Report from Dagstuhl Seminar 13421

Algorithms for Optimization Problems in Planar Graphs

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 13421 "Algorithms for Optimization Problems in Planar Graphs". The seminar was held from October 13 to October 18, 2013. This report contains abstracts for the recent developments in planar graph algorithms discussed during the seminar as well as summaries of open problems in this area of research.

Seminar 13.–18. October, 2013 – www.dagstuhl.de/13421
1998 ACM Subject Classification F.2 Analysis of Algorithms and Problem Complexity
Keywords and phrases Algorithms, planar graphs, theory, approximation, fixed-parameter tractable, network flow, network design, kernelization
Digital Object Identifier 10.4230/DagRep.3.10.36
Edited in cooperation with Kyle Fox

1 Executive Summary

Glencora Borradaile
Philip Klein
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Planar graphs, and more generally graphs embedded on surfaces, arise in applications such as road map navigation and logistics, computational topology, graph drawing, and image processing. There has recently been a growing interest in addressing combinatorial optimization problems using algorithms that exploit embeddings on surfaces to achieve provably higher-quality output or provably faster running times. New algorithmic techniques have been discovered that yield dramatic improvements over previously known results. In addition, results have been generalized to apply to other families of graphs: excluded-minor, bounded-genus and bounded-treewidth graphs.

This Dagstuhl seminar brought together researchers who have been working in these areas to present recent research results, consolidate and share understanding of the emerging basic techniques, and collaborate to move past the current barriers.

- **Polynomial-time solvable problems.** There is a long tradition of finding fast algorithms for poly-time problems in planar graphs. In 1956, the first paper on maximum *st*-flow addressed the case where the network is planar (and s and t are adjacent). In 1976, a linear-time algorithm was given for minimum spanning trees in planar graphs. In 1979, the paper introducing generalized nested dissection gave a fast algorithm for shortest paths in planar graphs with positive and negative lengths. The past couple of decades has witnessed the discovery of fast algorithms for a wide range of polynomial-time problems in planar graphs: variants of max flow, multicommodity flow, variants of shortest paths, Gomory-Hu cut trees, global min-cut, girth, matching, and min-cost flow. It seems, however, there is a long way yet to go; for many promising problems, no planarity-exploiting algorithm is known or there is reason to believe faster algorithms can be obtained.

- **Approximation schemes.** Research on polynomial-time approximation schemes (PTAS) for optimization problems in planar graphs goes back to the pioneering work of Lipton and Tarjan (1977) and Baker (1983), who introduced linear-time algorithms for certain problems in which the constraints were quite local, e.g. maximum-weight independent set and minimum-weight dominating set. For many years, little progress was made on problems with non-local constraints. In the mid-nineties, polynomial-time approximation schemes were developed for the traveling-salesman problem (TSP) in planar graphs, but in these the degree of the polynomial running time depended on the desired accuracy. A decade later, a linear-time approximation scheme was found for TSP. Shortly afterwards, the first polynomial time approximation schemes were found for problems, e.g. Steiner tree, in which the solution was much smaller than the input graph. Since then approximation schemes have been found for several other problems in planar graphs, such as two-connected spanning subgraph, Steiner forest, survivable network design, k-terminal cut, and k-center. Important new techniques have emerged, but we still lack fast approximation schemes for many important problems (e.g. facility location). The area of approximation schemes for planar graphs is ripe for further exploration.
- **Fixed-parameter tractable algorithms.** Another way to cope with computational intractability of some planar graph problems is through the lens of fixed-parameter tractability. The theory of bidimensionality and algorithms exploiting tree decompositions of planar graphs give a general methodology of dealing with planar problems. One way to obtain fixed-parameter tractability results is to show that there is a polynomial-time preprocessing algorithm that creates a "problem kernel" by reducing the size of the instance such that it is bounded by a function of the parameter k. Research on kernelization for planar graph problems has been a very active topic recently, culminating in a meta-theorem that gives problem kernels for a wide range of problems (2009).

The scientific program of the seminar consisted of 24 talks. Five of these talks were longer (60-90 minute) tutorials overviewing the three main areas of the seminar: Jeff Erickson ("Flows in planar and surface graphs") and Christian Wulff-Nilsen ("Separators in planar graphs with applications") covered polynomial-time algorithms; Philip Klein ("Some techniques for approximation schemes on planar graphs") covered approximation schemes; and Dániel Marx ("The square-root phenomenon in planar graphs") and Daniel Lokshtanov ("Kernels for planar graph problems") covered fixed-parameter tractability. One of the main goals of the seminar was to encourage collaboration between the three communities, and these well-received tutorials were very helpful by introducing the basics of each of these topics. The rest of the talks were 25-minute presentations on recent research of the participants.

The time between lunch and the afternoon coffee break was left open for individual discussions and collaborations in small groups. Two open-problem sessions were organized (on Monday evening and Wednesday evening). Notes on the presented problems can be found in this report.

2 Table of Contents

3 Overview of Talks

3.1 A polynomial-time approximation scheme for planar multiway cut

MohammadHossein Bateni (Google – New York, US)

Joint work of Bateni, MohammadHossein; Hajiaghayi, Mohammad Taghi; Klein, Philip; Mathieu, Claire
Main reference M. Bateni, M. Hajiaghayi, P. N. Klein, C. Mathieu, "A Polynomial-time Approximation Scheme for Planar Multiway Cut," in Proc. of the 23rd Annual ACM-SIAM Symp. on Discrete Algorithms (SODA'12), pp. 639–655, SIAM, 2012.
URL http://dl.acm.org/citation.cfm?id=2095116.2095170

Given an undirected graph with edge lengths and a subset of nodes (called the terminals), the multiway cut (also called the multi-terminal cut) problem asks for a subset of edges, with minimum total length, whose removal disconnects each terminal from all others. The problem generalizes minimum *s*-*t* cut, but is NP-hard for planar graphs and APX-hard for general graphs [2]. In this paper, we present a PTAS for multiway cut on planar graphs.

