

Realistic Cost for the Model of Coherent Computing

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Abstract

For the model of so-called coherent computing recently proposed by Yamamoto *et al.* [Y. Yamamoto *et al.*, New Gen. Comput. 30 (2012) 327–355], a theoretical analysis of the success probability is given. Although it was claimed as their prospect that the Ising spin configuration problem would be efficiently solvable in the model, here it is shown that the probability of finding a desired spin configuration decreases exponentially in the number of spins for certain hard instances. The model is thus physically unfeasible for solving the problem within a polynomial cost.

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1 Introduction

It has been of long-standing interest to study the ability of analog computing systems to solve computationally difficult problems [1, 2]. It is recently of growing interest to investigate the power of quantum adiabatic time evolution in this direction [3]. Nevertheless, it has been commonly believed, with strong theoretical and numerical evidences, that a desired solution should not be obtained with a sufficiently large probability within polynomial time owing to the exponential decrease in the energy gap between desired and undesired eigenstates during an adiabatic change of Hamiltonians [4, 5, 6, 7, 8, 9].

Recently, Yamamoto *et al.* wrote a series of papers [10, 11, 12] on their model—so called the coherence computing model—of an injection-locked slave laser network, which uses quantum states to some extent in contrast to conventional classical optical computing models [14, 15]. It was claimed to be promising in solving the Ising spin configuration problem [16] and those polynomial-time reducible to this problem faster than known conventional models.

The Ising spin configuration problem has been well-known as a typical NP-hard problem described by an Ising-type Hamiltonian [16]. A typical description is as follows.

Ising spin configuration problem: Given a graph $G = (V, E)$ with set V of vertices and set E of edges, and weighting functions $J : E \rightarrow \{0, \pm 1\}$ and $B : V \rightarrow \{0, \pm 1\}$, find the minimum eigenvalue λ_g of the Hamiltonian $H = \sum_{(ij) \in E} J_{ij} \sigma_{z,i} \sigma_{z,j} + \sum_{i \in V} B_i \sigma_{z,i}$. Here, $\sigma_{z,i}$ is the Pauli Z operator acting on the space of the i th spin (there are $n = |V|$ spin-1/2's).

In an intuitive point of view, the problem is difficult in the sense that the number of given parameters grows quadratically while the number of eigenvalues including multiplicity grows exponentially. Although the Hamiltonian is diagonal in the Z basis, writing it in the matrix form itself takes exponential time. Hereafter, we employ n for representing the input length

of an instance although, precisely speaking, the bit length of an encoded instance is $O(n^2)$. We do not go into the controversy on the definition of the input length [17]. As for known results on the complexity of the problem, it becomes P in case the graph is a planer graph and $B_i = 0 \forall i$ (see Ref. [18]); for nonplaner graphs, it is in general NP-hard, and it is so under many different conditions [18]. In addition, a planer graph together with nonzero B_i 's also makes the problem NP-hard [16]. It is also worthwhile to mention that the typical value of λ_g is $c_g n$ with coefficient c_g (so-called the ground-state energy density) typically between -2 and $-1/2$ when the values of J_{ij} are chosen in a certain random manner and B_i are set to zero [19, 20, 21, 22, 23, 24, 25, 26] (c_g is between -1.5 and -1 when the graph is a ladder and J_{ij} and B_i are randomly chosen from $\{\pm 1\}$ [27]). Furthermore, it should be mentioned that the distribution of eigenenergies of H (namely, the envelope of the multiplicity of eigenenergies with a normalization) is a normal distribution with mean zero and standard deviation proportional to \sqrt{n} in the random energy model [22, 33, 25]. Here, the important observation is that the standard deviation increases with n in spite of the exponentially increasing number of spin configurations.

Let us also introduce the NP-complete variant of the Ising spin configuration problem as follows.

