

# Reachability Problems for Infinite-State Systems

Edited by

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## Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 14141 “Reachability Problems for Infinite-State Systems”, held from March 30th until April 4th, 2014. The seminar gathered 44 participants and the program consisted of 34 presentations. Participants were asked to contribute open questions prior and during the seminar. A list of these open questions appears in a separate section of the present report. This list generated collaborations among participants and gave rise to research publications solving (partially), for example, question 5.13, namely “what functions are computable by VASS?”

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**Edited in cooperation with** Michael Blondin

## 1 Executive Summary

*Javier Esparza*

*Alain Finkel*

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Many standard verification problems can be rephrased as reachability problems, and there exist powerful methods for determining reachability in infinite-state systems. However, applications require not only decidability results, but provably optimal algorithms. The seminar focussed on complexity and algorithmic issues for the verification of infinite-state systems, with special emphasis on reachability problems.

Verification of finite-state systems can be illustrated by considering the case of counter systems, i. e., computational models combining a finite-state control with counters. Counter systems have been used to model distributed protocols, programs with recursive parallel threads, programs with pointers, broadcast protocols, replicated finite-state programs, asynchronous programs, etc. If zero-tests are allowed – one speaks of “Minsky machines” –, counter systems have the power of Turing machines, and so all their verification problems



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are undecidable. On the other hand, many problems can be decided when zero-tests are forbidden – one speaks of VASS, for “vector addition systems with states”, or equivalently “Petri nets”. In particular, reachability for VASS was shown decidable in 1982, and this can be leveraged into many more positive results. Moreover, researchers developed techniques that, while necessarily incomplete, allow analysing many questions: reversal-bounded analysis à la Ibarra, accelerations à la FAST, or well-structured extensions of VASS, see e. g., the forward analysis procedure. In turn, these techniques have led to many new theoretical results. For instance, it has been shown that the reachability sets of both reversal-bounded counter automata and flat counter automata are effectively definable in Presburger arithmetic (assuming some additional conditions).

The seminar addressed the following topics:

- Complexity of reachability on various models: parameterized counter systems, lossy channel systems, lossy counter systems, at counter systems, reversal-bounded counter systems, and other.
- Decidability and complexity of reachability problems for Petri nets extensions: timed Petri nets, Petri nets with one zero-test, with one unbounded counter, linear dynamical systems, BVASS, data nets, and other.
- Recent development and uses of the theory of well-structured transition systems.
- Decidability and complexity of reachability for systems with multiple (constraints) stacks: multiphase, reversal-bounded, and other.
- Games on infinite-state systems: counter automata, timed systems, weighted automata. Games with energy constraints.
- Monadic logics with costs.
- New developments in the algorithmics of Presburger logics; SMT-solvers.

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### Open Problems


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### 3 Overview of Talks

#### 3.1 Cut-offs on Parameterized Systems

*Parosh Aziz Abdulla (Uppsala University, SE)*

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We present a simple and efficient framework for automatic verification of systems with a parameteric number of communicating processes.

The processes may be organized in various topologies such as words, multisets, rings, or trees.

Our method needs to inspect only a small number of processes in order to show correctness of the whole system. It relies on an abstraction function that views the system from the perspective of a fixed number of processes. The abstraction is used during the verification procedure in order to dynamically detect cut-off points beyond which the search of the state space need not continue.

Our experimentation on a variety of benchmarks demonstrate that the method is highly efficient and that it works well even for classes of systems with undecidable verification problems. (*Preliminary abstract, March 30 2014*)

#### 3.2 Timed Pushdown Automata

*Mohamed Faouzi Atig (Uppsala University, SE)*

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**Joint work of** Abdulla, Parosh Aziz; Atig, Mohamed Faouzi; Stenman, Jari  
**Main reference** P. A. Abdulla, M. F. Atig, J. Stenman, “Zenoness for Timed Pushdown Automata,” in Proc. of the 15th Int’l Workshop on Verification of Infinite-State Systems (INFINITY’14), EPTCS, Vol. 140, pp. 35–47, 2014.  
**URL** <http://dx.doi.org/10.4204/EPTCS.140.3>

Timed pushdown automata are pushdown automata extended with a finite set of real-valued clocks. Additionally, each symbol in the stack is equipped with a value representing its age. The enabledness of a transition may depend on the values of the clocks and the age of the topmost symbol. Therefore, dense-timed pushdown automata subsume both pushdown automata and timed automata. In this talk, I will show that the reachability and zenoness problems are EXPTIME-complete.

#### 3.3 The Linear Ranking-Function Problem For Linear-Constraint Loops with a Precondition

*Amir M. Ben-Amram (Academic College of Tel Aviv, IL)*

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The linear ranking-function problem for (single-path) linear-constraint loops has been well-studied and its complexity established for loop that compute over integers or over the rational numbers. That is, when one assumes that any initial state is possible. What if one is also given part of the initial state? Then the problem is much harder. I can prove some lower bounds but it is not known if the problems are even decidable.

### 3.4 Handling Infinitely Branching WSTS

*Michael Blondin (ENS Cachan, FR and Université de Montréal, CA)*

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**Joint work of** Blondin, Michael; Finkel, Alain; McKenzie, Pierre

**Main reference** M. Blondin, A. Finkel, P. McKenzie, “Handling Infinitely Branching WSTS,” in Proc. of the 41st Int’l Colloquium on Automata, Languages, and Programming (ICALP’14), LNCS, Vol. 8573, pp. 13–25, Springer, 2014.

**URL** [http://dx.doi.org/10.1007/978-3-662-43951-7\\_2](http://dx.doi.org/10.1007/978-3-662-43951-7_2)

Most decidability results concerning well-structured transition systems apply to the finitely branching variant. Yet some models (inserting automata, omega-Petri nets, ...) are naturally infinitely branching. Here we develop tools to handle infinitely branching WSTS by exploiting the crucial property that in the (ideal) completion of a well-quasi-ordered set, downward-closed sets are finite unions of ideals. Then, using these tools, we derive decidability results and we delineate the undecidability frontier in the case of the termination, the control-state maintainability and the coverability problems. A new forward algorithm for deciding coverability is obtained and boundedness is also shown decidable.