This work has been published in SODA [1].

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1 MohammadHossein Bateni, MohammadTaghi Hajiaghayi, Philip N. Klein and Claire Mathieu. *A polynomial-time approximation scheme for planar multiway cut*. In SODA. SIAM, Philadelphia, PA, USA, 2012.
2 E. Dahlhaus, D. S. Johnson, C. H. Papadimitriou , P. D. Seymour, M. Yannakakis. *The Complexity of Multiterminal Cuts*. SIAM Journal on Computing: 23, 864–894, 1994.

3.2 Triangulating planar graphs under constraints

Therese Biedl (University of Waterloo, CA)

Main reference T. Biedl, "On triangulating k-outerplanar graphs," arXiv:1310.1845v2 [cs.DM] , 2013.
URL http://arxiv.org/abs/1310.1845v2

Most planar graph drawing algorithms operate by first triangulating the planar graph. If we want to use this approach, but draw the planar graph with approximately the optimal height, then we'll need to be careful about how we triangulate. It is easy to see that this can be done without increasing the area-requirement (at least if we don't insist on triangulating the outerface): take an optimal drawing and triangulate it in the computational geometry sense. But how can we find the edges to add without knowing the optimal drawing? This remains open.

In this talk, I will present two results that are close, in that they maintain some graph parameters that are closely related to the optimal height of a planar drawing. Namely, I will show how to triangulate a planar graph such that the outer-planarity remains (roughly) the same, and I will show how to triangulate a planar graph such that the pathwidth remains (asymptotically) the same.

3.3 The ball cover problem

Glencora Borradaile (Oregon State University, US)

Joint work of Borradaile, Glencora; Chambers, Erin

A recent result of Chepoi, Estellon and Vaxes [Disc. Comp. Geom. '07] states that any planar graph of diameter at most $2R$ can be covered by a constant number of balls of size R; put another way, there are a constant-sized subset of vertices within which every other vertex is distance half the diameter. We generalize this result to graphs embedded on surfaces of fixed genus with a fixed number of apices, making progress toward the conjecture that graphs excluding a fixed minor can also be covered by a constant number of balls. To do so, we develop two tools which may be of independent interest. The first gives a bound on the density of graphs drawn on a surface of genus g having a limit on the number of pairwise-crossing edges. The second bounds the size of a non-contractible cycle in terms of the Euclidean norm of the degree sequence of a graph embedded on the surface.

3.4 Parameterized complexity of 1-planarity

Sergio Cabello (University of Ljubljana, SI)

Joint work of M. J. Bannister; Cabello, Sergio; D. Eppstein
Main reference M. J. Bannister, S. Cabello, D. Eppstein, "Parameterized complexity of 1-planarity," in Proc. of the 13th Int'l Symp. on Algorithms and Data Structures (WADS'13), LNCS, Vol. 8037, pp. 97–108, Springer, 2013; also available as pre-print as arXiv:1304.5591v1 [cs.DS].
URL http://dx.doi.org/10.1007/978-3-642-40104-6_9
URL http://arxiv.org/abs/1304.5591v1

We consider the problem of finding a 1-planar drawing for a general graph, where a 1-planar drawing is a drawing in which each edge participates in at most one crossing. Since this problem is known to be NP-hard we investigate the parameterized complexity of the problem with respect to the vertex cover number, tree-depth, and cyclomatic number. For these parameters we construct fixed-parameter tractable algorithms. However, the problem remains NP-complete for graphs of bounded bandwidth, pathwidth, or treewidth.

3.5 Multiple source shortest paths in embedded graphs

Erin Moriarty Wolf Chambers (St. Louis University, US)

Let G be a graph with non-negative edge lengths, embedded on an orientable surface of genus g, and let f be an arbitrary face of G. We describe an algorithm to preprocess the graph in $O(gn \log n)$ time, so that the shortest-path distance from any vertex on the boundary of f to any other vertex in G can be retrieved in $O(\log n)$ time. Our result directly generalizes the $O(n \log n)$-time algorithm of Klein [Proc. SODA 2005] for multiple-source shortest paths in planar graphs. Intuitively, our preprocessing algorithm maintains a shortest-path tree as its source point moves continuously around the boundary of f. As an application of our algorithm, we describe algorithms to compute a shortest non-contractible or non-separating cycle in G in $O(g^2 n \log n)$ time.