NPC Ising spin configuration problem:

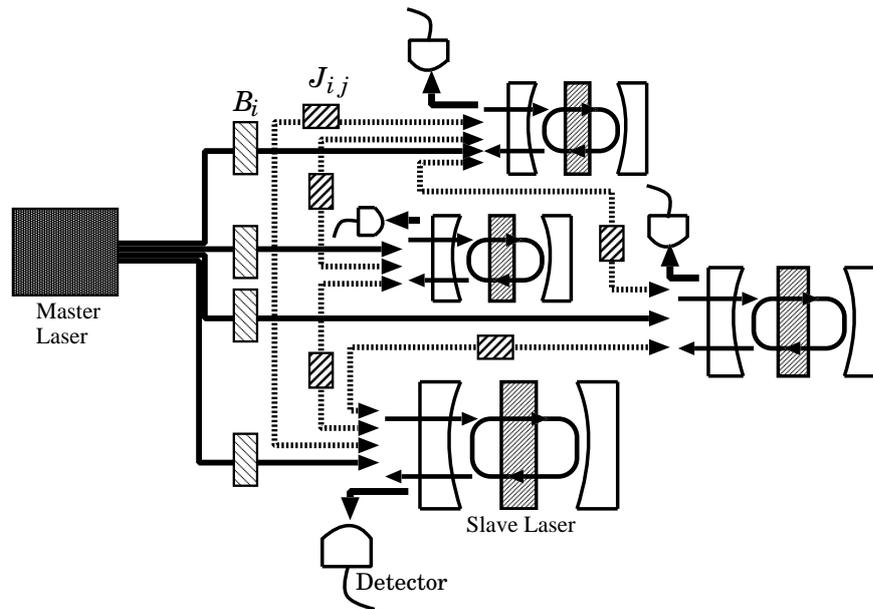
Instance: Positive integer n , integer K , and parameters $J_{ij} \in \{0, \pm 1\}$ ($i < j$) and $B_i \in \{0, \pm 1\}$ for integers $0 \leq i, j \leq n - 1$.

Question: Is there an eigenvalue λ of the Hamiltonian $H = \sum_{i,j=0;i<j}^{n-1} J_{ij} \sigma_{z,i} \sigma_{z,j} + \sum_{i=0}^{n-1} B_i \sigma_{z,i}$ such that $\lambda < K$?

This is the problem we are going to investigate in this contribution as for its computational difficulty under the coherent computing model.

Let us now briefly look into Yamamoto *et al.*'s coherent computing model [10, 11, 12] which is schematically depicted as Fig. 1. It has one master laser whose output is split into n paths and injected to n slave lasers. Each slave laser is initially locked to the superposed state $(|R\rangle_i + |L\rangle_i)$ where $|R\rangle$ and $|L\rangle$ are the right and left circular polarized states (see, *e.g.*, Refs. [28, 29] for physics of the injection-locked laser system). The initial state of the n slave lasers is therefore $\bigotimes_{i=0}^{n-1} (|R\rangle_i + |L\rangle_i)$. The laser network is a macroscopic system; thus initially it holds many photons in this same state. The computational basis is set to $\{|R\rangle, |L\rangle\}^n$ and σ_z is written as $|R\rangle\langle R| - |L\rangle\langle L|$. The i th slave laser and the j th slave laser are connected for nonzero J_{ij} . At time $t = 0$, they mutually inject a small amount of horizontally polarized signal via an attenuator, a phase shifter, and a horizontal linear polarizer, which determine the amplitude attenuation coefficient that is regarded as J_{ij} . Among the three instruments, the attenuator's transmission coefficient controls $|J_{ij}|$ and the other instruments controls $\text{sgn} J_{ij}$. In addition, a small amount of injection of horizontally polarized signal is also made from the master laser to each slave laser at $t = 0$. This amount corresponds to B_i for the i th slave laser. It is controlled by the combination of a half-wave plate and a quarter wave plate. For more details of implementation of the coefficients, see section 7 of Utsunomiya *et al.* [10].

Then one waits for a small time duration t_{st} to let the system evolve. Laser modes satisfying the matching condition with the above-mentioned setting grow rapidly and other modes are suppressed. For $t > t_{\text{st}}$, the system is thought to be in a steady state. Then for each slave laser its output is guided to a polarization beam splitter and the right and the left polarization components are separately detected by photodetectors. By a majority vote of photon number counting, the computational result of each slave laser, $|a\rangle_i \in \{|R\rangle, |L\rangle\}$, is



■ **Figure 1** Schematic description of the coherent computing model. See the text for how J_{ij} and B_i are realized by optical instruments.

retrieved. The steady state $|a\rangle_0 \cdots |a\rangle_{n-1}$ is thus determined. Once this is determined, it takes only polynomial time to calculate the corresponding eigenvalue since there are only $O(n^2)$ terms in the Hamiltonian (here, we do not use its matrix form).

