Report from Dagstuhl Perspectives Workshop 14182

Categorical Methods at the Crossroads

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 14182 "Perspectives Workshop: Categorical Methods at the Crossroads". The aim of the meeting was to investigate the potential of category theory as a paradigm for mathematical modeling and applied science. The envisaged application areas included computation, physics, biology, complex systems, social and cognitive science and linguistics. Many of these areas were indeed tackled in the variety of topics dealt with during the workshop.

Each working day followed the same structure: two survey lectures during the morning, followed by three shorter talks in the afternoon, and closed by a working group session. During these sessions the attendants split into several groups according to the main thematic areas that had been identified on the first day. Both surveys and talks are reported in the "Overview" section of the report, while a wrap-up of the discussions that occurred inside the working groups is reported in the "Working Groups" section.

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1 Executive Summary

Fabio Gadducci

Since the 1960s, category theory has been recognised as a powerful conceptual framework and a flexible specification language. The range of research areas where categorical methods found application is quite wide: from physics, economics, and linguistics to many branches of mathematics, especially algebraic geometry, algebraic topology, and logic. And, of course, computer science: possibly the discipline, apart from mathematics, where these methods have been most wholeheartedly adopted. Indeed, they have become part of the standard "tool-box" in many areas of theoretical informatics, from programming languages to automata, from process calculi to type theory.

Despite their flexibility and expressiveness, a more general acceptance of categorical methods has been hindered by the perceived difficulties of the formalism. As a consequence, many researchers in different communities share the feeling of under-exploitation of the potentialities of category theory to their areas of interest. This Dagstuhl Perspectives Workshop seminar brought together people from various disciplines that are interested in the application of categorical tools in their research area, from computer science towards venues such as economics, mathematics, and physics.

Besides the benefits to the understanding of each topic that came from a plurality of voices in a discussion, the meeting tried to address more general concerns. As far as some disciplines are concerned, the workshop helped the reconciliation of different research strands that uses categorical tools. Most importantly though, the workshop aimed at building bridges between disciplines, by reviewing the variety of uses of categorical methods in different fields and trying to find common abstractions that allow the same structures and concepts to be recognized as they arise in different settings, and to be transferred from one area to another.

In order to put on firm grounds the foundations of a common language, each working day included two survey lectures during the morning, which presented a variety of topics where categorical methods play a major role. These were followed by three shorter talks in the early afternoon, which presented active areas and innovative application for these methods. The day was closed by a working group session: during these sessions the attendants split into several groups according to the main thematic areas that had been identified on the first day. The variety of topics dealt with in the workshop was large, and the suggested application areas included (quantum) computation, physics, biology, complex systems, economic, social and cognitive science, and linguistics. Indeed, some of the items span more than one discipline, e. g. game theory, and the list is definitively not exhaustive.

Although the scope of the workshop was broad, the over-arching research theme was to develop categorical methods as a unified approach to the modeling of complex systems, and category theory as a paradigm for mathematical modeling and applied science. To this end, the overall purpose of the workshop was to start developing a coherent research community applying categorical methods to a wide range of disciplines. Under these terms, the workshop has been indeed successful. Laying out a common mathematical language and finding analogies among apparently distant concepts from unrelated disciplines provided a basis for fruitful cross-disciplinary interactions also by facilitating a “technology transfer”. Concretely, this led to the fostering of new collaborations among the participants, and the preliminary exploration of new directions and research themes.

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3 Overview of Talks

3.1 Categories at the crossroads

Samson Abramsky (University of Oxford, GB)

This talk laid out some of the themes for the workshop. The main aim of the meeting was formulated as considering category theory as a new paradigm for mathematical modelling and applied science. The envisaged application areas include computation, physics, biology, complex systems, social and cognitive science and linguistics. Much of the work using category theory within Computer Science to date has been within the field of semantics of programming languages. It was suggested that the potential scope for the application of categorical methods within CS is much wider than this, and it may be fruitful to loosen the traditional role of a preconceived syntax. Examples of current work in this spirit includes coalgebra and the use of diagrammatic and graphical formalisms. New territories to conquer include algorithms and complexity, intensional computation, and logic in AI. As an example of the last of these, valuation algebras were discussed as a foundational formalism for local computation and generic inference, and it was shown how they could be formulated very naturally in categorical terms. As another case study, the application of categorical methods in physics, in particular to the study of contextuality and non-locality, was described. This use of categorical methods leads to the recognition of general structures which arise in many situations outside the quantum realm, and to a general notion of contextual semantics, with a wide range of applications.