3.6 Catalan structures & embedded dynamic programming – a survey

Frederic Dorn (SINTEF – Trondheim, NO)

Joint work of Dorn, Frederic; Fomin, Fedor; Penninkx, Eelko; Bodlaender, Hans; Thilikos, Dimitrios
Main reference F. Dorn, E. Penninkx, H. L. Bodlaender, F. V. Fomin, "Efficient Exact Algorithms on Planar Graphs: Exploiting Sphere Cut Decompositions," Algorithmica 58(3):790–810, 2010.
URL http://dx.doi.org/10.1007/s00453-009-9296-1
Main reference F. Dorn, "Planar Subgraph Isomorphism Revisited," in Proc. of the 27th Int'l Symp. on Theoretical Aspects of Computer Science (STACS'10), LIPIcs, Vol. 5, pp. 263–274, Dagstuhl Publishing, 2010.
URL http://dx.doi.org/10.4230/LIPIcs.STACS.2010.2460

The topic of this talk is a survey on tools for solving NP-hard planar graph problems exactly. For basic problems such as Planar Vertex Cover a commonly used exact algorithm goes as follows: find a structure of the input graph with small separators which are used to do some Myhill-Nerode equivalence class type of dynamic programming. Planarity only plays a role for finding such structure, typically tree- or branch-decompositions, and for proving the bounded separator size. Sphere-cut decompositions represent a novel tool that allows one to do dynamic programming which explicitly exploits planarity. The small separators are connected by simple, closed curves in the plane and thereby obtaining a circular order of the separator vertices. For problems like Planar Longest Cycle, where the solution intersects the separator as a set non-crossing paths, one can bound the number of equivalent solutions by the Catalan numbers. Embedded dynamic programming goes one step further. For problems such as Planar Subgraph Isomorphism one looks at how a separator curve may intersect the pattern. The latter problem for patterns of size k and input graphs with n vertices may be solved in solved in time $2^{O(k)}n$.

3.7 Node-weighted network design in planar and minor-free graphs

Alina Ene (Princeton University, US and University of Warwick, GB)

Joint work of Chekuri, Chandra; Ene, Alina; Vakilian, Ali
Main reference C. Chekuri, A. Ene, A. Vakilian, "Node-weighted network design in planar and minor-closed families of graphs," in Proc. of the 39th Int'l Colloquium on Automata, Languages, and Programming (ICALP'12), Part I, LNCS, Vol. 7391, pp. 206–217, Springer, 2012.
URL http://dx.doi.org/10.1007/978-3-642-31594-7_18

We consider node-weighted network design problems in planar graphs. In particular, we focus on the survivable network design problem (SNDP). The input consists of a node-weighted undirected graph G and connectivity requirements $r(uv)$ for each pair of nodes uv. The goal is to find a minimum weight subgraph H of G such that, for each pair uv of nodes, H contains $r(uv)$ edge-disjoint paths between u and v. In this talk, we describe an $O(k)$-approximation algorithm for the problem when the graph is planar; here k is the maximum requirement of a pair. This improves the $O(k \log n)$-approximation known for node-weighted SNDP in general graphs [Nutov '10].

3.8 Minimum cuts and maximum flows in planar and surface graphs

Jeff Erickson (University of Illinois – Urbana Champaign, US)

This talk is a survey of the state of the art for the classical minimum cut and maximum flow problems for planar and surface-embedded graphs.

3.9 Online node-weighted Steiner forest in planar graphs and extensions

MohammadTaghi Hajiaghayi (University of Maryland, US)

Consider a graph $G = (V, E)$ with a weight value $w(v)$ associated with each vertex v. A demand is a pair of vertices (s, t). A subgraph H satisfies the demand if s and t are connected in H. In the (offline) node-weighted Steiner forest problem, given a set of demands the goal is to find the minimum-weight subgraph H which satisfies all demands. In the online variant, the demands arrive one by one and we need to satisfy each demand immediately; without knowing the future demands.

In the online variant of the problem, we give a randomized $O(\log^3(n))$-competitive algorithm. The competitive ratio is tight to a logarithmic factor. This result generalizes the recent result of Naor, Panigrahi, and Singh for the Steiner tree problem, thus answering one of their open problems. When restricted to planar graphs (and more generally graphs excluding a fixed minor) we give a deterministic primal-dual algorithm with a logarithmic competitive ratio which is tight to a constant factor.

3.10 Some techniques for approximation schemes in planar graphs

Philip N. Klein (Brown University, US)

An approximation scheme for an optimization problem gives a $(1 + \varepsilon)$-approximation algorithm for every $\varepsilon > 0$. I survey some techniques for obtaining approximation schemes for optimization problems in planar graph. I briefly illustrate Baker's framework in addressing vertex cover. I then turn to the traveling salesman problem (TSP) in planar graphs with edge-weights. I discuss a framework in which one can obtain a linear-time approximation scheme for TSP. The framework has been used to obtain approximation schemes for a variety of problems. The key step is computing a spanner designed for the specific optimization problem. I outline the spanner construction for TSP and for Steiner tree, and finish with a few words on the tools needed to extend the Steiner tree result to Steiner forest.

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3 Glencora Borradaile, Philip N. Klein, and Claire Mathieu. An $O(n \log n)$ approximation scheme for Steiner tree in planar graphs. *ACM Trans. Algorithms* 5(3):article 31, 2009.
4 MohammadHossein Bateni, Mohammadtaghi Hajiaghayi, and Dániel Marx. Approximation schemes for Steiner forest on planar graphs and graphs of bounded treewidth. *J. ACM*, 58(5):21:1–21:37, October 2011.
5 David Eisenstat, Philip N. Klein, and Claire Mathieu. An efficient polynomial-time approximation scheme for Steiner forest in planar graphs. *Proceedings of the 23nd ACM-SIAM Symposium On Discrete Algorithms*, 626–638, 2012.