Thus, in short, the state starts from $(|R\rangle + |L\rangle)^{\otimes n}$ and eventually reaches a steady state representing a configuration that corresponds to the minimum energy of the given Hamiltonian. Yamamoto *et al.* [10, 11, 12] employed rate equations involving several factors characterizing each oscillator and connections with other oscillators to analyze photon numbers of the right and left polarization components for each slave laser; they concluded that the system reaches a steady state within 10 nano seconds without obvious dependence on n .

It has been unknown so far if the coherent computing model is a valid computer model in view of a rigid and fair description of computational costs. Conventional analog computing models do not solve NP-hard problems within a polynomial cost; they require either exponentially long convergence time or exponentially fine accuracy [13]. Thus it should be natural to be skeptical against the power of the coherent computing model. In this contribution, we investigate the signal per noise ratio in the output of the coherent computer when the NPC Ising spin configuration problem is handled. We will reach the fact that for certain hard instances, the relative signal intensity corresponding to solutions is bounded above by a function decreasing exponentially in n . This is because the number of modes that are possibly locked in the laser network increases rapidly in n owing to the fact that the locking range of the laser network does not shrink as n grows considering imperfectness of optical instruments.

The analysis of computational difficulty is described in Sec. 2. The result is discussed in Sec. 3 and summarized in Sec. 4.

2 Computational difficulty in the coherent computing model

The coherent computing model illustrated in Fig. 1 was so far analyzed by Utsunomiya *et al.* [10, 11, 12] on the basis of the assumption that given coefficients J_{ij} and B_i are exactly implemented by optical instruments although fluctuations and quantum noise in the system were considered in their analyses of time evolutions using rate equations, which led to a quite ideal convergence taking only 10 nano seconds.

Here, we assume that individual optical instruments are imperfect¹ so that there are errors in J_{ij} and B_i , which are due to calibration errors and/or thermal fluctuations. Then the following proposition is achieved.

► **Proposition 1.** *Consider the NPC Ising spin configuration problem. Suppose calibration errors and/or thermal fluctuations of optical instruments cause nonzero physical deviations,¹ $\varepsilon_{ij} \in \mathbf{R}$ for nonzero J_{ij} and $\kappa_i \in \mathbf{R}$ for nonzero B_i . We assume that ε_{ij} are i.i.d. random variables with mean zero and a certain standard deviation σ_ε and κ_i are i.i.d. random variables with mean zero and a certain standard deviation σ_κ . Then, for large n , there exist YES instances such that the probability to obtain a spin configuration corresponding to one of λ 's $< K$ using the coherent computer is $\leq \text{poly}(n)2^{-n}$.*

Proof. Here we consider instances generated in the way that J_{ij} 's and B_i 's are independent uniformly distributed random variables with values in $\{0, \pm 1\}$. Since a problem instance is a given data set, the standard deviation for J_{ij} and that for B_i intrinsic to the problem instance itself are not of our concern. We only consider physical deviations as errors.

As the model is a sort of a bulk model (there are many photons), it is convenient to consider populations of individual configurations. Let $P_{\lambda, l_\lambda}(t)$ be the population of eigenstate $|\varphi_{\lambda, l_\lambda}\rangle$ ($l_\lambda \in \{0, \dots, d_\lambda - 1\}$) corresponding to eigenenergy λ of the Hamiltonian (specified by the problem instance), where t stands for time and d_λ is the multiplicity of λ . We also introduce $P_\lambda(t) = \sum_{l_\lambda=0}^{d_\lambda-1} P_{\lambda, l_\lambda}(t)$. It should be kept in mind that we do not start from the thermal distribution; for the initial state, we have identical copies of $\sum_\lambda \sum_{l_\lambda} |\varphi_{\lambda, l_\lambda}\rangle = (|R\rangle + |L\rangle)^{\otimes n}$. In the present setting, the random-energy model [22, 33] is valid² and hence, for large n , with an appropriate scaling factor M , one can write $P_\lambda(0) = MN(0, \sigma_\lambda^2)$ with $\sigma_\lambda = \Theta(\sqrt{n})$ where $\mathcal{N}(\mu, \sigma^2)$ is the density function of the normal distribution with mean μ and standard deviation σ . Here, we have $M = 2^n P_{\lambda_g, 0}(0)$ with λ_g the ground state energy because the initial population is same for all the configurations.

