P. Schlicht, V. Selivanov, "Wadge-like reducibilities on arbitrary quasi-Polish spaces," arXiv:1204.5338v2 [math.LO], 2013; accepted for publication in Mathematical Structures in Computer Science.
URL http://arxiv.org/abs/1204.5338v2

Wadge reducibility has been introduced as a tool for classifying subsets of zero-dimensional Polish spaces according to their (topological) complexity, and it has found many applications in both set theory (a branch of mathematical logic) and computer science (it is e.g. crucial for the classification of omega-regular languages recognized by various type of automata). In the last years, many mathematicians have worked on extending the analysis of the classical Wadge hierarchy to other kinds of reducibilities between subsets of various topological spaces, including arbitrary Polish spaces and spaces relevant to computer science, like ω-continuous domains. This naturally led to the project (initiated in the Dagstuhl seminar "Computing with infinite data: topological and logical foundations", 2011) of systematically classifying the degree-hierarchies that one obtains by considering Wadge-like reducibilities on arbitrary quasi-Polish spaces, a wide class which contains all the above mentioned spaces. So far we were able to obtain a quite nice picture, covering many interesting cases in a quite uniform way. In this talk I will survey the know results and put in evidence the main open problems in this area of research.

3.22 Epistemic updates on algebras, dynamic logics on an intuitionistic base, and their sequent calculi

Alessandra Palmigiano (University of Amsterdam, NL)

In this talk, I will give an overview of very recent results on dynamic epistemic logic, stemming from the research line which investigates dynamic epistemic logic with the duality toolkit. Each of these results stands on its own and could be presented in isolation; however, I will try and convey the outline of an emerging nonclassical theory of dynamic updates.

3.23 Pervin spaces and duality

Jean-Eric Pin (University Paris-Diderot, FR)

Joint work of Gehrke, Mai; Grigorieff, Serge; Pin, Jean-Eric

This lecture is based on a joint work with M. Gehrke and S. Grigorieff.

A Pervin space is simply a set equipped with a lattice of subsets. This notion suffices to define a natural notion of completion which is equal to the Stone dual of the lattice. Pervin spaces are actually a very special case of quasi-uniform space (to be precise, a quasi-uniform space is a Pervin sapce iff it is transitive and totally bounded). However, the theory becomes much simpler than for quasi-uniform spaces. For instance, a function f from (X, L_X) to (Y, L_Y) (L_X and L_Y are lattices of subsets) is uniformly continuous iff for each L in L_Y, $f^{-1}(L)$ belongs to L_X.

3.24 Relational semantics for the Lambek-Grishin calculus and extensions

Lorijn van Rooijen (University of Bordeaux, FR)

Joint work of Chernilovskaya, Anna; Gehrke, Mai; van Rooijen, Lorijn
Main reference A. Chernilovskaya, M. Gehrke, L. van Rooijen, "Generalised Kripke semantics for the Lambek-Grishin calculus," Logic Journal of the IGPL, 20(6):1110–1132, 2012.
URL http://dx.doi.org/10.1093/jigpal/jzr051

We present relational semantics for a substructural logic called the Lambek-Grishin calculus and various extensions. Following the approach of generalised Kripke semantics described in [2], we consider semantics based on the generalised Kripke frames naturally associated with the algebraic semantics of the logics in question via their representation theory. This approach is based on canonicity and correspondence as in the classical modal logic setting.

The approach via canonical extensions of LG-algebras provides the possibility of a modular treatment of various extensions of the Lambek-Grishin calculus. Additional axioms that lift to the canonical extension give additional first-order properties on the frame, whereas additional connectives modularly slot in as additional relational components.

All groups of additional axioms presented by Grishin in [3] are canonical, and we obtain correspondence results for each of these. The modular set-up allows us to augment these results by the correspondence results for associativity, commutativity, weakening and contraction from [1], and by results for additional connectives such as lattice operations and linear logic-type negation. This allows a clear comparison of the various logics and a fully modular family of completeness results.