3.11 All-or-nothing multicommodity flow problem with bounded fractionality in planar graphs

Yusuke Kobayashi (University of Tokyo, JP)

Joint work of Kawarabayashi, Ken-ichi; Kobayashi, Yusuke
Main reference K.-I. Kawarabayashi, Y. Kobayashi, "All-or-nothing multicommodity flow problem with bounded fractionality in planar graphs," in Proc. of the 54th Annual IEEE Symp. on Foundations of Computer Science (FOCS'13), pp. 187–196, IEEE CS, 2013.
URL http://dx.doi.org/10.1109/FOCS.2013.28

We study the following all-or-nothing multicommodity flow problem in planar graphs.

Input: A graph G with n vertices and k pairs of vertices $(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)$ in G.

Find: A largest subset W of $\{1, \ldots, k\}$ such that for every i in W, we can send one unit of flow between s_i and t_i.

This problem is different from the well-known maximum edge-disjoint paths problem in that we do not require integral flows for the pairs. This problem is APX-hard even for trees, and a 2-approximation algorithm is known for trees. For general graphs, Chekuri et al. (STOC'04) give a poly-logarithmic factor approximation algorithm and show that a natural LP-relaxation has a poly-logarithmic integrality gap. This result is in contrast with the integrality gap $\Omega(\sqrt{n})$ for the maximum edge-disjoint paths problem.

Our main result considerably strengthens this result when an input graph is planar. Namely, for the all-or-nothing multicommodity flow problem in planar graphs, we give an $O(1)$-approximation algorithm and show that the integrality gap is $O(1)$. In particular, in polynomial time, we can find an index set W with $|W| = \Omega(\mathrm{OPT})$ and eight s_i-t_i paths for each $i \in W$ such that each edge is used at most eight times in these paths (with multiplicity), where OPT is the optimal value of the LP-relaxation of the all-or-nothing multicommodity flow problem.

Our result can be compared to the recent result by Séguin-Charbonneau and Shepherd (FOCS'11) who give an $O(1)$-approximation algorithm for the maximum edge-disjoint paths problem in planar graphs with congestion 2 (but not implied by this result).

3.12 Prize-collecting network design in planar graphs

Nitish Korula (Google – New York, US)

Joint work of Bateni, MohammadHossein; Chekuri, Chandra; Ene, Alina; Hajiaghayi, MohammadTaghi; Korula, Nitish; Marx, Dániel
Main reference M. Bateni, C. Chekuri, A. Ene, M. T. Hagjiaghayi, N. Korula, D. Marx, "Prize-collecting Steiner problems on planar graphs," in Proc. of the 22nd ACM-SIAM Symp. on Discrete Algorithms (SODA'11), pp. 1028–1049, SIAM, 2011.
URL http://dx.doi.org/10.1137/1.9781611973082.79

In this talk, based on work in [1], we describe reductions from Prize-Collecting Steiner TSP (PCTSP), Prize-Collecting Stroll (PCS), Prize-Collecting Steiner Tree (PCST), Prize-Collecting Steiner Forest (PCSF), and more generally Submodular Prize-Collecting Steiner Forest (SPCSF), on planar graphs (and also on bounded-genus graphs) to the corresponding problems on graphs of bounded treewidth. We show that for each of the mentioned problems, an α-approximation algorithm for the problem on graphs of bounded treewidth implies an $(\alpha + \epsilon)$-approximation algorithm for the problem on planar graphs (and also bounded-genus graphs), for any constant $\epsilon > 0$. PCS, PCTSP, and PCST can be solved exactly on graphs of bounded treewidth and hence we obtain a PTAS for these problems on planar graphs and bounded-genus graphs.

In contrast, we show that PCSF is APX-Hard on series-parallel graphs, which are planar graphs of treewidth at most 2. Apart from ruling out a PTAS for PCSF on planar graphs and bounded-treewidth graphs, this result is also interesting since it gives the first provable hardness separation between the approximability of a problem and its prize-collecting version.

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1 MohammadHossein Bateni, Chandra Chekuri, Alina Ene, Mohammad Taghi Hagjiaghayi, Nitish Korula and Dániel Marx. Prize- collecting Steiner problems on planar graphs. *SODA*, 1028–1049, 2011.

3.13 Kernels for planar graph problems

Daniel Lokshtanov (University of Bergen, NO)

Joint work of Fomin, Fedor; Lokshtanov, Daniel; Saurabh, Saket
Main reference F. V. Fomin, D. Lokshtanov, S. Saurabh, "Efficient Computation of Representative Sets with Applications in Parameterized and Exact Algorithms," to appear in Proc. of the 25th Symp. on Discrete Algorithms (SODA'14), 2014.
URL http://www.ii.uib.no/~daniello/papers/EfficientRepSet.pdf

Bollobás' lemma and its generalization to matroids, due to Lovász, are classical results in extremal combinatorics. In this talk we will discuss algorithmic variants of these lemmas, and survey some recent applications in parameterized and exact algorithms.

3.14 The square-root phenomenon in planar graphs

Dániel Marx (Hungarian Academy of Sciences, HU)

Most of the classical NP-hard problems remain NP-hard when restricted to planar graphs, and only exponential-time algorithms are known for the exact solution of these planar problems. However, in many cases, the exponential-time algorithms on planar graphs are significantly faster than the algorithms for general graphs: for example, 3-Coloring can be solved in time $2^{O(\sqrt{n})}$ in an n-vertex planar graph, whereas only $2^{O(n)}$-time algorithms are known for general graphs. For various planar problems, we often see a square-root appearing in the running time of the best algorithms, e.g., the running time is often of the form $2^{O(\sqrt{n})}$, $n^{O(\sqrt{k})}$, or $2^{O(\sqrt{k})} \cdot n$. By now, we have a good understanding of why this square-root appears. On the algorithmic side, most of these algorithms rely on the notion of treewidth and its relation to grid minors in planar graphs (but sometimes this connection is not obvious and takes some work to exploit). On the lower bound side, under a complexity assumption called Exponential Time Hypothesis (ETH), we can show that these algorithms are essentially best possible, and therefore the square root has to appear in the running time.