Let us denote the set of solution states (spin configurations corresponding to λ 's $< K$) as Y . The total population of solution states at t is given by $P_Y(t) = \sum_{\lambda < K} P_\lambda(t)$. Similarly, the total population of nonsolution states is given by $P_X(t) = \sum_{\lambda \geq K} P_\lambda(t)$; here, $X = \{|\varphi_{\lambda, l_\lambda}\rangle \mid \lambda \geq K\}$. Ideally, only $|\varphi_{\lambda, l_\lambda}\rangle$'s $\in Y$ will enjoy population enhancement by mode selections. However, there exists $v \geq K$ such that $P_\lambda(t > t_{\text{st}}) \gg 0$ for $\lambda \leq v$. This is because

¹ It is a common case that each optical instrument has a few permil uncertainty in the calibration of each property (see Ref. [30]). In addition, there is a quantum limit in any classical instrument [31, 32] so that a manufacturing error and a manipulation error cannot be made arbitrarily small.

² Let us pick up a certain configuration $|\varphi\rangle$. Suppose, by applying m bit flips, its energy changes by $\Delta E(\varphi \xrightarrow{m} \varphi')$ with $|\varphi'\rangle$ a resultant configuration. This process should obey the random energy change and hence for large m , $\Delta E(\varphi \xrightarrow{m} \varphi')$ should obey the normal distribution with mean zero and a standard deviation proportional to \sqrt{m} by the central limit theorem (in regard with a sum of random variables). In addition, the most typical number of bit flips is $n/2$ when we generate all other configurations from $|\varphi\rangle$. Typical bit flips generate a dominant number of configurations. Thus the distribution of energies is approximated by the normal distribution with mean zero and a standard deviation proportional to \sqrt{n} . In this way, we have just obtained the energy distribution function in the random-energy model.

the matching condition is imperfect in reality; the locking range is broader than the ideal range considering errors in optical instruments.³ Let us write $P_Z(t) = \sum_{K \leq \lambda \leq v} P_\lambda(t)$; here, $Z = \{|\varphi_{\lambda,l_\lambda}\rangle \mid K \leq \lambda \leq v\}$.

By assumption, we are considering physical deviations (including calibration errors and thermal fluctuations), ε_{ij} for nonzero J_{ij} and κ_i for nonzero B_i . The Hamiltonian implemented on the laser network is written as $\tilde{H} = \sum_{i < j | J_{ij} \neq 0} (J_{ij} + \varepsilon_{ij}) \sigma_{z,i} \sigma_{z,j} + \sum_{i | B_i \neq 0} (B_i + \kappa_i) \sigma_{z,i}$. This suggests that $v = K + K'(n)$ with $K'(n) \simeq \sigma_\varepsilon \sqrt{n(n-1)/3} + \sigma_\kappa \sqrt{2n/3}$ by the central limit theorem in regard with a sum of random variables (see, e.g., Refs. [35, 36]), considering the expected number of nonzero J_{ij} 's and that of nonzero B_i 's. Therefore, $P_Z(0) = M \int_K^{K+K'(n)} \mathcal{N}(0, \sigma_\lambda^2) d\lambda$.

Let us write $H = H_J + H_B$ with $H_J = \sum_{i < j} J_{ij} \sigma_{z,i} \sigma_{z,j}$ and $H_B = \sum_i B_i \sigma_{z,i}$. As we have mentioned, it is known [19, 20, 21, 22, 23, 24, 25, 26] that the ground state energy of H_J is typically $c_g n$ with $-2 < c_g < -1/2$. Therefore, for any normalized vector $|v\rangle$ in the Hilbert space of the system of our concern, $\langle v | H | v \rangle$ is typically bounded below by $-3n$. Thus, for typical instances we can choose $K = K(n)$ with $-K(n) = O(n)$. Recall that $K'(n) = \Theta(n)$ and $\sigma_\lambda = \Theta(\sqrt{n})$. We find that $\int_K^{K+K'(n)} \mathcal{N}(0, \sigma_\lambda^2) d\lambda = \left[\frac{1}{2} \operatorname{erf}\left(\frac{\lambda}{\sqrt{2}\sigma_\lambda}\right) \right]_K^{K+K'(n)}$ is a monotonically increasing function of n . Hence, for a certain constant $b > 0$, $P_Z(0) \geq b2^n P_{\lambda_g,0}(0)$.