Some general methodological principles and advantages of category theory as a methodology for mathematical modelling were articulated. Finally, some practical issues involved in developing the community working in this area, and achieving fruitful interactions with and impact upon the wider community, were discussed.

3.2 Network theory

John C. Baez (University of California – Riverside, US)

URL http://math.ucr.edu/home/baez/networks_dagstuhl/

Nature and the world of human technology are full of networks. Researchers in many fields draw diagrams of networks: flow charts, electrical circuit diagrams, signal-flow graphs, Bayesian networks, Feynman diagrams and the like. Mathematicians and computer scientists know that in principle these diagrams fit into a common framework: category theory. But we are still far from a unified theory of networks. We give an overview of the theory as it stands now, with an emphasis on topics for future research.

3.3 Introduction to differential and tangent categories

Robin Cockett (University of Calgary, CA)

Differential categories come in two complementary colours: as *tensor* differential categories and as *Cartesian* differential categories. Tensor differential categories have their roots in linear logic and the work of Thomas Ehrhard who provided several key examples of these settings. The coKleisli category of a tensor differential category is a Cartesian differential category – although it is not the case that a Cartesian differential category need to arise. Cartesian differential categories provide the categorical semantics for the "resource λ-calculus" (Curian, Boudel) which is equivalently the "differential λ-calculus" (Ehrhard, Regnier) – both these systems arose from computer science considerations. Similarly, the differentiation of combinatorial species (Joyal) can be organized to provide a Cartesian differential category – and the differentiation there is the same as the differentiation of datatypes (Huet, Abbott, McBride). These applications are significant as they illustrate an key feature of differential categories: they are "additive" settings which do not assume the presence of negatives. Of course, the standard differential from elementary calculus – which absolutely assumes negatives – also gives an example of a Cartesian differential category.

While differential categories collect a wide range of examples they fail to explain examples arising from differential geometry. Furthermore, there are now a number of important variations on differential geometry. For example: synthetic differential geometry, known as SDG (Kock, Lawvere), convenient differential geometry (Kriegl, Michor), and tropical differential geometry (Mikhalkin). This last example is of special significant to this development as negatives are not assumed. In order to capture these settings – and the standard settings from differential geometry and algebraic geometry – Geoff Cruttwell and I introduced the notion of a "tangent category". It turned out that we were not the first to have developed these ideas. Rosicky some thirty years earlier had introduced the notion of a tangent functor. His excellent but brief paper on the subject had received one citation in that time and, indeed, it would have escaped our notice but for Anders Kock who brought it to our attention.

A tangent category is essentially a category with a tangent functor in the sense of Rosicky – although reformulated so that negatives are not required. This associates naturally to each object a tangent bundle, $p : T(A) \to A$, satisfying certain axioms. Significantly the notion not only captures all the settings mentioned above, but also the notion of a Cartesian differential category. Even more significantly, once one reformulates the notion of "vector bundles" for the setting, the category of such bundles over any object naturally forms a Cartesian differential category – this prompted us to name this abstract reformulation a "differential bundle" to emphasize this connection.

At the time that Rosicky's paper was written many of the above connections could not have been made – the surrounding geography of ideas had not yet been developed. In retrospect, however, one can see that his notion – suitably generalized to the additive setting – unifies the broad subject matter of differential geometry. This makes it reasonable to speak of the subject embodied by tangent categories as the subject of "abstract differential geometry".