3.15 Approximating k-center in planar graphs

Claire Mathieu (Brown University, US)

Joint work of Eisenstat, David; Klein, Philip; Mathieu, Claire
Main reference D. Eisenstat, P. N. Klein, C. Mathieu, "Approximating k-center in planar graphs," to appear in the Proc. of the 25th Symp. on Discrete Algorithms (SODA'14), 2014.
URL http://www.davideisenstat.com/cv/EisenstatKM14.pdf

We consider variants of the metric k-center problem. Imagine that you must choose locations for k rehouses in a city so as to minimize the maximum distance of a house from the nearest rehouse. An instance is specified by a graph with arbitrary nonnegative edge lengths, a set of vertices that can serve as rehouses (i.e., centers) and a set of vertices that represent houses. For general graphs, this problem is exactly equivalent to the metric k-center problem, which is APX-hard. We give a polynomial-time bicriteria approximation scheme when the input graph is a planar graph [1].

References

1 David Eisenstat, Philip N. Klein, and Claire Mathieu. *Approximating k-center in planar graphs*. In Proceedings of the Twenty-Fifth Symposium on Discrete Algorithms (SODA), January 2014. To be published.

3.16 Lower and upper bounds for long induced paths in 3-connected planar graphs

Tamara Mchedlidze (KIT – Karlsruhe Institute of Technology, DE)

Joint work of Mchedlidze, Tamara; Di Giacomo, Emilio; Liotta, Giuseppe
Main reference E. Di Giacomo, G. Liotta, T. Mchedlidze, "Lower and Upper Bounds for Long Induced Paths in 3-connected Planar Graphs," in Proc. of the 39th Int'l Workshop on Graph-Theoretic Concepts in Computer Science (WG'13), LNCS, Vol. 8165, pp. 213–224, Springer, 2013.
URL http://dx.doi.org/10.1007/978-3-642-45043-3_19

Let G be a 3-connected planar graph with n vertices and let $p(G)$ be the maximum number of vertices of an induced subgraph of G that is a path. We prove that $p(G) \geq \frac{\log n}{12 \log \log n}$. To demonstrate the tightness of this bound, we notice that the above inequality implies $p(G) \in \Omega((\log_2 n)^{1-\varepsilon})$, where ε is any positive constant smaller than 1, and describe an infinite family of 3-connected planar graphs for which $p(G) \in O(\log n)$.

As a byproduct of our research, we prove a result of independent interest: Every 3-connected planar graph with n vertices contains an induced subgraph that is outerplanar and connected and that contains at least $\sqrt[3]{n}$ vertices. The proofs in the paper are constructive and give rise to $O(n)$-time algorithms.

3.17 Large independent sets in triange-free planar graphs

Matthias Mnich (Universität des Saarlandes, DE)

Joint work of Dvorak, Zdenek; Mnich, Matthias
Main reference Z. Dvorak, M. Mnich, "Large Independent Sets in Triangle-Free Planar Graphs," arXiv:1311.2749v1 [cs.DM], 2013.
URL http://arxiv.org/abs/1311.2749v1

Every triangle-free planar graph on n vertices has an independent set of size at least $(n+1)/3$, and this lower bound is tight. We give an algorithm that, given a triangle-free planar graph G on n vertices and an integer $k >= 0$, decides whether G has an independent set of size at least $(n+k)/3$, in time $2^{O(\sqrt{k})}n$. Thus, the problem is fixed-parameter tractable when parameterized by k. Furthermore, as a corollary of the result used to prove the correctness of the algorithm, we show that there exists $\varepsilon > 0$ such that every planar graph of girth at least five on n vertices has an independent set of size at least $n/(3-\varepsilon)$.

3.18 Multiple-source multiple-sink maximum flow in directed planar graphs in near-linear time

Shay Mozes (Interdisciplinary Center Herzliya, IL)

Joint work of Borradaile, Glencora; Klein, Philip; Mozes, Shay; Nussbaum, Yahav; Wulff-Nilsen, Christian
Main reference G. Borradaile, P. Klein, S. Mozes, Y. Nussbaum, C. Wulff-Nilsen, "Multiple-Source Multiple-Sink Maximum Flow in Directed Planar Graphs in Near-Linear Time," in Proc. of the IEEE 52nd Annual Symp. on Foundations of Computer Science (FOCS'11), pp. 170–179, IEEE, 2011.
URL http://dx.doi.org/10.1109/FOCS.2011.73

In this talk I describe an $O(n \log^3 n)$ algorithm that, given an n-node directed planar graph with arc capacities, a set of source nodes, and a set of sink nodes, finds a maximum flow from the sources to the sinks. I give an overview of the algorithm and go into the details of a procedure to redistribute flow among nodes of a cycle separator. The procedure is based on a representation of circulations via face potentials and efficiently computing shortest paths using an extension of Fakcharoenphol and Rao's fast Dijkstra on dense distance graphs.

3.19 Min-cost flow duality in planar networks

Yahav Nussbaum (Tel Aviv University, IL)

Joint work of Kaplan, Haim; Nussbaum, Yahav
Main reference H. Kaplan, Y. Nussbaum, "Min-Cost Flow Duality in Planar Networks," arXiv:1306.6728v1 [cs.DM] , 2013.
URL http://arxiv.org/abs/1306.6728v1

In this talk we will discuss the minimum-cost flow problem in planar graphs.