Let us assume that locked modes have equally enhanced intensities for $t > t_{\text{st}}$. This leads to the signal per noise ratio for $t > t_{\text{st}}$: $P_Y(t > t_{\text{st}})/P_Z(t > t_{\text{st}}) = P_Y(0)/P_Z(0)$. (In case one can assume that only one of $|\varphi_{\lambda,l_\lambda}\rangle$'s in $Y \cup Z$ survives, the ratio of the probability of finding $|\varphi_{\lambda,l_\lambda}\rangle$ originated from Y and that of finding $|\varphi_{\lambda,l_\lambda}\rangle$ originated from Z at $t > t_{\text{st}}$ is given by the same equation.)

Consider some typical instances for which d_g is small and is not clearly dependent on n (d_g is the multiplicity in the ground level). This is a typical situation because the multiplicity of λ obeys the distribution $\mathcal{N}(0, \sigma_\lambda^2)$ with $\sigma_\lambda = \Theta(\sqrt{n})$ in the present setting, as we have explained. It is always possible to choose⁴ the value of K such that all $|\varphi_{\lambda,l_\lambda}\rangle \in Y$ are configurations with at most a constant number of bits different from one of the ground states. In this case, $P_Y(0) = \text{poly}(n) P_{\lambda_g,0}(0)$ and thus, for large n , $P_Y(t > t_{\text{st}})/P_Z(t > t_{\text{st}}) \leq \text{poly}(n) 2^{-n}$. ◀

► **Remark.** It is trivial to find a similar proof for the existence of hard instances of the Ising spin configuration problem for finding a ground level in the coherent computing model.

By Proposition 1, it is now easy to prove the following theorem.

► **Theorem 1.** *There exists an instance of the NPC Ising spin configuration problem such that a decision takes $\Omega(2^n/\text{poly}(n))$ time in the coherent computing model when nonzero physical deviations,¹ $\varepsilon_{ij} \in \mathbf{R}$ for nonzero J_{ij} and $\kappa_i \in \mathbf{R}$ for nonzero B_i , are considered. Here, ε_{ij} (κ_i) are assumed to be i.i.d. random variables with zero mean and a certain standard deviation σ_ε (σ_κ).*

Proof. By Proposition 1, there exists an YES instance such that the probability p_s for a single trial of coherent computing to find $\lambda < K$ is $\leq \text{poly}(n) 2^{-n}$. The success probability after τ trials is given by $1 - (1 - p_s)^\tau$. In order to make this probability larger than a certain constant $c > 0$, we need $\tau > \log(1 - c) / \log(1 - p_s) = (\log \frac{1}{1-c}) / [p_s + \mathcal{O}(p_s^2)] = \Omega(2^n/\text{poly}(n))$. ◀

³ See, e.g., Ref. [34] for an experimental gain curve.

⁴ Recall that we are proving the *existence* of hard instances.

3 Discussion

We have theoretically shown a weakness of the coherent computing model for the problem to examine the existence of a suitably small (large negative) eigenvalue of an Ising spin glass Hamiltonian. As the number n of spins grows, the desired signal decreases exponentially for certain hard instances because exponentially many undesired configurations obtain gains in a realistic setting.

Indeed, Yamamoto *et al.* made numerical simulations [10, 11, 12] to examine their prospect that a desired configuration would be found efficiently in the coherent computing model. But, in general, the following points should be taken into account whenever a computer simulation of a physical system is performed.

First, in classical computing, exponentially fine accuracy is achievable by linearly increasing the register size of a variable or an array size of combined variables. Nevertheless, in physical systems, noise decreases as $\propto 1/\sqrt{T}$ with T the number of trials or the number of identical systems according to the well-known central limit theorem. In the field of quantum computing, this has been well-studied in the context of NMR bulk-ensemble computation at room temperature which suffers from exponential decrease of signal intensity corresponding to the computation result as the input size grows (see, *e.g.*, [37, 38]). In the coherent computing model, the ratio of the population of correct configurations and that of wrong configurations at the steady state should not decrease in a super-polynomial manner if the model were physically feasible for solving the problem efficiently. So far, Yamamoto *et al.* reported [10, 11, 12] that each slave laser maintains a sufficiently large discrepancy between the populations of $|R\rangle$ and $|L\rangle$ at the steady state for some instances with a small number of spins ($n \leq 10$), using a simulation based on rate equations. They also showed their simulation results for $n = 1000$ for a very restricted type of instances such that J_{ij} 's take the same value and B_i 's for odd i take the same value and so do for even i . Nevertheless, the populations (in other words, the joint probabilities) of correct and wrong configurations and how they scale for large n were not reported. Recently, Wen [39] showed his simulation results for the case where the graph was a two-layer lattice for n up to 800. Although it was reported that his simulations of the coherent computer found eigenvalues lower than those found by a certain semidefinite programming method, the populations of correct and wrong configurations were not shown. Thus, it is difficult to discuss the power of the coherent computing model on the basis of presently known simulation results.