3.4 Category theory for the masses: A tale of food, spiders and Google

Bob Coecke (University of Oxford, GB)

We will demonstrate the following. Category theory, usually conceived as some very abstract form of meta-mathematics, is present everywhere around us. Explicitly, we show how it provides a kindergarten version of quantum theory as well as a new process-based foundation of it, how it helps to automate quantum reasoning, and how it will help Google to understand sentences given the meaning of their words.

3.5 Categories in cognition: An integrative model for multi-systems

Andrée Ehresmann (University of Amiens, FR)

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Biological, cognitive or socio-economic systems are evolutionary, self-organized, multi^3 systems meaning: multi-level hierarchy, multi-agent cooperation/concurrence, multi-temporality. While most models retain only some of these properties, the *Memory Evolutive Systems* (developed with J.-P. Vanbremeersch since several years) propose an integrative approach to such systems, accounting for their local/global internal dynamics "in the making" and their emergence properties. MES rely on various categorical tools: "partial" (pre)sheaves, (co)limits and hierarchical categories, (pro-)sketches, leading to the Emergence and Double Complexification Theorems. The talk gives (http://ehres.pagesperso-orange.fr/Dagstuhl.pdf) give a brief summary of the main characteristics of MES, emphasizing some difficult computational problems. For more details, cf. our book *Memory Evolutive Systems: hierarchy, emergence, cognition* (Elsevier, 2007). For recent applications of MES to cognition, to creativity, to innovation in design and to anticipation, cf. the site http://ehres.pagesperso-orange.fr.

3.6 Intermodelling, queries and Kleisli categories

Tom Maibaum (McMaster University – Hamilton, CA)

After a brief review of some CT based approaches used in software engineering research, we explore the specification and maintenance of relationships between models, which are vital for Model Driven Engineering. We show that a wide class of such relationships can be specified in a compact and precise manner if inter-model mappings involve derived model elements computed by corresponding queries. Composition of such mappings is not straightforward and requires specialized algebraic machinery. We present a formal framework, in which such machinery can be generically defined for a wide class of meta-model definitions, and thus important inter-modeling scenarios can be algebraically specified and formalized.

3.7 Coalgebraic methods in epistemic game theory

Martin Meier (Institute for Advanced Studies – Vienna, AT)

Economic game theory has be concerned with incomplete and imperfect information since the seminal papers of Harsany in the Sixties. Harsanyi already then has been thinking about the problem of modelling expectations of players and concretely about the expectations about expectations, i. e. hierarchies of mutual expectations of the players involved. He considered the mathematical constructions too difficult and the problem was not tackled until much later in the work of Mertens, Zamit or Heifetz. In 2004 finally Moss and Viglizzo showed how to construct the type space of the hierarchical expectations as final coalgebras. This close connection of categorical methods with traditional modelling problems in economic game theory is an example where recent research begins to consider how category theory can help to construct the complicated informational or epistemic structures about who knows and beliefs what in epistemic game theory.

3.8 Some categorical approaches to concurrency theory

Ugo Montanari (University of Pisa, IT)

Concurrency can be studied from two different viewpoints: operational and abstract. In the former sense, concurrency defines equivalence classes of computations such that concurrent events can be executed in any order. Abstract concurrency observes causality dependencies between events and consequently defines certain notions of bisimilarity. Category theory is useful in both cases, offering natural notions of: (i) deterministic and non-deterministic concurrent computations; and (ii) coalgebraic transition systems labelled with event dependencies. About (i) we summarize the theory of P/T nets and pre-nets as developed by Meseguer, Sassone, Baldan, Bruni and the author. About (ii) we consider the notion of causal tree by Darondeau and Degano as the prototypical definition of causality, and derive from it a coalgebra in a presheaf category whose index category contains partial orders on events and monotone mappings. Applying a result by Ciancia, Kurz and the author, it is then possible to collapse such a coalgebra on a much smaller, often finite one, corresponding to the notion of causal automaton by Pistore, developed long time ago, with some ingenuity.