We begin with a minimum-cost flow problem in a planar graph and modify the problem using the following two transformations. First, we express the problem as a problem in the geometric dual graph. Then, we find the linear programming dual of this problem. The result is a minimum-cost flow problem in a related planar graph, such that the balance constraints are defined by the costs of the original problem, and the costs are defined by the capacities of the original problem.

As an application for our transformation, we show an $O(n \log^2 n)$ time algorithm for the minimum-cost flow problem in an n-vertex outerplanar graph, which takes advantage of the simple structure of the dual graphs of outerplanar graphs.

3.20 Network sparsification for Steiner problems on planar and bounded-genus graphs

Marcin Pilipczuk (University of Warsaw, PL)

Joint work of Pilipczuk, Marcin; Pilipczuk, Michał; Sankowski, Piotr; van Leeuwen, Erik Jan
Main reference M. Pilipczuk, M. Pilipczuk, P. Sankowski, E. J. van Leeuwen, "Network Sparsification for Steiner Problems on Planar and Bounded-Genus Graphs," arXiv:1306.6593v1 [cs.DS], 2013.
URL http://arxiv.org/abs/1306.6593v1

We propose polynomial-time algorithms that sparsify planar and bounded-genus graphs while preserving optimal solutions to Steiner problems. Our main contribution is a polynomial-time algorithm that, given a graph G embedded on a surface of genus g and a designated face f bounded by a simple cycle of length k, uncovers a set $F \subseteq E(G)$ of size polynomial in g and k that contains an optimal Steiner tree for *any* set of terminals that is a subset of the vertices of f.

We apply this general theorem to prove that:

- given a graph G embedded on a surface of genus g and a terminal set $S \subseteq V(G)$, one can in polynomial time find a set $F \subseteq E(G)$ that contains an optimal Steiner tree T for S and that has size polynomial in g and $|E(T)|$;
- an analogous result holds for the Steiner Forest problem;
- given a planar graph G and a terminal set $S \subseteq V(G)$, one can in polynomial time find a set $F \subseteq E(G)$ that contains an optimal edge multiway cut C separating S (i.e., a cutset that intersects any path with endpoints in different terminals from S) and that has size polynomial in $|C|$.

In the language of parameterized complexity, these results imply the first polynomial kernels for Steiner Tree and Steiner Forest on planar and bounded-genus graphs (parameterized by the size of the tree and forest, respectively) and for Edge Multiway Cut on planar graphs (parameterized by the size of the cutset).

3.21 The k-disjoint paths problem on directed planar graphs

Michał Pilipczuk (University of Bergen, NO)

Joint work of Cygan, Marek; Marx, Dániel; Pilipczuk, Marcin; Pilipczuk, Michał
Main reference M. Cygan, D. Marx, M. Pilipczuk, M. Pilipczuk, "The Planar Directed K-Vertex-Disjoint Paths Problem Is Fixed-Parameter Tractable," in Proc. of the 54th Annual Symp. on Foundations of Computer Science (FOCS'13), pp. 197–206, IEEE CS, 2013; also available as pre-print as arXiv:1304.4207v1 [cs.DM].
URL http://dx.doi.org/10.1109/FOCS.2013.29
URL http://arxiv.org/abs/1304.4207

It was shown in the 90s by Schrijver that the k-Disjoint Paths problem can be solved in $n^{O(k)}$ time on directed planar graphs [1]. In this work we present an FPT algorithm for the problem working in time $f(k) * n^{O(1)}$ for a doubly-exponential function f.

References

1 Alexander Schrijver. *Finding k Disjoint Paths in a Directed Planar Graph*. SIAM Journal on Computing 23(4), 1994

3.22 A near-optimal planarization algorithm

Saket Saurabh (The Institute of Mathematical Sciences – Chennai, IN)

Joint work of Jansen, Bart M. P.; Lokshtanov, Daniel; Saurabh, Saket

The problem of testing whether a graph is planar has been studied for over half a century, and is known to be solvable in $O(n)$ time using a myriad of different approaches and techniques. Robertson and Seymour established the existence of a cubic algorithm for the more general problem of deciding whether an n-vertex graph can be made planar by at most k vertex deletions, for every fixed k. Of the known algorithms for k-Vertex Planarization, the algorithm of Marx and Schlotter (WG 2007, Algorithmica 2012) running in time $2^{k^{O(k^3)}} \cdot n^2$ achieves the best running time dependence on k. The algorithm of Kawarabayashi (FOCS 2009), running in time $f(k)n$ for some $f(k) \in \Omega\left(2^{k^{\Omega(k^3)}}\right)$ that is not stated explicitly, achieves the best dependence on n.

In this paper we present an algorithm for k-Vertex Planarization with running time $2^{O(k \log k)} \cdot n$, significantly improving the running time dependence on k without compromising the linear dependence on n. Our main technical contribution is a novel scheme to reduce the treewidth of the input graph to $O(k)$ in time $2^{O(k \log k)} \cdot n$. It combines new insights into the structure of graphs that become planar after contracting a matching, with a Baker-type subroutine that reduces the number of disjoint paths through planar parts of the graph that are not affected by the sought solution. To solve the reduced instances we formulate a dynamic programming algorithm for Weighted Vertex Planarization on graphs of treewidth w with running time $2^{O(w \log w)} \cdot n$, thereby improving over previous double-exponential algorithms.