References

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3.25 Intrinsic Sobriety

Giuseppe Rosolini (University of Genova, IT)

Joint work of Rosolini, Giuseppe; Bucalo, Anna
Main reference A. Bucalo, G. Rosolini, "Sobriety for equilogical spaces," to appear in Theoret. Comput. Sci.

The category of equilogical spaces, originally introduced by Dana Scott in his fundamental paper on Data Types as Lattices, is a locally cartesian closed extension of the category of topological spaces. Hence in that category, it is straightforward to consider (equilogical) spaces of continuous functions without bothering about suitable topologies. We test the power of this extension with the notion of sober topological space, producing an intrinsic characterization of those topological spaces which are sober in terms of a construction on equilogical spaces of functions.

3.26 Visser topology and other topologies from lambda calculus

Antonino Salibra (University of Venezia, IT)

Of great relevance for the lambda calculus is a topology, introduced by Visser in 1980, for sets which have an "effective" flavor. In the first part of the talk we describe applications of the Visser topology to the syntax and semantics of lambda calculus, and we present connections between the Visser topology and the Priestley topology. In the second part of the talk we introduce new separation axioms for topological algebras which are n-subtractive. n-subtractivity was introduced by the speaker to study the order-incompleteness problem of the lambda calculus.

3.27 Kleene-Kreisel Representations

Matthias Schröder (Universität Wien, AT)

A recent approach to Complexity Theory in Computable Analysis proposes to use representations that have Kleene-Kreisel spaces as their underlying spaces of names. By Kleene-Kreisel spaces we mean those spaces that can be constructed from the discrete natural numbers N by forming finite products, subspaces and function spaces.

We define and investigate Kleene-Kreisel representations. By constructing a Wadge-complete open set in the Kleene-Kreisel space N^{N^N} we prove that QCB-spaces embed into the category of Kleene-Kreisel representations via a functor that preserves binary products and function spaces.

3.28 A Universal Krull-Lindenbaum Theorem

Peter M. Schuster (University of Leeds, UK)

We formulate a natural common generalisation of Krull's theorem on prime ideals and of Lindenbaum's lemma on complete consistent theories. Our theorem not only has instantiations in diverse branches of algebra but also covers Henkin's approach to Gödel's completeness theorem. Following Scott we put the theorem in universal rather than existential form, which allows us to prove it with Raoult's Open Induction in place of Zorn's Lemma. By reduction to the corresponding theorem on irreducible ideals due to Noether, McCoy, Fuchs, and Schmidt, we further shed light on why transfinite methods occur at all.

3.29 Fine Hierarchies via Priestley Duality

Victor Selivanov (A. P. Ershov Institute – Novosibirsk, RU)

Main reference V. Selivanov, "Fine hierarchies via Piestley duality," Annals of Pure and Applied Logic, 163(8):1075–1107, 2012.
URL http://dx.doi.org/10.1016/j.apal.2011.12.029

In applications of the fine hierarchies [1], their characterizations in terms of the so called alternating trees are of principal importance. Also, in many cases a suitable version of many-one reducibility (m-reducibility for short) exists that fits a given fine hierarchy. With a use of Priestley duality we obtain a surprising result that suitable versions of alternating trees and of m-reducibilities may be found for any given fine hierarchy, i.e. the methods of alternating trees and m-reducibilities are quite general, which is of some methodological interest.

Along with the hierarchies of sets, we consider also more general hierarchies of k-partitions and in this context propose some new notions and establish new results, in particular extend the above-mentioned results for hierarchies of sets. Some preliminary results on the hierarchies of k-partitions may be found in [2, 3, 4].

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3.30 Overt Subspaces of $\mathbb{R}^n$

Paul Taylor (UK)

URL http://www.paultaylor.eu/drafts/overtrn.pdf
URL http://www.paultaylor.eu/drafts/overt.bib
URL http://www.paultaylor.eu/drafts/overt-brainstorm
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Overt spaces and subspaces have arisen under different names in several constructive settings: open locales, positive formal spaces and locates subspaces in analysis. However, each discipline has presented it in its own formal notation with little attempt to draw the bigger picture for the benefit of colleagues, students or classical mathematicians.