3.9 The next 700 logics

Till Mossakowski (University of Magdeburg, DE)

In this talk we analyse and clarify the method to establish and clarify the scope of logic theorems offered within the theory of institutions The method presented pervades a lot of abstract model theoretic developments carried out within institution theory. The power of the proposed general method is illustrated with the examples of (Craig) interpolation and (Beth) definability, as they appear in the literature of institutional model theory. Both case studies illustrate a considerable extension of the original scopes of the two classical theorems. Our presentation is rather narrative with the relevant logic and institution theory concepts introduced and explained gradually to the non-expert audience.

3.10 Abstract nonsense about gaming

Dusko Pavlovic (University of Hawaii – Manoa, US)

Game theory has largely been developed as the theory of equilibria that emerge from interactions between rational players. Since equilibria invariably arise as fixed points of diverse dynamics, which are conveniently presented as the morphisms in various categories, it is not surprising that the categorical presentations of a great variety of gaming processes turn out to be natural, and even fairly succinct. E. g., Moss and Viglizzo showed in CMCS 2004 how to present Harsanyi's type spaces as final coalgebras, and I proposed in CALCO 2009 (also http://arxiv.org/abs/0905.3548) a framework for deriving and analyzing categorically a broad range of gaming equilibria, old and new. These treatments, however did not lead to any new results in either game theory or category theory, but perhaps pointed to a convenient language. In this workshop talk, I discuss a dark alley of game theory where the categorical presentation seems to shine some light. The dark alley is the notion of bounded rationality, as gaming where the players are logically and computationally limited, so that they may fail to reach the optimizations that are too hard to construct. This is not only a very natural concept, going back to Herbert Simon's work from the late 1950s, but also a tremendously important practical problem, fully in the focus of experimental and behavioral game theory. Yet the underlying theory seems surprisingly thin. The main technical tool of the theory of bounded rationality is the view of strategies as state machines, and the technical results are in the tune of "this state machine strategy is the best response in the populations that consist of these 6 (or so) state machine strategies". The more desirable proofs that a state machine strategy is the best response against *all possible* state machine strategies generally remain beyond reach, because quantifying over all state machines is hard. And if we cannot calculate the best response strategies, then it is of course even harder to calculate any equilibria. A natural way beyond this obstacle is to look at strategies as Turing machines, or computable functions of some other form. This seems much better, because there are universal Turing machines, and they make quantifying over all Turing machines into an effective operation. Alas, it is fairly easy to construct games such that the best response to computable strategies is not computable. The point of my talk was to present a categorical model that seems to show the way out of this impasse. It captures the games in which the players do not just optimize following their respective utilities, but can also use deceit to outsmart the opponent, thus leading to *learning equilibria*. Some of the presented ideas originated in conversations with Viktor Winschel during his visits to Egham and Honolulu.

3.11 Equational Logic for PROPs

Gordon Plotkin (University of Edinburgh, GB)

We would like to program with various forms of graphs. For example, they occur in rule-based modelling in biology, representing individual proteins or populations of them. The various forms of dags used include: Milner's place graphs, link graphs and bigraphs, and Danos *et alii*'s kappa graphs. We would particularly like a general algebraic framework including textual notation. We could then consider equational technology: matching, rewriting, etc, whether qualitative or quantitative. In this talk we begin such a project by seeking an equational logic for dags. From a categorical point of view these form certain symmetric monoidal theories (aka PROPS), and so we approach the problem by analogy with standard equational logic and Lawvere theories. As a warm-up we consider linear equational logic and operads, which are intermediate between Lawvere theories and PROPs.

3.12 Categories in quantum theory

Peter Selinger (Dalhousie University – Halifax, CA)

I started by reviewing the basic postulates of quantum mechanics, and what they mean in the context of quantum computing. Then I reviewed the graphical languages of symmetric monoidal categories and of compact closed categories. The central concept of categorical quantum mechanics is that of a dagger compact closed category, first defined by Abramsky and Coecke. Many properties of finite-dimensional Hilbert spaces (i. e., complex linear algebra, or finite-dimensional quantum mechanics) can be axiomatised within arbitrary dagger compact closed categories. Finally, I reviewed completeness theorems and completely positive maps, which unify the concepts of unitary transformations and measurements.