While Kawarabayashi's planarization algorithm relies heavily on deep results from the graph minors project, our techniques are elementary and practically self-contained. We expect them to be applicable to related edge-deletion and contraction variants of planarization problems.

3.23 Approximation algorithms for Euler genus, and related problems

Anastasios Sidiropoulos (University of Illinois – Urbana Champaign, US)

Joint work of Chekuri, Chandra; Sidiropoulos, Anastasios

The Euler genus of a graph is a fundamental and well-studied parameter in graph theory and topology. Computing it has been shown to be NP-hard by [Thomassen '89 & '93], and it is known to be fixed-parameter tractable. However, the approximability of the Euler genus is wide open. While the existence of an $O(1)$-approximation is not ruled out, only an $O(\sqrt(n))$-approximation [Chen, Kanchi, Kanevsky '97] is known even in bounded degree graphs. In this paper we give a polynomial-time algorithm which on input a bounded-degree graph of Euler genus g, computes a drawing into a surface of Euler genus $poly(g, log(n))$. Combined with the upper bound from [Chen, Kanchi, Kanevsky '97], our result also implies a $O(n^{1/2-\alpha})$-approximation, for some constant $\alpha > 0$. Using our algorithm for approximating the Euler genus as a subroutine, we obtain, in a unified fashion, algorithms with approximation ratios of the form $poly(\text{OPT}, log(n))$ for several related problems on bounded degree graphs. These include the problems of orientable genus, crossing number, and planar edge and vertex deletion problems. Our algorithm and proof of correctness for the crossing number problem is simpler compared to the long and difficult proof in the recent breakthrough by [Chuzhoy 2011], while essentially obtaining a qualitatively similar result. For planar edge and vertex deletion problems our results are the first to obtain a bound of form $poly(\text{OPT}, log(n))$.

We also highlight some further applications of our results in the design of algorithms for graphs with small genus. Many such algorithms require that a drawing of the graph is given as part of the input. Our results imply that in several interesting cases, we can implement such algorithms even when the drawing is unknown.

3.24 Separators in planar graphs with applications

Christian Wulff-Nilsen (University of Copenhagen, DK)

For planar graphs, we have separators with small size and/or nice structural properties. I present some of these and give applications to shortest path and min cut/max flow problems. Minor-free and shallow minor-free graps also have good separators but we do not know how to compute them in linear time. I will present faster separator algorithms for these graph classes and give applications to shortest paths and maximum matching.

4 Open Problems

4.1 Planarization by vertex deletion

Proposed by Peter Rossmanith (rossmani@cs.rwth-aachen.de)

Given a graph with m edges, can it be made planar by deleting at most $m/6$ vertices?

4.2 All-pair distances on the infinite face

Proposed by Shay Mozes (smozes@idc.ac.il)

Here is a special case of multiple-source shortest paths: Given a planar graph, one can compute all the shortest path trees rooted at vertices on the infinite face f_∞, in total time $O(n \log n)$ (Klein, Cabello-Chambers-Erickson), and there is a matching lower bound. Can one compute all-pair distances between vertices of f_∞ in time $O(n + |f_\infty|^2)$?

4.3 Two-edge-connected planar subgraph

Proposed by Philip Klein (klein@brown.edu)

Given a planar graph G with edge weights, find a subgraph H such that $V(H) = V(G)$, H is 2-edge-connected, and the total weight of H is minimum. Does there exist an efficient PTAS, that is, a PTAS with running time $f(\epsilon)n^c$, where c is an absolute constant (independent of ϵ)?

The problem has linear-time PTAS (Grigni) in the special case when all edge weights are 1, and there is a PTAS with running time $n^{f(\epsilon)}$ for general weights. The problem is APX-hard for general graphs.

A similar problem is two-edge-connected Steiner subgraph where $V(H)$, instead of being equal to $V(G)$, must contain a specified set T of terminal vertices. Is there a PTAS?

4.4 Euclidean multiway cut with unit disks

Proposed by Sergio Cabello (sergio.cabello@fmf.uni-lj.si)

Given k points $s_1 s_2, \ldots s_k$ in the Euclidean plane, and a collection of unit disks (none of which contains any s_i), find a minimum cardinality set of disks that separates every s_i from every s_j. Is there a PTAS?

The problem is NP-hard and has a constant factor approximation. In the special case where $k = 2$, the problem is in P. In the generalization where disks have weights, is there a constant factor approximation? In the variant where the goal is to separate s_1 from every s_i for $i \neq 1$, is the problem FPT?

4.5 Minimum stretch shape shifting

Proposed by Erin Chambers (echambe5@slu.edu)

Given an unweighted triangulated planar graph and two curves ℓ and r, "morph" ℓ into r by a sequence of elementary moves while minimizing the length of the longest intermediate curve. There are two types of elementary moves: either replace one edge of a triangle t by the other two edges of t, or vice versa, or, when the curve goes through a vertex x, insert $(x, y), (y, x)$ into the curve, where $\{x, y\}$ is an edge.

A logarithmic approximation is known by divide-and-conquer using a shortest path, even if the edges have weights. It is not known whether the problem is in NP, nor whether it is NP-hard. In the unweighted triangulation case it is also not known whether the optimal sequence is monotonic, that is, never traverses the same triangle twice. (There is an example where the motion is not purely monotonic, but it is in a graph with appropriate weights.)