One difficulty with this notion is that it dissolves into nothing in the classical setting. However, the way in which classical mathematicians handle the issues that we would consider constructively is to make things continuous in a parameter, so my starting point is to say that an overt subspace is a fibre of an open map.

A function $f : X \to Y$ between locally compact metric spaces is continuous and open iff

$$d_{y(x)} \equiv \inf\{d(x, a) \mid f(a) = y\}$$

defines a jointly continuous function $d : X \times Y \to \mathbb{R}$, thereby linking open maps to located subspaces. An overt subspace may be characterised in the same way but without the parameter y.

The basic technical result is that predicates on the lattice of open subspaces of a locally compact metric space X take take unions to disjunctions correspond to upper semicontinuous functions $d : X \to \mathbb{R}$ with the properties that

$$d(x) < r \iff \exists r'.d(x) < r' < r$$
$$d(x) < r \implies \forall \epsilon > 0.\exists x'.d(x') < \epsilon \wedge d(x, x') < r$$
$$d(x) < r \wedge d(x, y) < s \implies d(y) < r + s$$

The second property invites iteration, converging to a limit a, so that

$$d(x) < r \text{ iff } \exists a.d(x, a) < r \wedge d(a) = 0$$

where $d(a) = 0$ means $\forall r > 0.d(a) < r$, and we call such a an *accumulation point*.

If we think of $\diamond U$ as saying that the open subspace "ought" to contain a solution of a problem encoded by the operator $\diamond$ then this result says that it does contain such a solution (accumulation point).

I call this the *tangency theorem*. It is a topological (special) case of the *existence property*.

My characterisation above does not assume that the distance d takes (Euclidean) real values or that the subspace of accumulation points is closed. However, these two additional conditions are equivalent.

The first two of the three properties of d above are satisfied by the estimate that the Newton–Raphson algorithm provides for (the next iteration of) the solution of an equation involving a differentiable function.

This account of overt subspaces is valid

- but trivial in classical point–set topology,
- in intuitionistic local theory,
- in Formal Topology and
- in Abstract Stone Duality.

However, these theorems have different logical strengths, because each of these theories defines a different class of functions.

3.31 Computability, lattices, and logic

Sebastiaan A. Terwijn (Radboud University Nijmegen, NL)

The Medvedev lattice is a structure from computability theory that was introduced in order to obtain a connection with intuitionistic logic, but that also turned out to be interesting in its own right. Nowadays the notion of Medvedev reducibility is used to classify sets of reals from various areas, such as computable analysis, algorithmic randomness, reverse mathematics, and Π^0_1 classes. In this talk we discuss the logics connected to factors of the Medvedev lattice and the closely related Muchnik lattice. Skvortsova proved that there is a factor of the Medvedev lattice that captures intuitionistic propositional logic IPC. In joint work with Sorbi we recently obtained an analog of this theorem for the Muchnik lattice. This makes use of an earlier characterization of the finite intervals of the Muchnik lattice. Finally, we discuss natural factors and a conjecture about the set of complete extensions of PA.

3.32 The separation problem for regular languages

Marc Zeitoun (University of Bordeaux, FR)

Joint work of Place, Thomas; van Rooijen, Lorijn; Zeitoun, Marc
Main reference T. Place, L. van Rooijen, M. Zeitoun, "Separating Regular Languages by Piecewise Testable and Unambiguous Languages," in Proc. of the 38th Int'l Symp. on Mathematical Foundations of Computer Science (MFCS'13), LNCS, Vol. 8087, pp. 729–740, Springer, 2013.
URL http://dx.doi.org/10.1007/978-3-642-40313-2_64

Separation is a classical problem asking whether, given two sets belonging to some class, it is possible to separate them by a set from another class. We discuss the separation problem, and we present separation algorithms of regular languages for several classes of separators.

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