Second, the coefficients of a problem Hamiltonian cannot be implemented as they are, in reality. Seemingly negligible errors in the coefficients might be crucial in complexity analyses for a large input size. This point has not been considered in conventional simulation studies [11, 12, 39] of the coherent computing model. In the coherent computing model, nonzero J_{ij} 's and nonzero B_i 's in the Ising spin glass Hamiltonian should accompany calibration errors and/or thermal fluctuations. In particular, optical instruments usually have nonnegligible calibration errors [30]. As we have written in the proof of Proposition 1, a well-known application of the central limit theorem for the sum of random variables [35, 36] indicates the important observation that the sum of such physical deviations is an increasing function of the number of spins. This fact has led to our conclusion that the relative population of desired configurations decreases exponentially in n for certain hard instances.

The second point is also usually overlooked in computer simulations [3] of adiabatic quantum computing. Discussions on the complexity of adiabatic time evolution are usually made as to how long time should be spent in light of a minimum energy gap between the ground state and the nearest excited state during adiabatically changing the Hamiltonian

toward its final form. The coefficients in the starting and the final Hamiltonians are quite often considered to be given accurate numbers [9]. Nevertheless, they should have certain errors due to imperfect calibrations [30] and/or fluctuations in reality, as we have discussed. The target state will not appear as a stable state if a nontarget state of the final Hamiltonian becomes a ground state of the Hamiltonian owing to the errors. A real physical setup for adiabatic quantum computing should suffer from the demand of considerably fine tuning of individual apparatus to implement desired coupling for large n . So far, n has not been very large in physical implementations [40, 41, 42] so that this problem has not been significant. (In addition, even under the setting without error in Hamiltonian coefficients, adiabatic quantum computing tends to suffer from exponentially decreasing energy gap when random instances of certain NP-hard problems are tried, according to the numerical analysis by Farhi *et al.*[9])

A possible way to avoid very fine tuning is to use error correction schemes similar to those for standard circuit-model quantum computing. There have been several studies on error correction codes [43] and dynamical decoupling [44, 45] in the context of adiabatic quantum computing. It is of interest if similar schemes apply to the coherent computing model. As for error correction codes, each Pauli operator in an original Hamiltonian should be encoded to a certain multi-partite coupling term in an encoded Hamiltonian. Thus one needs to find a scheme to implement such a term in the coherent computing model. It is highly nontrivial to introduce, *e.g.*, a four-partite coupling among slave lasers. Further investigation is needed for the usability of error correction codes. Another scheme is dynamical decoupling. This scheme looks effective for suppressing thermal fluctuations at a glance. Consider the minimum gap between two distinct eigenvalues of a problem Hamiltonian and normalize it with the maximum gap. This decreases only polynomially in n for any instance of the Ising spin configuration problem by the definition of the problem. Thus the minimum operation interval of dynamical decoupling required for an effective noise suppression decreases only polynomially in n according to Eq. (52) of Ref. [46]. One problem is how to use this scheme for cancelling calibration errors. In addition, we need to find an implementation of the scheme such that the scheme itself does not introduce an uncontrollable noise. This will be difficult for large n because imperfections in decoupling operations probably lead to a similar argument as Proposition 1.

As we have proved, there are hard instances of the NPC Ising spin configuration problem for which one cannot efficiently achieve a correct decision in the coherent computing model (Theorem 1). This is a reasonable result in light of the fact that no known conventional computer model could solve an NP-complete problem within a polynomial cost. It is still an open problem if an unreasonable computational power is achievable by combining error protection schemes with the coherent computing model.

4 Conclusion

The model of coherent computing has been theoretically investigated in view of computational cost under a realistic setting. It has been proved that there exist hard instances of the NPC Ising spin configuration problem, which require exponential time for a correct decision in the model.