4.6 Vertex-capacitated max flow in directed planar graphs

Proposed by Jeff Erickson (jeffe@illinois.edu)

Here is a table showing best known strongly-polynomial running times for some variants of max flow in directed planar graphs:

	source s, sink t	sources s, s', sink t
edge capacities	$O(n \log n)$	$O(n \log n)$
vertex capacities	$O(n \log n)$	$O(n^2 \log n)$

The top-right variant can be addressed by first finding a max st-flow and then finding a max $s't$-flow in the residual graph. The bottom-left variant can be addressed by using a reduction from vertex-capacities to edge-capacities.

For the fourth variant, there is no planarity-exploiting algorithm known!

1. Cannot use the reduction because it leads to violation at one vertex.
2. Cannot use the residual graph because there isn't a residual graph with respect to vertex capacities.

4.7 *k-minimum spanning tree* problem

Proposed by Alina Ene (aene@cs.princeton.edu)

- *input:* undirected edge-weighted graph, integer k
- *output:* subgraph tree T with at least k vertices
- *goal:* minimize the weight of the tree

In general graphs, this is SNP-hard, and there is a 2-approximation algorithm due to Garg, using primal-dual techniques. There exists a PTAS for the Euclidean case. What happens in planar graphs? Is there a PTAS? Is the problem APX-hard? Indications are that the spanner approach will not work.

4.8 Prize-collecting Steiner forest

Proposed by Alina Ene (aene@cs.princeton.edu)

Prize-collecting Steiner forest is APX-hard in series-parallel graphs.

The best approximation we have for bounded treewidth is one for general graphs (the approximation ratio is 2.54). Can we do better? If so, it would yield an improved approximation ratio for planar graphs as well.

4.9 Achieving subexponential time for several parameterized problems in planar graphs

Proposed by Marcin Pilipczuk (malcin@mimuw.edu.pl)

1. directed k-path
2. weighted (undirected) k-path: path on k vertices of minimum weight
3. Exact k-cycle
4. Steiner tree parameterized by number of terminals
5. k-MSR parameterized by k
6. Subgraph isomorphism parameterized by size of subgraph

All but the fourth can be solved in $(1+\varepsilon)^k n^{\varepsilon^{-2}}$ so we don't expect a ETH lower bound.

They should have running times of the form $2^{\sqrt{k}\text{polylog } k} n^c$ but it would be interesting to have $2^{o(k)} n^c$.

- For (undirected) planar disjoint paths, the bound is $f(k) \cdot \text{poly}$ where $f(k)$ is triply exponential in k. Can we get $2^{\text{poly } k} \cdot \text{poly}$?
- Finding an independent set of size $\frac{m}{4} + k$ – is this fixed-parameter tractable?
- Kernels for feedback vertex set, dominating set. Can the dependence of the kernel size on k be reduced, from around $100k$ to, say, $10k$?
- Vertex cover. There is a $2k$-vertex kernel. It is believed that one cannot do better because it might imply a better approximation than 2, which we don't expect. But what about planar graphs?
- k-vertex deletion to get a planar graph, or, more generally, to get a graph that excludes a fixed graph H as a subgraph. Hitting all minors leads to a bound that is doubly exponential in treewidth. Can we get a bound of $2^{\text{poly } tw} \cdot \text{poly(input graph)}$?

4.10 Recognizing map graphs

Proposed by Daniel Lokshtanov (daniello@ii.uib.no)

A map graph derives from a modified notion of planarity in which two (connected) regions of a map are considered adjacent when they share a point of their boundaries (not an edge, as standard planarity requires) (Chen, Grigni, Papadimitriou, STOC'98). How quickly can we recognize them? Thorup has $n^{\geq 120}$. Can $O(n^{10})$ be achieved?

4.11 Planar local TSP

Proposed by Rolf Niedermeier (rolf.niedermeier@tu-berlin.de)

- *input:* edge-weighted planar graph, Hamilton cycle C, given by a permutation π of the vertices, integer $k \geq 0$

- *output:* Does there exist a permutation π' such that $\lambda(\pi, \pi') \leq k$ and such that π' gives a shorter tour?

Here λ is a funtion measuring distance between permutations. There are two versions of λ that are the same up to a factor of 2:

- number of edges in symmetric difference
- number of reversals

For λ =number of swaps, the problem is known to be FPT.

4.12 Subgraph isomorphism in planar graphs with a twist

Proposed by Dániel Marx (dmarx@cs.bme.hu)

Testing whether H is isomorphic to a subgraph of G, where the parameter k is the difference $|E(G)| - |E(H)|$. Is this problem FPT?

For the parameter being zero, the problem is isomorphism, which is solvable in polynomial time.

Can easily achieve n^kpoly (n). If H is also 3-connected, the problem is FPT.

4.13 Mincost flow in planar graphs

Proposed by Jeff Erickson (jeffe@illinois.edu)

- Every vertex has a supply value (could be negative or zero).
- Every arc has a capacity and a cost.

Assign a flow value to every arc such that net flow out of a vertex equals the supply of the vertex. Flow values are nonnegative but no more than capacity.

Goal: minimize cost.

- Without loss of generality, can assume each vertex's supply is zero , which yields the problem *min-cost circulation.*
- Alternatively, can assume capacities are infinite, which yields the *transshipment* problem.

Can you give an algorithm whose running time is strongly polynomial-time and beats $O(n^2 \log n)$?

If maximum cost is $O(1)$ or or max capacity is $O(1)$, can achieve $O(n^{1.5})$ time (Cornelsen and Karrenbauer; Cornelsen, Karrenbauer, Li).

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