### 3.5 Symbolic Hybrid Transduction

*Bernard Boigelot (Université de Liège, BE)*

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This work addresses the exact computation of the set of reachable configurations of systems modeled by linear hybrid automata. The contribution is an original approach to accelerating control cycles, which consists in computing symbolically the effect of iterating such cycles any number of times. This is achieved by first modeling the data transformation labeling a cycle as a Linear Hybrid Relation, which is a set of linear constraints describing the transformation applied to the variables when this cycle is followed. A LHR maps individual variable values onto a set corresponding to a convex polyhedron in  $n$ -dimensional space, and such a polyhedron into a polyhedron of the same form. We introduce a data structure called linear hybrid transducer that represents a LHR and makes it possible to reason about the possible transformations that such a LHR induces on polyhedra. Accelerating a LHR can then be performed by computing symbolically the transitive closure of its corresponding linear hybrid transducer. This effectively reduces the acceleration of LHR to a purely discrete problem.

### 3.6 Sets with Atoms

*Mikolaj Bojańczyk (University of Warsaw, PL)*

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**Joint work of** Bojańczyk, Mikolaj; Slawomir Lasota; Bartek Klin; Luc Segoufin; Szymon Toruńczyk; Joanna Ochremiak

**URL** <http://atoms.mimuw.edu.pl>

The talk is about sets with atoms, and how they can be used to compute some things about infinite sets. Included in the talk is a demonstration of a tool, written by Eryk Kopczynski, which allows one to compile and run the following program.

```
for every rational numbers x, y, z
  if (x < y) and (y < z) and not (x < z) then print ‘‘error’’
```

The program does not print “error”. In general, instead of the rational numbers, there can be any logical structure with a decidable first-order theory. The tool is an implementation, and a theoretical extension, of a programming language for sets with atoms.

### 3.7 On the Hardness of Solving Ordinary Differential Equations

*Olivier Bournez (École Polytechnique – Palaiseau, FR)*

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We prove that a particular class of ordinary differential equations (ordinary differential equations with polynomial right hand side) can be solved in polynomial time.

We prove that conversely that polynomial time can be characterized by ordinary differential equations with polynomial right hand side.

This yields both an implicit characterization of polynomial time in terms of ordinary differential equations, and a completeness result on the reachability problems for the corresponding class. (*Preliminary abstract, March 30 2014*)

### 3.8 Zero-Reachability in Probabilistic Multi-Counter Automata

*Tomas Brazdil (Masaryk University – Brno, CZ)*

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We study the qualitative and quantitative zero-reachability problem in probabilistic multi-counter systems. We identify the undecidable variants of the problems, and then we concentrate on the remaining two cases. In the first case, when we are interested in the probability of all runs that visit zero in some counter, we show that the qualitative zero-reachability is decidable in time which is polynomial in the size of a given pMC and doubly exponential in the number of counters. In the second case, when we are interested in the probability of all runs that visit zero in some counter different from the last counter, we show that the qualitative zero-reachability is decidable. In both cases we show that the probability of all zero-reaching runs can be effectively approximated up to an arbitrarily small given error  $\epsilon > 0$ . (*Preliminary abstract, March 30 2014*)



### 3.9 Secure Equilibria in Weighted Games


Véronique Bruyère (*University of Mons, BE*)

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We consider two-player non zero-sum infinite duration games played on weighted graphs. We extend the notion of secure equilibrium introduced by Chatterjee et al., from the Boolean setting to this quantitative setting. As for the Boolean setting, our notion of secure equilibrium refines the classical notion of Nash equilibrium. We prove that secure equilibria always exist in a large class of weighted games which includes common measures like sup, inf, lim sup, lim inf, mean-payoff, and discounted sum. Moreover we show that one can synthesize such strategy profiles that are finite-memory and use few memory. We also prove that the constrained existence problem for secure equilibria is decidable for sup, inf, lim sup, lim inf and mean-payoff measures. Our solutions rely on new results for zero-sum quantitative games with lexicographic objectives that are interesting on their own right.

### 3.10 Infinite-state Model Checking with Data Recycling

Giorgio Delzanno (*University of Genova, IT*)

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We present a model checking algorithm for infinite-state systems based on a garbage collection procedure for dynamically recycling unused data representations. The goal of data recycling is to maintain within a finite range the set of representations of distinct values needed for a complete state space exploration. The verification procedure takes inspiration from recent decidability results obtained for classes of data-centric systems with dynamic generation of fresh data. We have implemented a prototype version of the algorithm using the Spin model checker and applied it to some classical examples of concurrent systems with unbounded data, like Lamport's bakery and ticket mutual exclusion protocols with a fixed number of process instances. (*Preliminary abstract, March 30 2014*)

### 3.11 Reachability in Partial-Observation Stochastic Games

Laurent Doyen (*ENS Cachan, FR*)

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**Joint work of** Doyen, Laurent; Chatterjee, Krishnendu

**Main reference** K. Chatterjee, L. Doyen, "Partial-Observation Stochastic Games: How to Win when Belief Fails," in Proc. of the 27th Annual IEEE Symp. on Logic in Computer Science (LICS'12), pp. 175–184, IEEE, 2012; pre-print available as arXiv:1107.2141 [cs.GT].

**URL** <http://dx.doi.org/10.1109/LICS.2012.28>

**URL** <http://arxiv.org/abs/1107.2141v1>

The talk presents a quick survey of the main results about the complexity of two-player partial-observation finite-state stochastic games with a reachability objective (as well as some subclasses and extensions).

While several (recent) results have enriched our knowledge about this framework, it is not known whether the existence of a winning strategy is decidable for reachability games with pure strategies.

Special cases of this problem are decidable, and show that the memory requirement is at least non-elementary (tower of exponentials) although the games are finite-state and with a simple reachability objective (and finite memory is sufficient).

This surprising result has connections with difficult problems for certain classes of counter systems. The talk glances through this connection and open problem.

### 3.12 Reasoning About Data Repetitions with Counter Systems

*Diego Figueira (University of Edinburgh, GB)*

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**Joint work of** Demri, Stéphane; Figueira, Diego; Praveen, M.

**Main reference** S. Demri, D. Figueira, M. Praveen, “Reasoning about Data Repetitions with Counter Systems,” in Proc. of the 28th Annual ACM/IEEE Symp. on Logic in Computer Science, pp. 33–42, IEEE, 2013.