3.13 Brzozowski's Algorithm (Co)Algebraically

Alexandra Silva (Radboud University Nijmegen, NL)

We show a proof of correctness of Brzozowski's minimization algorithm using a combined algebraic and coalgebraic view on automata. We discuss how this is an example of a potentially wider application of categorical methods to algorithms.

3.14 A topos of probabilistic concepts

Alex Simpson (University of Edinburgh, GB)

In *A review of probability theory*, Terence Tao stresses that 'probabilistic concepts' are distinguished by being preserved under 'extensions' to the sample space. Such extensions allow one, for example, to freely introduce new independent random variables. The general principle can be reformulated from a category-theoretic perspective: 'probabilistic concepts' organise themselves into *presheaves* over the category of sample spaces and extensions.

In this talk, we take the category of measure-preserving maps (identified mod 0) between standard probability spaces as the category of sample spaces. We observe that presheaves of probabilistic interest (e. g., presheaves corresponding to individual probability spaces, presheaves of random elements) form *sheaves* for the atomic Grothendieck topology. Thus we are led to the stronger idea of 'probabilistic concepts' as sheaves. Various constructions on (pre)sheaves are discussed, including a notion of independent product. The main result is a characterisation of the sheaf property in terms of conditional independence.

As future work, we hope to develop the idea that the resulting boolean sheaf topos offers a mathematical universe combining set-theoretic and probabilistic primitives, somewhat in the spirit of the universe envisaged in David Mumford's *The dawning of the age of stochasticity.*

4 Working Groups

4.1 Network theory

John C. Baez (University of California – Riverside, US)

Besides the author of the abstract, the participants included Fabio Gadducci, Barbara König, Ugo Montanari, Pawel Sobociński, and Jamie Vicary.

In the working group sessions on Network Theory, we discussed a general framework for networks with inputs and outputs, such as electrical circuit diagrams, signal flow diagrams, and the like. The basic idea is to consider a category where objects are labelled finite sets and morphisms are labelled graphs having designated sets of "input" and "output" vertexes. In fact, this kind of category should extend to a bi-category where the 2-morphisms are graph rewrites. Furthermore, this bi-category should be symmetric monoidal, allowing us to compose networks "in parallel" as well as in series.

We investigated how to prove these claims. Jamie noted that Michael Shulman's idea for constructing symmetric monoidal bi-categories should be useful, at least in the simpler case where the 2-morphisms are all isomorphisms. We explored that and found no obstacle. For bi-categories where the 2-morphisms are graph rewrites, Andrea and Fabio's work may prove useful. Pawel pointed out that the technique of double pushout rewriting, used to study graph rewriting, makes use of a concept that has a nice 2-categorical formulation: namely, adhesive categories.

Wrap up. In short, researchers from rather different areas met and discovered a body of common techniques, which could be used to solve some open problems.

4.2 Game theory

Viktor Winschel and Philipp Zahn (University of Mannheim, DE)

Besides the authors of the abstract, the participants included Robin Cockett, Martin Meier, Jeffrey C. Morton, Dusko Pavlovic, and Peter Selinger.

Viktor notes that reflexivity in economics and computer science is at the core of both sciences. In computer science this is due to the need to discuss code as data or the universality of Turing machines. In economics it arises because the modeler of the society is in the society hence the model and the modeled interact. There are many existing modeling issues that are ultimately in need of a reflexive treatment. An example is the learning equilibrium he defined with Dusko where they use the existence of the universal Turing machine to define a learning equilibrium in a game theoretical setting where strategies, the learning mechanism, process strategies, the actual ones for choosing the moves of a player in a game.

Dusko asserts that the minority game is an interesting example for applying the concept of a learning equilibrium because traditional analysis is not really helpful.

After a brief discussion of mixed strategy equilibria and their interpretation, Martin remarks that there are alternatives to viewing players as really randomising. One prominent example being "purification" by Harsanyi.