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References

- 1 T. Roska, L. O. Chua, The CNN Universal Machine: An Analogic Array Computer, *IEEE Trans. Circuits Sys. II* 40 (1993) 163-173.
- 2 M. Ercsey-Ravasz, T. Roska, and Z. Néda, Cellular neural networks for NP-hard optimization, in the 11th International Workshop on Cellular Neural Networks and their Applications (CNNA2008), Santiago de Compostela, Spain, 14-16 July 2008, pp.52-56; *ibid.*, *EURASIP J. Adv. Signal Process.* 2009 (2009) 646975.
- 3 E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, and D. Preda, A Quantum Adiabatic Evolution Algorithm Applied to Random Instances of an NP-Complete Problem, *Science* 292 (2001) 472-475.
- 4 W. van Dam, M. Mosca, and U. Vazirani, How Powerful is Adiabatic Quantum Computation? in: *Proceedings of the 42nd IEEE Symposium on Foundations of Computer Science (FOCS 2001)*, Las Vegas, NV, 14-17 October 2001 (IEEE, New York, 2001) pp.279-287.
- 5 M. Žnidarič, Scaling of the running time of the quantum adiabatic algorithm for propositional satisfiability, *Phys. Rev. A* 71 (2005) 062305.
- 6 M. Žnidarič and M. Horvat, Exponential complexity of an adiabatic algorithm for an NP-complete problem, *Phys. Rev. A* 73 (2006) 022329.
- 7 M. H. S. Amin, Effect of Local Minima on Adiabatic Quantum Optimization, *Phys. Rev. Lett.* 100 (2008) 130503.
- 8 I. Hen and A. P. Young, Exponential complexity of the quantum adiabatic algorithm for certain satisfiability problems, *Phys. Rev. E* 84 (2011) 061152.
- 9 E. Farhi, D. Gosset, I. Hen, A. W. Sandvik, P. Shor, A. P. Young, and F. Zamponi, Performance of the quantum adiabatic algorithm on random instances of two optimization problems on regular hypergraphs, *Phys. Rev. A* 86 (2012) 052334.
- 10 S. Utsunomiya, K. Takata, and Y. Yamamoto, Mapping of Ising models onto injection-locked laser systems, *Opt. Exp.* 19 (2011) 18091.
- 11 K. Takata, S. Utsunomiya, and Y. Yamamoto, Transient time of an Ising machine based on injection-locked laser network, *New J. Phys.* 14 (2012) 013052.
- 12 Y. Yamamoto, K. Takata, and S. Utsunomiya, Quantum Computing v.s. Coherent Computing, *New Gen. Comput.* 30 (2012) 327-355.
- 13 S. Aaronson, Guest Column: NP-complete problems and physical reality, *ACM SIGACT News*, 36(1) (2005) 30-52.
- 14 N. T. Shaked, S. Messika, S. Dolev, and J. Rosen, Optical solution for bounded NP-complete problems, *Appl. Opt.* 46 (2007) 711-724.
- 15 S. Dolev, T. Haist, and M. Olteans, Eds., *Optical Supercomputing*, 1st Int. Workshop, Vienna, Austria, 26 August 2008, *Proceedings, LNCS 5172* (Springer, Berlin, 2008).
- 16 F. Barahona, On the computational complexity of Ising spin glass models, *J. Phys. A: Math. Gen.* 15 (1982) 3241-3253.
- 17 T. E. O'Neil, The Importance of Symmetric Representation, in: *Proceedings of the 2009 International Conference on Foundations of Computer Science (FCS'09)*, Las Vegas, NV, 13-16 July 2009 (CSREA Press, USA, 2009) pp.115-119.
- 18 S. Istrail, Statistical mechanics, three-dimensionality and NP-completeness: I. Universality of intractability for the partition function of the Ising model across non-planar surfaces, in: *Proceedings of the Thirty-Second Annual ACM Symposium on Theory of Computing (STOC'00)*, Portland, OR, 21-23 May 2000 (ACM, New York, 2000) pp.87-96.
- 19 J. Vannimenus and G. Toulouse, Theory of the frustration effect: II. Ising spins on a square lattice, *J. Phys. C: Solid State Phys.* 10 (1977) L537-L542.
- 20 S. Kirkpatrick, Frustration and ground-state degeneracy in spin glasses, *Phys. Rev. B* 16 (1977) 4630-4641.