**URL** <http://dx.doi.org/10.1109/LICS.2013.8>

We study linear-time temporal logics interpreted over data words with multiple attributes. We demonstrate correspondences between satisfiability problems for logics and reachability-like decision problems for counter systems. We show that allowing/disallowing atomic formulas expressing repetitions of values in the past corresponds to the reachability/coverability problem in Petri nets. This gives us 2-EXPSpace upper bounds for several satisfiability problems. We prove matching lower bounds by reduction from a reachability problem for a newly introduced class of counter systems. This new class is a succinct version of vector addition systems with states in which counters are accessed via pointers, a potentially useful feature in other contexts. We strengthen further the correspondences between data logics and counter systems by characterizing the complexity of fragments, extensions and variants of the logic.

### 3.13 The VJGL Lemma

*Jean Goubault-Larrecq (ENS Cachan, FR)*

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The VJGL Lemma generalizes a famous useful theorem by Valk and Jantzen (1985). It allows one to compute finite bases of upward closed subsets  $U$  of well-quasi-ordered sets  $X$  under two simple assumptions: the so-called effective complement property, and the assumption that it is decidable whether a given element of the completion of  $X$  meets  $U$ . We give a short proof of it, and applications to several (in fact infinitely many) well-quasi-ordered sets. The effective complement property is true in all these cases.

### 3.14 Temporal Logics and Automata on Multi-attributed Data Words with Ordered Navigation

*Peter Habermehl (Université Paris-Diderot, FR)*

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We study temporal logics and automata on multi-attributed data words. Recently, BD-LTL was introduced as a temporal logic on data words extending LTL by navigation along

positions of single data values. It is known that allowing for navigation wrt. tuples of data values renders the logic undecidable. We therefore introduce ND-LTL, an extension of BD-LTL by tuple-navigation that is only restricted in terms of a certain order on the attributes. While complete ND-LTL is still undecidable, the two natural fragments allowing for either future or past navigation along data values are shown to be Ackermann-hard, yet decidability is obtained by reduction to nested multi-counter systems. To this end, we introduce and study nested variants of data automata as an intermediate model simplifying the constructions. Interestingly, on BD-LTL, the same restrictions have a significant impact on the decision procedure. While satisfiability of BD-LTL is as hard as reachability in Petri nets, our restrictions yield two 2-EXPSPACE-complete fragments. (*Preliminary abstract, March 30 2014*)

### 3.15 Subclasses of Presburger Arithmetic and the Weak EXP Hierarchy

*Christoph Haase (ENS Cachan, FR)*

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**Main reference** C. Haase, “Subclasses of Presburger Arithmetic and the Weak EXP Hierarchy,” arXiv:1401.5266v2 [cs.LO], 2014.

**URL** <http://arxiv.org/abs/1401.5266v2>

I will show that for any fixed  $i > 0$ , the  $\Sigma_{i+1}$ -fragment of Presburger arithmetic, i. e., its restriction to  $i + 1$  quantifier alternations beginning with an existential quantifier, is complete for the  $i^{\text{th}}$  level of the weak EXP hierarchy. This result completes the computational complexity landscape for Presburger arithmetic, a line of research which dates back to the seminal work by Fischer & Rabin in 1974. Moreover, I will discuss bounds on sets of naturals definable in the existential fragment of Presburger arithmetic: given an existential formula  $p(x)$ , I will show that the set of solutions is an ultimately periodic set whose period can be doubly-exponentially bounded from below and above.

### 3.16 Reachability Problems are NP-complete for Flat Counter Systems with Octagonal Loops

*Radu Iosif (VERIMAG – Gières, FR)*

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**Joint work of** Iosif, Radu; Bozga, Marius; Konecny, Filip


**Main reference** M. Bozga, R. Iosif, F. Konecny, “Safety Problems are NP-complete for Flat Integer Programs with Octagonal Loops,” arXiv:1307.5321v3 [cs.CC], 2013.

**URL** <http://arxiv.org/abs/1307.5321v3>

This paper proves the NP-completeness of the reachability problem for the class of flat counter machines with difference bounds and, more generally, octagonal relations, labeling the transitions on the loops. The proof is based on the fact that the sequence of powers of such relations can be encoded as a periodic sequence of matrices, and that both the prefix and the period of this sequence are simply exponential in the size of the binary encoding of the relation. This result allows to characterize the complexity of the reachability problem for one of the most studied class of counter machines, and has a potential impact for other problems in program verification.

### 3.17 Equivalences of Deterministic Systems as Reachability Problems

*Petr Jančár (TU – Ostrava, CZ)*

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In a labelled transition system (LTS), a pair  $(s, t)$  of states is in the trace-preorder if each trace (a sequence of actions) that is enabled by  $s$  is also enabled by  $t$ . States  $s, t$  are trace-equivalent if both  $(s, t)$  and  $(t, s)$  are in the trace preorder. For a deterministic LTS  $L$ , the complement of the trace-preorder problem can be naturally formulated as a reachability problem in the product deterministic LTS  $L \times L$ :  $(s, t)$  is not in the trace-preorder if there is a pair  $(s', t')$  that is reachable from  $(s, t)$  and where  $s'$  enables an action that is not enabled by  $t'$ . We focus on these (reachability) problems for deterministic pushdown automata (DPDA) where the trace-preorder is undecidable while the trace-equivalence is known to be in TOWER. The precise complexity of the latter remains a challenging problem, since only P-hardness is known regarding the lower bound.

### 3.18 Analysis of Probabilistic Parallel Processes

*Stefan Kiefer (University of Oxford, GB)*

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**Joint work of** Bonnet, Remi; Kiefer, Stefan; Lin, Anthony W.

**Main reference** R. Bonnet, S. Kiefer, A. W. Lin, “Analysis of Probabilistic Basic Parallel Processes,” in Proc. of the 17th Int’l Conf. on Foundations of Software Science and Computation Structures (FOSSACS’14), LNCS, Vol. 8412, pp. 43–57, Springer, 2014.

**URL** [http://dx.doi.org/10.1007/978-3-642-54830-7\\_3](http://dx.doi.org/10.1007/978-3-642-54830-7_3)


Basic Parallel Processes (BPPs) are a well-known subclass of Petri Nets. They are the simplest common model of concurrent programs that allows unbounded spawning of processes.

In the probabilistic version of BPPs, every process generates other processes according to a probability distribution. We study the decidability and complexity of fundamental qualitative problems over probabilistic BPPs – in particular reachability with probability 1 of different classes of target sets (e. g. upward-closed sets).

Our results concern both the Markov-chain model, where processes are scheduled randomly, and the MDP model, where processes are picked by a scheduler.