Viktor emphasizes that there have been already several projects in between economics and computer science where reflexivity has been addressed. He has been involved in one with Samson Abramksy that uses coalgebras to define infinite games and coinduction to proof equilibria. In another with Achim Blumensath coalgebras are used to compose games from simpler units and which allows a reflexive feature of game being played in networks about the very structure of the network. he mentions an ongoing project with Paulo Oliva and Julian Hedges from computer science where higher order functions are used to define very general games. It allows to define a reflexive structure of players in a game solving the game in order to coordinate or differentiate in a very general sense. Currently the project encompasses a game simulation, equilibrium checking and solving engine for this general kind of games.

Dusko asks what is the result of Escardo/Oliva that is used.

Philipp replies that for the first paper there is no result being used but a mere clarification of concepts. The crucial elements are quantifiers and selection functions, both higher order functions. The first result is that one can represent standard game utility functions as a special case. As an example one can consider an agent choosing between different petrol stations looking for the cheapest fuel. Here the environment is the cost functions linking petrol station and cost of fuel. The quantifier is then a function mapping from this cost function into the minimal price whereas the selection function is mapping from the cost function into the optimal choice, i. e. the cheapest petrol station. Besides being able to instantiate the standard approach one can go much beyond. In particular, one can have outcomes with less structure but even more importantly one can consider goals of agents that comprising the whole environment and not only the outcomes, respectively the image of the outcome function. One example is a beauty contest where one agent is only interested in voting for the winner and not for one particular candidate. An example is a game with three judges and two candidates, judge three being an agent only motivated by voting for the winner of the voting contest. This agent can be represented by a fixed point quantifier, respectively selection function.

Viktor remarks that in some sense this is capturing context-dependency.

Jeffrey wonders how dynamic games fit into that framework, and how is the standard framework able to capture reflexivity?

Dusko replies that equilibrium modelling is one thing but the dynamics beyond are important and not addressed so far – von Neumann himself was opposing the Nash program.

To this effect, Jeffrey mentions a game, Nomic, introduced by Peter Suber, a lawyer, aiming at creating policy rules that are implementing good policies where the issue of constitutional rules versus other rules become important.

Dusko further affirms that in some sense the learning equilibrium is a step in this direction because there learning is a change of strategy. But moreover, there can be dynamics beyond the point where everything is learnt.

Philipp notes that in "rock, scissors, paper" there are empirical frequencies different from the standard result of equally randomising between the three different moves.

Jeffrey remarks that an interesting personal experience of playing Nomic was that loopholes turned out to become important and most often policies ended up being under attack by people successfully finding loopholes.

Robin asks where category theory occurs, i. e., how a categorical game theory looks like.

Philipp states that to some degree there is composition already in the selection function approach because a game theoretic setting is reduced to a decision problem at all stages.

Along this line, Martin states that you can do the same with standard game theory by looking at pathological games of one person.

Jeffrey wonders how can there be reflexivity in normal games.

And Peter further asks if there is a use of categories in economics besides game theory.

Viktor replies that in general category theory should be very suited also to macro economics as a framework for synthesis and system analysis. For example there might be a use of sheaf theory as a general approach for the transition from the local to the global level, hence macroeconomics. Here reflexivity arises explicitly as the Lucas critique by the need to consider that agents take the theory and the model of the politician into account, which the politician need to take into account as well. Or a so to say vertical reflection where the macro aggregates like interest rate depend on the discussion of the agents who in turn decide based on the interest rate. Similarly the general issue of institutional dynamics is about the context being depended on but also ruling the content.

Dusko notes that category theory in quantum physics was not "demanded" by the experimental side, it clearly came from the theoretical side.

As before, Martin asks if in physics peoples use categories outside of quantum physics.

Peter states that it is most prominent is quantum physics but there is also use outside of it. And people such as John Baez promoting the view that, for instance, higher order categories should play a more important role.

Martin notes that in physics in a situation with many particles it possible to focus on the nearest particles; those farther apart can be neglected. This is hardly the case in economics.