- 21 I. Morgenstern and K. Binder, Magnetic correlations in two-dimensional spin-glasses, *Phys. Rev. B* 22 (1980) 288-303.
- 22 B. Derrida, Random-energy model: Limit of a family of disordered models, *Phys. Rev. Lett.* 45 (1980) 79-82.
- 23 B. Derrida, Random-energy model: An exactly solvable model of disordered systems, *Phys. Rev. B* 24 (1981) 2613-2626.
- 24 C. De Simone, M. Diehl, M. Jünger, P. Mutzel, G. Reinelt, and G. Rinaldi, Exact ground states of Ising spin glasses: New experimental results with a branch-and-cut algorithm, *J. Stat. Phys.* 80 (1995) 487-496.
- 25 A. Andreatov, F. Barbieri, and O.C. Martin, Large deviations in spin-glass ground-state energies, *Eur. Phys. J. B* 41 (2004) 365-375.
- 26 S. Boettcher, Simulations of ground state fluctuations in mean-field Ising spin glasses, *J. Stat. Mech.* 2010 (2010) P07002.
- 27 T. Kadowaki, Y. Nonomura, and H. Nishimori, Exact ground-state energy of the Ising spin glass on strips, *J. Phys. Soc. Jpn.* 65 (1996) 1609-1616.
- 28 H. Haken, H. Sauermann, Ch. Schmid, and H. D. Vollmer, Theory of Laser Noise in the Phase Locking Region, *Z. Phys.* 206 (1967) 369-393.
- 29 H. A. Haus and Y. Yamamoto, Quantum noise of an injection-locked laser oscillator, *Phys. Rev. A* 29 (1984) 1261-1274.
- 30 The SP 250 Series on NIST Measurement Services, http://www.nist.gov/calibrations/sp250_series.cfm, see *e.g.*, SP 250-64: R. W. Leonhardt, Calibration Service for Low-level Pulsed-Laser Radiometers at 1.06 μm : Pulse Energy and Peak Power.
- 31 A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Introduction to quantum noise, measurement, and amplification, *Rev. Mod. Phys.* 82 (2010) 1155-1208.
- 32 M. D. LaHaye, O. Buu, B. Camarota, and K. C. Schwab, Approaching the Quantum Limit of a Nanomechanical Resonator, *Science* 304 (2004) 74-77.
- 33 B. Derrida, A generalization of the Random Energy Model which includes correlations between energies, *J. Physique Lett.* 46 (1985) L-401-L-407.
- 34 S. Kobayashi and T. Kimura, Injection Locking in AlGaAs Semiconductor Laser, *IEEE J. Quant. Ele. QE-17* (1981) 681-689.
- 35 A. N. Shiryaev, *Probability* (2nd ed., translated by R. P. Boas, Springer-Verlag, New York, 1996).
- 36 A. Klenke, *Probability Theory: A Comprehensive Course* (Springer-Verlag, London, 2008).
- 37 E. Knill, I. Chuang, and R. Laflamme, Effective pure states for bulk quantum computation, *Phys. Rev. A* 57 (1998) 3348-3363.
- 38 A. SaiToh and M. Kitagawa, Matrix-product-state simulation of an extended Brüscheiler bulk-ensemble database search, *Phys. Rev. A* 73 (2006) 062332.
- 39 K. Wen, Injection-locked laser network for solving NP-complete problems, PhD Thesis, Stanford University (2012), <http://purl.stanford.edu/xp446hc0861>.
- 40 M. Steffen, W. van Dam, T. Hogg, G. Breyta, and I. Chuang, Experimental Implementation of an Adiabatic Quantum Optimization Algorithm, *Phys. Rev. Lett.* 90 (2003) 067903.
- 41 X. Peng, Z. Liao, N. Xu, G. Qin, X. Zhou, D. Suter, and J. Du, Quantum Adiabatic Algorithm for Factorization and Its Experimental Implementation, *Phys. Rev. Lett.* 101 (2008) 220405.
- 42 M. W. Johnson *et al.*, Quantum annealing with manufactured spins, *Nature* 473 (2011) 194-198.
- 43 S. P. Jordan, E. Farhi, and P. W. Shor, Error-correcting codes for adiabatic quantum computation, *Phys. Rev. A* 74 (2006) 052322.

- 44 D. A. Lidar, Towards Fault Tolerant Adiabatic Quantum Computation, *Phys. Rev. Lett.* 100 (2008) 160506.
- 45 G. Quiroz and D. A. Lidar, High-fidelity adiabatic quantum computation via dynamical decoupling, *Phys. Rev. A* 86 (2012) 042333.
- 46 H. K. Ng, D. A. Lidar, and J. Preskill, Combining dynamical decoupling with fault-tolerant quantum computation, *Phys. Rev. A* 84 (2011) 012305.