### 3.19 Well-Structured Graph Transformation Systems

*Barbara König (Universität Duisburg-Essen, DE)*

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**Joint work of** König, Barbara; Stueckrath, Jan

Graph transformation systems (GTSSs) can be seen as well-structured transition systems (WSTSSs), thus obtaining decidability results for certain classes of GTSSs. It was shown that well-structuredness can be obtained using the minor ordering as a well-quasi-order. We extend this idea to obtain a general framework in which several types of GTSSs can be seen as (restricted) WSTSSs. We instantiate this framework with the subgraph ordering and the induced subgraph ordering. Furthermore we present the tool UNCOVER and discuss runtime results.

### 3.20 Adversarial Patrolling Games

Antonín Kučera (Masaryk University – Brno, CZ)

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Patrolling is one of the central problems in operational security. Formally, a *patrolling problem* is specified by a set  $U$  of vulnerable *targets* and a function  $d$  which to every target  $u$  assigns the time  $d(u) \in \mathbb{N}$  needed to complete an intrusion at  $u$ . The goal is to design an optimal strategy for a *defender* who is moving from target to target and aims at detecting possible intrusions. The defender can detect an intrusion at  $u$  only by visiting  $u$  before the intrusion is completed. The goal of the *attacker* is to maximize the probability of a successful attack. We assume that the attacker is *adversarial*, i. e., he knows the strategy of the defender and can observe her moves. We assume that all targets are equally important and that each move from target to target takes one unit of time. The set of admissible moves is specified by an *environment*  $E \subseteq U \times U$ , where the fully connected digraph (incl. all loops) models the *unrestricted environment*.

We prove that the defender has an optimal strategy for every patrolling problem and every environment. Then, we give an upper bound for the *Stackelberg value*, i. e., the maximal probability of successfully defended attacks that can be achieved by the defender against an arbitrary strategy of the attacker. The bound is valid for an arbitrary environment. Further, we show that if for every attack length  $k$  the total number of all targets  $u$  with  $d(u) = k$  is divisible by  $k$ , then the bound is achievable in the unrestricted environment by a simple *modular* strategy of the defender which is computable in polynomial time. We also give an exact classification of all *sufficiently connected* environments  $E$  where the bound is achievable – we show that there is a *characteristic digraph* computable in polynomial time such that  $E$  is sufficiently connected if and only if it contains a subdigraph isomorphic to the characteristic digraph. Hence, the problem whether a given environment is sufficiently connected is in **NP**, and we provide a matching lower bound which is valid also for restricted subclasses of patrolling problems.

### 3.21 Turing Machines Over Infinite Alphabets

Slawomir Lasota (University of Warsaw, PL)

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**Main reference** M. Bojańczyk, B. Klin, S. Lasota, S. Torunczyk, “Turing Machines with Atoms,” in Proc. of the 28th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS’13), pp. 183–192, IEEE CS, 2013.

**URL** <http://dx.doi.org/10.1109/LICS.2013.24>

In sets with atoms, finiteness is relaxed to “finiteness up to permutation”, known as orbit-finiteness. The talk will be devoted to Turing machines where state space, input alphabet and transition relation are orbit-finite. We will focus on two main results.

First, we show that deterministic machines are weaker than nondeterministic ones; in particular,  $P$  is not equal  $NP$  in sets with atoms. The separating language is closely related to the Cai-Fuierer-Immerman graphs used in descriptive complexity theory. The second result is an effective characterization of those input alphabets for which Turing machines determinize (called standard alphabets). To this end, the determinization problem is expressed as a Constraint Satisfaction Problem, and a characterization is obtained from deep results in CSP theory.

### 3.22 Non-Elementary Complexities for Branching VASS, MELL, and Extensions

*Ranko Lazić (University of Warwick, GB)*

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**Joint work of** Lazić, Ranko; Schmitz, Sylvain

**Main reference** R. Lazić, S. Schmitz, “Non-Elementary Complexities for Branching VASS, MELL, and Extensions,” arXiv:1401.6785v1 [cs.LO], 2014.

**URL** <http://arxiv.org/abs/1401.6785v1>

We study the complexity of reachability problems on branching extensions of vector addition systems, which allows us to derive new non-elementary complexity bounds for fragments and variants of propositional linear logic. We show that provability in the multiplicative exponential fragment is Tower-hard already in the affine case – and hence non-elementary. We match this lower bound for the full propositional affine linear logic, proving its Tower-completeness. We also show that provability in propositional contractive linear logic is Ackermann-complete.

### 3.23 Vector Addition System Toolbox

*Jerôme Leroux (Université de Bordeaux, FR)*

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Vector addition systems, a class equivalent to the Petri nets, form a well-known class of models with many decidable properties. In this presentation, we present a well partial order on runs. This partial order is shown to be central for deriving many simple proof of decidability for various problems.

### 3.24 Deciding Coverability in Non-Monotone Infinite-State Systems

*Richard Mayr (University of Edinburgh, GB)*

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**Joint work of** Mayr, Richard; Abdulla, Parosh Aziz

**Main reference** P. A. Abdulla, R. Mayr, “Priced Timed Petri Nets,” Logical Methods in Computer Science, 9(4):10, 51 pages, 2013.

**URL** [http://dx.doi.org/10.2168/LMCS-9\(4:10\)2013](http://dx.doi.org/10.2168/LMCS-9(4:10)2013)

We describe a general construction for solving reachability/coverability problems for infinite-state systems under some weak abstract conditions. In particular, we do not assume that the behavior of these systems is fully monotone w.r.t. some well-quasi-order, unlike in many related works on well-structured/well-quasi-ordered transition systems (e.g., Abdulla et al., Finkel et al.). This technique, called the phase construction, has been described in [1, 2, 3] and instantiated to solve a question about infinite-state real-time systems. However, it is fully abstract and can be instantiated in many different ways. We think that this technique deserves to be more widely known, since it can be useful for other people working in the field of infinite-state system verification. In particular, it includes a technique for computing the minimal elements of upward-closed sets in arbitrary well-quasi-ordered domains. This

is the most general form of the Valk-Jantzen construction in arbitrary well-quasi-ordered domains, as described in [1, 2].

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## 3.25 Decision Problems for Discrete Linear Dynamical Systems: A Survey

*Joel Ouaknine (University of Oxford, GB)*

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Joint work of Ouaknine, Joel; Worrell, James

I will survey the state of the art regarding decision problems for discrete linear dynamical systems, including the Skolem Problem, the Positivity and Ultimate Positivity problems for linear recurrence sequences, the Orbit and Polyhedron-hitting problem, etc. I will also discuss applications to verification, e.g. the termination of simple linear loops and reachability and invariance in Markov chains.