Peter remarks that in markets everything seems to be related to everything.

Dusko recalls that there is a branch of research that wants to use concepts from quantum physics to apply them to markets, in quantum economics and econophysics.

4.3 Semantics

Alexandra Silva (Radboud University Nijmegen, NL)

Besides the author of the abstract, the participants included Samson Abramsky, Andrea Corradini, Bartek Klin, Francois Metayer, Andy Pitts, Alex Simpson, Peter Selinger, and Sam Staton.

Francois asks if there are many parts of CT that are actually applied in CS, and wonders if one can enumerate the main areas that are applied. Andy thinks it is too wide a subject and that there is no such thing as the CT that is applied in CS. However, Samson points that the general approach is the same even if different sub-areas of CT are applied to different areas of CS/Math. Samson mentions the work by Baez on Higher Categories in Math and mentions that the fact that these are so hard to work with reduces the impact ... It is noted that homotopy type theory is attracting a lot of interest. A new trend seems to be to make models intensional, both in type theory and concurrency.

Peter asks if CT in semantics is on the rise. Alexandra replies that in programming languages there is more and more of a trend to use CT as a base for designing. Other examples mentioned by people are homotopy type theory, an area where people apply CT unapologetically, and theorem proving systems, which are becoming more and more usable and thus more widespread. and there CT will survive and thrive. It is noted that automated theorem proving is a good way to bring CT to the mainstream.

It is noted that coalgebra came from the CT community but has a life of its own ... they found interesting applications in automata theory. There seems however to be a lot of inbreeding, while on the contrary the communication to other communities is an important point. Learning to talk with different communities is an import an point in the quest of finding open problems in other areas where CT can make a contribution. Coinduction in theorem provers is still an open problem and though the communication issue with the theorem proving community is not such a big issue one still does not see enough cross-fertilization. Peter mentions that communication problems are also present in quantum computation though there one already sees contributions. He also argues that a weakness of semantics/CT is that the definitions play a key role. Having the right definitions makes the theorems trivial, which is the opposite of hard subjects where they have combinatorial proofs of theorems (and simple definitions). CT is very organized, good for compiler design with complicated inter-dependencies (and not so complicated algorithmic problems). In general, the audience agrees that people sees category theorists only as reconstructing the things they knew already, and that is a disadvantage, because we do not give them a good reason to care enough.

Andy wonders about linguistics. Samson tells that there are successful stories there. Word semantics based on a vector representation of words. This is used in information retrieval. The nice observation is that using tensor product you can compose information. Goes back to Lambek grammars. This is the subject of Bob Coecke's talk.

The audience consider crucial the two following questions: How will we promote CT? And should we be training pure CT? Also relevant is to understand the need of a venue, i. e., of a focused forum. There is of course a CT conference. but is this enough? Bartek proposes a forum like Highlights: 8 min talks for around 100 talks, with no claim of originality. As a general remark, it is true that there are several areas where CT is a commonly used tool: quantum, type theory, coalgebra, ... Each of these areas has its own workshop, but this may not be enough for further spreading the use of CT.

A few more areas are identified where it would be good to have an impact: security, algorithms & complexity, machine learning. Andy asks about the NSA take on category theory and Peter mentions they are actually interested in formal methods. Concerning security: proving the safety of a property means imagining all possible ways that an attacker can use. Could the attacker's behavior be abstracted by means of CT tools? How about security compositionally? Related to this, and a main conference such as ICALP, Samson mentions that going into track A might give fruits. In his talk he had an example related to oracles and this seems to be a good way to bring track A subjects to track B.

Wrap up. Category theory started as re-writing arguments of other areas. Now we have a very powerful toolkit. Compositionality is one of the strengths. Applied CT can use the toolkit in other areas, and in its spread computational tools will play a key role.

The book of Spivac is an interesting result of someone who came into CT from the outside. There is still need to have books/tutorials on how to use basic category theory.

Having a conference or a training network could create the synergy to produce the tutorials needed for a widespread adoption of CT.

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