## 3.26 Software Model Checking via Petri Net Language Inclusion

*Andreas Podelski (Universität Freiburg, DE)*

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We present an approach to software model checking for parametrized concurrent programs. In this approach, one reduces the validity of a candidate proof to checking the inclusion between two Petri net languages (which is checked via non-reachability).

We then extend the approach to the case where the proof needs to account for the local variables of each of the threads in a parametrized concurrent program. Here, one reduces the validity of a candidate proof to checking the inclusion between two languages recognized by a kind of data automata. The inclusion is decidable thanks to an argument based on well-quasi orderings.

The open problem is to extend the approach to parametrized concurrent programs where the proof needs to account for the relation between local variables of threads.

### 3.27 Reachability Problems for Braids, Knots and Links – New Challenges for Computer Science

Igor Potapov (*University of Liverpool, GB*)

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In this talk I will introduce a few challenging reachability problems in computational topology opening new connections between mathematical knots, braid groups, combinatorics on words, words over infinite alphabets, complexity theory and provide solutions for some of these problems by application of several techniques from automata theory, matrix semigroups and algorithms.

### 3.28 Unordered Data Nets Are as Hard as Lossy Channel Systems

Fernando Rosa-Velardo (*University Complutense of Madrid, ES*)

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We characterize the exact ordinal-recursive complexity of Unordered Data Nets (UDN), a subclass of Data Nets in which the data carried by tokens belong to an unordered domain. We use the techniques developed by Schmitz and Schnoebelen to bound the length of bad sequences in well-quasi orderings of finite multisets over tuples of naturals. These bounds imply hyper-Ackermannian upper bounds for the termination and the coverability problems for UDN. Then we prove that the previous bounds are tight, by constructing UDNs that weakly compute fast-growing functions and their inverses.

### 3.29 Complexity Hierarchies Beyond Elementary

Sylvain Schmitz (*ENS Cachan, FR*)

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**Main reference** S. Schmitz, “Complexity Hierarchies Beyond Elementary,” arXiv:1312.5686v2 [cs.CC], 2013.

**URL** <http://arxiv.org/abs/1312.5686v2>

The talk first introduces a hierarchy of fast-growing complexity classes and show its suitability for completeness statements of many non elementary problems. This hierarchy allows the classification of many decision problems with a non-elementary complexity, which occur naturally in logic, combinatorics, formal languages, verification, etc., with complexities ranging from simple towers of exponentials to Ackermannian and beyond.

The second part of the talk gives a quick overview of so-called *length function theorem* for well-quasi-orders. These are combinatorial statements that provide explicit upper bounds on the length of *controlled* bad sequences over a given wqo.



### 3.30 Ackermann-hardness for Monotone Counter Systems

*Philippe Schnoebelen (ENS Cachan, FR)*

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**Main reference** P. Schnoebelen, “Revisiting Ackermann-Hardness for Lossy Counter Machines and Reset Petri Nets,” in Proc. of the 35th Int’l Symp. on Mathematical Foundations of Computer Science (MFCS’10), LNCS, Vol. 6281, pp. 616–628, Springer, 2010.

**URL** [http://dx.doi.org/10.1007/978-3-642-15155-2\\_54](http://dx.doi.org/10.1007/978-3-642-15155-2_54)

We explain the key ideas behind the proof of Ackermann-hardness of reachability/coverability for Lossy Counter Machines, aka Lossy Minsky Machines.

The presentation follows the plan of Schnoebelen MFCS 2010 paper, with some recent minor simplifications like the use of Hardy functions.

### 3.31 Critical Exponents of $k$ -automatic Words and Generalizations

*Jeffrey O. Shallit (University of Waterloo, CA)*

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**URL** <https://cs.uwaterloo.ca/~shallit/Talks/dag2.pdf>

A (fractional) repetition of exponent  $l/p$  is a word of length  $l$  and period  $p$ . For example, the French word **entente** is a repetition of exponent  $7/3$ . The supremum over the exponents of all factors of an infinite word  $w$  is known as the critical exponent of  $w$ . Thue proved, for example, that the critical exponent of the Thue-Morse word is 2. In a recent paper with Luke Schaeffer, we showed that the critical exponent of  $k$ -automatic words is (i) rational or infinite and (ii) computable. Can a similar result be proved for some classes of infinite-state automata? For example, it is easy to see that the “ruler sequence” has critical exponent 2 and the “infinity series” of the Danish composer Per Nørgård has critical exponent  $4/3$ .

### 3.32 Infinite Games and Reachability

*Wolfgang Thomas (RWTH Aachen, DE)*

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In this survey talk we present the fundamentals of the algorithmic theory of infinite games, starting with Church’s Problem (1957), and emphasizing the central role of reachability problems in the study of such games. The first part explains the solution of games defined by a requirement formulated in MSO-logic (Büchi-Landweber). This solution is obtained via a transformation of an MSO-formula into a finite-state game with the Muller winning condition, followed by a transformation into a parity game, which is then solved inductively, with an essential use of reachability games. In a second part we address two more recent tracks of research, the study of parity games over infinite arenas (in particular, pushdown graphs), and the approach of McNaughton to determine the winner of a finite-state game by a scoring process in the progress of a play, which results in a reduction of the problem to solving a reachability (or dually to a safety) game.

### 3.33 Models of the lambdaY-Calculus for Weak Monadic Second-Order Logic

*Igor Walukiewicz (Université de Bordeaux, FR)*

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**Joint work of** Salvati Sylvain; Walukiewicz, Igor

**Main reference** S. Salvati, I. Walukiewicz, “Models of the lambdaY-calculus for weak monadic second-order logic,” unpublished manuscript.

LambdaY-calculus is simply typed lambda calculus with fixpoint operators. This calculus faithfully models control in higher-order programs. The semantic of a program represented by a lambdaY-term is a tree reflecting the control flow of the program.

We describe a model construction calculating properties of trees generated by lambdaY-terms: the value of term in a model determines if the tree generated by the term has the given property. The construction works for all properties expressible in weak monadic second-order logic. This construction allows to obtain a (decidable) typing system for the model-checking problem. It gives a “verification by evaluation” approach to model-checking, and it opens a possibility of modular verification of large programs.

### 3.34 Ultimate Positivity is Decidable for Simple Linear Recurrent Sequences

*James Worrell (University of Oxford, GB)*

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We consider the decidability and complexity of the Ultimate Positivity Problem, which asks whether all but finitely many terms of a given rational linear recurrence sequence (LRS) are positive. Using lower bounds in Diophantine approximation concerning sums of S-units, we show that for simple LRS (those whose characteristic polynomial has no repeated roots) the Ultimate Positivity Problem is decidable in polynomial space. If we restrict to simple LRS of a fixed order then we obtain a polynomial-time decision procedure. As a complexity lower bound we show that Ultimate Positivity for simple LRS is at least as hard as the decision problem for universal sentences in the theory of real-closed fields. (*Preliminary abstract, March 30 2014*)

## 4 Open Problems

### 4.1 Linear-Ranking Function and Affine Functions

*Amir M. Ben-Amram (Academic College of Tel Aviv, IL)*

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#### 4.1.1 Linear-Ranking Functions Problems

- What is the complexity of this group of problems (termination, LRF existence, LRF verification) for linear-constraint loops over the rationals? Over the integers?
- What are the complexities for affine-linear loops in either domain?
- What are other special cases of interest?
- Are the three problems above just as hard?

#### 4.1.2 Affine Functions Problems

- In  $\mathbb{Z}^2$ , I ask whether mortality decidable for functions of a restricted form, where “mortality” means that every iteration sequence (trajectory) reaches  $(0,0)$ ; and the functions in question are defined by dividing  $\mathbb{Z}^2$  into a finite number of rectangular regions, i. e., by constraints of the form  $x_1 \geq a, x_1 \leq b, x_2 \geq c, x_2 \leq d$  (some regions will be infinite, having no upper bound or no lower bound on one or both variables) and defining the function on each region by  $f(x_1, x_2) = (x_1 + s, x_2 + t)$  where  $s, t$  depend on the region. One could also allow  $f$  to be defined to be zero on some regions.
- Over  $\mathbb{Z}^2$ , consider a function class which is bigger than the above since one can also define  $f(x_1, x_2) = (x_2 + s, x_1 + t)$  in some regions, i. e., switch the variables. Then mortality is shown undecidable in the paper; but it is a challenging open problem to settle the decidability when the number of regions is bounded by a constant (even if this constant is as low as 7).
- Given an arbitrary affine-linear function with integer coefficients  $f(x_1, \dots, x_n)$ , and a loop of the form while  $(Ax > b)$  do  $x := f(x)$  (here  $x$  ranges over  $\mathbb{Z}^n$ ), is the loop mortal (universally terminating)?

### 4.2 Completion and Infinitely Branching WSTS

*Michael Blondin (ENS Cachan, FR and Université de Montréal, CA)*

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- What applications has the WSTS completion?
- Boundedness, coverability, strong termination, and strong control-state maintainability are decidable for infinitely branching WSTS under some assumptions. Are there other problems decidable? Under which assumptions?
- For what families of WSTS and for what problems are the algorithms working on the completion more efficient?

### 4.3 Symbolic Hybrid Transduction

*Bernard Boigelot (Université de Liège, BE)*

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- Could this approach be employed for computing directly the reachability set (instead of only accelerating a given control cycle)?
- Is it possible to decide whether the computation of the closure of a linear hybrid transducer will terminate or not?
- How can linear hybrid relations over a large number of variables be accelerated efficiently?

### 4.4 The VJGL Lemma

*Jean Goubault-Larrecq (ENS Cachan, FR)*

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- Can completions be used to analyze Kosaraju’s algorithm in terms of the well-quasi-ordering on runs presented by Jérôme Leroux (with Sylvain Schmitz) on the first day of the seminar? The connection with bounded languages seems alluring. Here is why. The language  $L$  of traces of a Petri net is a subset of a set of words on a well-quasi-ordered alphabet  $A$ . The quasi-ordering looks very close to word embedding. We know that the completion of a poset of words with word embedding is a space of word-products, which are regular languages of a specific form, and which are certainly bounded. Is the completion of  $L$  of a similar form? Does it consist of bounded languages?
- The VJGL Lemma depends on showing the effective complement property for (effective) representations of elements of the completion. What would be such representations for graphs or hypergraphs under the minor embedding ordering, or other well quasi-orderings?

### 4.5 Subclasses of Presburger Arithmetic and the Weak EXP Hierarchy

*Christoph Haase (ENS Cachan, FR)*

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#### 4.5.1 Presburger arithmetic with divisibility

Lipshitz [2] has shown that the existential fragment of Presburger arithmetic with a full divisibility predicate is decidable. He moreover gave an outline of an NP upper bound in [3]. Can we find a complete rigorous proof of the NP bound?

#### 4.5.2 The complexity of inclusion for context-free commutative grammars

Given context-free grammars  $G$  and  $H$ , is the Parikh image of  $G$  included in the Parikh image of  $H$ ? This problem is known to be  $\Pi_2^P$ -hard and in NEXP [4]. If the size of the alphabet is fixed, the problem becomes  $\Pi_2^P$ -complete [5]. What is the precise complexity

of this problem when the alphabet is part of the input? Is there a syntactic fragment of Presburger arithmetic which yields an optimal upper bound of this problem?

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## 4.6 Equivalences of Deterministic Systems as Reachability Problems


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Can we improve the complexity gap described in the overview of the talk “Equivalences of deterministic systems as reachability problems”?

## 4.7 Turing machines over infinite alphabets

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- Are standard alphabets closed under union?
- Is every deterministic TM equivalent to a deterministic TM whose work alphabet is the same as the input alphabet?
- Is there a natural NP-complete problem for TMs with atoms?

## 4.8 Branching VAS

*Ranko Lazić (University of Warwick, GB)*

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- The reachability problem for branching VAS is now known to be TOWER-hard, but decidability is still open.
- A clear Karp-Miller procedure for the top-down direction in branching VAS is currently missing.
- One could investigate decidability and complexity of checking regularity of tree languages generated by branching VAS, in the top-down direction, and in the bottom-up direction.

## 4.9 Decision Problems for Discrete Linear Dynamical Systems

Joel Ouaknine (*University of Oxford, GB*)

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This area is replete with open problems, many with applications to other areas such as software model checking. For example, the question of Universal Termination of integer linear problems in the general (i. e. non-homogeneous) case, as stated by Mark Braverman in [1]:

Consider the program:

```
WHILE  $Bx > b$  do  $\{x := Ax + c\}$ 
```

where  $A, B$  are matrices and  $b, c$  are vectors, all over the integers.


Universal Termination is the assertion that this program terminates for all initial values of the vector of integer variables  $x$ . Decidability (for given  $A, B, b, c$ ) is open.

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## 4.10 Rewriting Over Infinite Alphabets

Igor Potapov (*University of Liverpool, GB*)

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Many computational processes can be described as rewriting systems over strings from a finite alphabet, where new words are generated following some rules for replacing, adding or deleting symbols. Nowadays there is a sufficiently broad research activity in the area of logic and automata for words and trees over infinite alphabets. It is many motivated by the need to analyse and verify infinite-state systems, which for example can use infinite alphabet of natural numbers  $\{1, 2, 3, \dots\}$  instead of finite number of symbols like  $\{a, b, c\}$ . In the seminal paper of M. Kaminski and N. Francez [1] a very restricted memory structure of the automaton (Register Automaton) working with words over infinite alphabets was introduced. The register automaton is operating by keeping a finite number of symbols (from the working tape) in its memory and making their comparison to other observed symbols. The model allows recognising a large class of languages over infinite alphabet and at the same time is not taking an advantage of its memory capabilities beyond what is needed for that purposes. A more complex system operating with words over infinite alphabet may require updating them in addition to the operations of comparison between symbols. Obviously, unrestricted and very general rules allowing rewriting over arbitrary infinite alphabet are too powerful making most of the computational problems to be undecidable. On the other hand there are existing fragments of rewriting systems over infinite alphabet with decidable word problem (i. e. algorithmic problem of deciding whether two given representatives represent the same element of the set). One of such example is unknotedness and equivalence of knots, where words over infinite alphabet are Gauss words (or Gauss diagrams) and the system of rewriting rules is a set of Reidemeister moves represented by insertion/deletion and swapping some of the symbols on Gauss words. While the set of the Reidemeister moves is quite powerful the

word problem for such rewriting rules on Gauss words is decidable following algorithms from combinatorial topology.

#### Open problems:

- Define the weakest rewriting rules over infinite alphabet with undecidable reachability problem (word problem).
- Define the most powerful (in computational sense) model of rewriting system over infinite alphabet with decidable reachability problem (word problem).
- Find upper/lower bounds on the length of reachability paths for rewriting systems over infinite alphabet with decidable reachability problem.

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### 4.11 Unordered Data Nets

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- Are UDN without broadcasts  $F_{\omega\omega}$ -hard?
- What is the exact complexity of coverability and termination for
- Unordered Petri Data nets (UDN without broadcasts or whole-place operations)?
- Is reachability decidable for Unordered Petri Data nets?

### 4.12 Cichoń's Principle Redux

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#### 4.12.1 Summary

Under which conditions can one bound the maximal length of  $(g, n)$ -controlled bad sequences over a normed wqo  $(A, \leq_A, |\cdot|_A)$  with the Cichoń function  $h_{o(A)}(n)$ , where  $o(A)$  is the maximal ordertype of  $A$  and  $h$  is a “reasonable” function of  $g$ ?

#### 4.12.2 Length Function Theorems

We refer to [5] for details on controlled bad sequences and normed wqos. The reader will also find there one positive answer to the problem: for  $A$  built from finite sets using disjoint unions, Cartesian products, and Kleene star, the result holds with  $h = p \circ g$  for a polynomial  $p$ . Another such answer was stated during the seminar: for  $A = \alpha$  an ordinal less than  $\varepsilon_0$  (and for a suitable norm), the maximal length is less than  $g_\alpha(n)$ . Of course one might want to consider other wqos, in particular with maximal order types higher than  $\varepsilon_0$ .

### 4.12.3 Cichoń Functions

The Cichoń hierarchy of functions [2] (aka “length hierarchy”) is a convenient way of measuring the length of controlled bad sequences. Given a monotone function  $g: \mathbb{N} \rightarrow \mathbb{N}$ , the Cichoń functions  $(g_\alpha)_\alpha$  are defined by transfinite induction by

$$g_0(x) = 0, \quad g_{\alpha+1}(x) = 1 + g_\alpha(g(x)), \quad g_\lambda(x) = g_{\lambda(x)}(x),$$

where  $\lambda(x)$  denotes the  $x^{\text{th}}$  element of a fundamental sequence of ordinals converging towards the limit  $\lambda$ ; typically for  $\lambda < \varepsilon_0$ :

$$\lambda(x) = \begin{cases} \gamma + \omega^\beta \cdot (x + 1) & \text{if } \lambda = \gamma + \omega^{\beta+1}, \\ \gamma + \omega^{\lambda'(x)} & \text{if } \lambda = \gamma + \omega^{\lambda'}. \end{cases}$$

### 4.12.4 Maximal Order Types

This is the ordinal height of the maximal linearisation of a wqo, and was defined in [3].

### 4.12.5 Why Redux?


A somewhat similar principle relating the order type of termination orderings with the maximal length of derivations using the so-called slow-growing functions was sometimes called “Cichoń’s Principle” after an observation in [1]. A related question appeared as Problem 23 of the *RTA List of Open Problems* and was solved in [4]; see <http://www.cs.tau.ac.il/~nachum/rtaloop/problems/23.html>.

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## 4.13 Weakly Computing Numerical Functions

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**Open problem:** What functions are weakly computable by VASS and extensions?

**Background:** Hack 1976 [1] uses weak computing of multivariate polynomials with positive coefficients to show the undecidability of the equivalence problem. Mayr and Meyer 1981 [2] use weak computing of fast-growing  $F_n$  functions to show the Ackermann-hardness of the finite containment problem. Weak computing of the inverse  $F_n^{-1}$  functions by Reset Petri nets,




i. e. VASS extended with reset operations, is instrumental in recent Ackermann-hardness results for monotone counter systems, see Schnoebelen 2010 [3].

### References

- 1 Michel Hack. *The Equality Problem for Vector Addition Systems is Undecidable*. Theor. Comput. Sci. Volume 2, Issue 4. 1976.
- 2 Ernst W. Mayr and Albert R. Meyer. *The Complexity of the Finite Containment Problem for Petri Nets*. J. ACM Volume 28, Issue 3, 1981.
- 3 Philippe Schnoebelen. *Revisiting Ackermann-Hardness for Lossy Counter Machines and Reset Petri Nets*. MFCS 2010.

## 4.14 Critical Exponents of $k$ -automatic Words and Generalizations

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URL <https://cs.uwaterloo.ca/~shallit/Talks/dag2.pdf>

1. Is there a decision procedure for determining the critical exponent (supremum over exponent of all factors) of a morphic word (i. e., the image, under a coding, of fixed point of an arbitrary morphism)?
2. Characterize the predicates for automatic sequences (e. g., squarefreeness) that are decidable in polynomial time. For example, Leroux has proved it for ultimate periodicity.
3. It is possible to prove that squarefreeness is not decidable, in general, for the obvious extension of automatic sequences to infinite alphabets (the  $k$ -regular sequences of Allouche and Shallit). But is it decidable for the special case of a fixed point of a morphism of the form  $i \rightarrow (ai + b, ci + d)$  where  $a, b, c, d$  are integers? E. g., for  $(a, b, c, d) = (0, 0, 1, 1)$  we get the “ruler sequence” 01020103 $\cdots$  with critical exponent 2 and for  $(a, b, c, d) = (-1, 0, 1, 1)$  we get the Nørgård “infinity sequence” recently proved to have critical exponent  $4/3$ .
4. Is  $\sup\{x/y : (x, y)_k \in L\}$  computable for context-free languages  $L$ ? Here by  $(x, y)_k$  we mean the representation of the pair of integers  $(x, y)$  in base  $k$ .
5. Given a regular language  $L \subseteq (\Sigma_k \times \Sigma_k)^*$  representing a set  $S \subseteq \mathbb{N} \times \mathbb{N}$  of pairs of natural numbers, is it decidable if  $S$  contains a pair  $(p, q)$  with  $p \mid q$ ?
6. Prove or disprove: if  $L$  is a regular language with  $\text{quo}_k(L) = \mathbb{Q}^{\geq 0}$ , then  $L$  contains infinitely many distinct canonical representations for infinitely many distinct rational numbers. Here by “canonical” we mean “no leading  $[0, 0]$ ’s” and by  $\text{quo}_k(L)$  we mean  $\{p/q : (p, q)_k \in L\}$ .

## 5 Program

### Monday 31 March

09:00	<b>Opening/Introduction</b>	
09:30	Survey: Jérôme Leroux	<i>Vector Addition System Toolbox</i>
10:30	<b>Coffee break</b>	
11:00	Michael Blondin	<i>Handling Infinitely Branching WSTS</i>
11:30	Christoph Haase	<i>Subclasses of Presburger Arithmetic and the Weak EXP</i>
12:00	Petr Jančar	<i>Equivalences of Deterministic Systems as Reachability Problems</i>
12:15	<b>Lunch and discussions</b>	
15:30	<b>Cake break</b>	
16:00	Giorgio Delzanno	<i>Infinite-State Model Checking with Data Recycling</i>
16:30	Barbara König	<i>Well-Structured Graph Transformation Systems</i>
17:00	<b>Break</b>	
17:30	Parosh Aziz Abdulla	<i>Cut-offs on Parameterized Systems</i>
18:00	<b>Dinner</b>	

### Tuesday 1 April

09:00	Survey: Sylvain Schmitz	<i>Complexity Hierarchies Beyond Elementary</i>
10:00	Philippe Schnoebelen	<i>Ackermann-hardness for Monotone Counter Systems</i>
10:30	<b>Coffee break</b>	
10:45	Ranko Lazić	<i>Non-Elementary Complexities for Branching VASS, MELL, and Extensions</i>
11:15	Richard Mayr	<i>Deciding Coverability in Non-monotone Infinite-state Systems</i>
11:45	Fernando Rosa-Velardo	<i>Unordered Data Nets are as Hard as Lossy Channel Systems</i>
12:15	<b>Lunch and discussions</b>	
15:30	<b>Cake break</b>	
16:00	Jeffrey Shallit	<i>Critical Exponents of <math>k</math>-automatic Words and Generalizations</i>
16:30	Igor Potapov	<i>Reachability Problems for Braids, Knots, and Links – New Challenges for Computer Science</i>
17:00	<b>Break</b>	
17:15	Mohammad Faouzi Atig	<i>Timed Pushdown Automata</i>
17:45	Diego Figueira	<i>Reasoning About Data Repetitions with Counter Systems</i>
18:00	<b>Dinner</b>	

**Wednesday 2 April**

09:00	Survey: Joel Ouaknine	<i>Decision Problems for Discrete Linear Dynamical Systems: A Survey</i>
10:00	Radu Iosif	<i>Safety Problems are NP-complete for Flat Integer Programs with Octagonal Loops</i>
10:30	<b>Coffee break</b>	
10:45	James Worrell	<i>Ultimate Positivity is Decidable for Simple Linear Recurrent Sequences</i>
11:15	Bernard Boigelot	<i>Symbolic Hybrid Transduction</i>
11:45	Amir Ben-Amram	<i>Linear Ranking Functions for Loops with a Precondition</i>
12:15	<b>Lunch and excursion</b>	
18:00	<b>Dinner</b>	

**Thursday 3 April**

09:00	Survey: Wolfgang Thomas	<i>Infinite Games and Reachability</i>
10:00	Véronique Bruyère	<i>Secure Equilibria in Weighted Games</i>
10:30	<b>Coffee break</b>	
10:45	Laurent Doyen	<i>Reachability in Partial-Observation Stochastic Games</i>
11:15	Mikolaj Bojańczyk	<i>Looping Over Infinite Structures</i>
11:45	Slawomir Lasota	<i>Turing Machines Over Infinite Alphabets</i>
12:15	<b>Lunch and discussions</b>	
15:30	<b>Cake break</b>	
16:00	Igor Walukiewicz	<i>Models of the lambdaY-calculus for Weak Monadic Second-Order Logic</i>
16:30	Olivier Bournez	<i>On the Hardness of Solving Ordinary Differential Equations</i>
16:45	<b>Break</b>	
17:15	Jean Goubault-Larrecq	<i>The VJGL Lemma</i>
17:45	Tomas Brazdil	<i>Zero-Reachability in Probabilistic Multi-Counter Automata</i>
18:00	<b>Dinner</b>	

**Friday 4 April**

09:00	Peter Habermehl	<i>Temporal Logics and Automata on Multi-attributed Data Words with Ordered Navigation</i>
09:30	Antonín Kučera	<i>Adversarial Patrolling Games</i>
10:00	Stefan Kiefer	<i>Analysis of Probabilistic Parallel Processes</i>
10:30	<b>Coffee break</b>	
11:00	Andreas Podelski	<i>Software Model Checking via Petri Net Language Inclusion</i>

## Participants

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