Solving the p-median location problem with the Erlenkotter approach in public service system design

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Abstract

This work deals with the problem of designing an optimal structure of a public service system. The problem can be often formulated as a weighted p-median problem. Real instances of the problem are characterized by big numbers of possible service center locations, which can take the value of several hundreds or thousands. The optimal solution can be obtained by the universal IP solvers only for smaller instances of the problem. The universal IP solvers are very time-consuming and often fail when solving a large instance. Our approach to the problem is based on the Erlenkotter procedure for solving of the uncapacitated facility location problem and on the Lagrangean relaxation of the constraint which limits number of the located center. The suggested approach finds the optimal solution in most of the studied instances. The quality and the feasibility of the resulting solutions of the suggested approach depends on the setting of the Lagrangean multiplier. A suitable value of the multiplier can be obtained by a bisection algorithm. The resulting multiplier cannot guarantee an optimal solution, but provides a near-to-optimal solution and a lower bound. If our approach does not obtain the optimal solution, then a heuristic improves the near-to-optimal solution. The resulting solution of our approach and the optimal solution obtained by the universal IP solver XPRESS-IVE are compared in the computational time and the quality of solutions.

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1 Introduction

The p-median location problem has became one of the most well-known and studied problems in the field of facility location. Solving the p-median location problem in the public service system design is the NP-hard problem [6, 7]. The public service system structure is formed by deployment of limited number of the service centers and the associated objective is to minimize costs. The family of public service systems includes medical emergency system [11], fire-brigade deployment, public administration system design and many others. Mathematical models of the public service system design problem are often related to the p-median problem, where the p-median problem is formulated as a task of determination of at most p network nodes as facility locations. In real problems, the number of serviced customers takes the value of several thousands and the number of the possible facility locations can take this value as well. The number of possible service center locations impacts the computational time. To obtain good decision on the facility location in any serviced area, a mathematical model of

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the problem can be formulated and some of the mathematical programming methods can be applied to find the optimal solution. Reese summarized the exact solution methods for the p-median problem in [4]. Mladenovic summarized the heuristic methods in [8]. Avella, Sassano and Vasil´ev presented a branch-and-price-and-cut algorithm to solve large-scale instances of the p-median problem in [9]. Balinski provided an early integer programming formulation of the plant location problem that has historically been adapted to the p-median problem in [10]. Erlenkotter designed the approach for solving of the location problem in [1]. Erlenkotter approach is based on the theory of duality and the branch and bound algorithm. The suggested approach realizes an algorithm DualLoc. Körkel continued and improved the Erlenkotter approach and designed an algorithm PDLoc in [2]. Janáček and Buzna [3] improved the Erlenkotter and Körkel approach and designed an algorithm BBDual for solving the uncapacitated facility location problem. BBDual was extended by the Lagrangean relaxation of the constraint which limits number of the located center. Usage of the Lagrangean relaxation allowed to solve the p-median problem with the Erlenkotter approach. The Algorithm pMBBDual [5] was designed for solving the p-median problem with the Erlenkotter approach and the Lagrangean relaxation. If the algorithm pMBBDual does not provide the optimal solution, then we would like to improve the near-to-optimal solution by some heuristic.

2 Problem formulation

The p-median location problem finds the optimal location of exactly p facilities, so that the sum of the distances between customers and their closest facilities, measured along the shortest paths, is minimized. The location problem consists of a placing facility in some sites of a given finite set I such as hospitals, police stations, warehouses and the customers from a given finite set J such as people, patients in hospital or villages and cities. The costs of the optimal deployment of facilities in the specific network constitute the fixed charges f_i and the costs c_{ij}. The fixed charges f_i introduce costs for the facility location at the location i. The costs c_{ij} introduce costs for the demand satisfaction of a j-th customer from the location i. The formulated p-median location problem can be modeled using of the following notation. Let the decision of the service center location at the place $i \in I$ be modeled by a zero-one variable $y_i \in \{0, 1\}$ which takes the value of 1, if the center is located at i, otherwise it takes the value of 0. In addition, the variables $z_{ij} \in \{0, 1\}$ for each $i \in I$ and $j \in J$ are introduced to assign a customer j to a possible location i by the value of one. The maximal number of the facility locations introduces a constant p. The p-median location model can be formulated as follows:

Minimize
$$\sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} z_{ij} \tag{1}$$

Subject to:
$$\sum_{i \in I} z_{ij} = 1 \qquad \forall i \in I \tag{2}$$
$$z_{ij} \leq y_i \qquad \forall i \in I, \forall j \in J \tag{3}$$
$$\sum_{i \in I} y_i \leq p \tag{4}$$
$$y_i \in \{0, 1\} \qquad \forall i \in I \tag{5}$$
$$z_{ij} \in \{0, 1\} \qquad \forall i \in I, \forall j \in J \tag{6}$$

The objective function (1) minimizes the total costs of the p-median location problem which consists of the fixed charges f_i and the costs c_{ij}. The constraints (2) ensure that each customer is assigned to the exactly one possible service center location. Binding constraints (3) enable to assign a customer to a possible location i, only if the service center is located at this location. The constraint (4) bounds the number of the located service centers. The obligatory conditions in the mathematical model are (5) and (6). This location problem without the condition (4) gives the uncapacitated facility location problem (UFLP). If the location problem contains the condition (4) and the fixed charges f_i is equal zero for each $i \in I$ then it becomes p-median problem. Our p-median location problem is the combination of the UFLP and the p-median problem.

3 Solution method

Algorithm pMBBDual [5] provides us a possibility of solving the p-median problem with an iterative approach. The main advantage of the algorithm pMBBDual is the transformation of the p-median location problem to the UFLP. The mathematical model (1–6) using the Lagrangean relaxation of the constraint (4) which limits number of located centers is modified as follows:

Minimize
$$\sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J} c_{ij} z_{ij} + lg(\sum_{i\in I} y_i - p) \tag{7}$$

Subject to:
$$\sum_{i\in I} z_{ij} = 1 \qquad \forall i \in I \tag{8}$$
$$z_{ij} \leq y_i \qquad \forall i \in I, \forall j \in J \tag{9}$$
$$y_i \in \{0,1\} \qquad \forall i \in I \tag{10}$$
$$z_{ij} \in \{0,1\} \qquad \forall i \in I, \forall j \in J \tag{11}$$

The solution of the mathematical model (7–11) represents one iteration of the algorithm pMBBDual. The quality and the feasibility of the solution of the suggested approach depends on a suitable setting of the Lagrangean multiplier lg. The suitable value of the multiplier can be obtained by a bisection algorithm. The comparison between XPRESS-IVE and the pMBBDual showed that the algorithm pMBBDual does not provide the optimal solution for the location problem all the time. We can obtain the optimal solution of the model (1–6) by repeating the solution of the model (7–11) with a change of the Lagrange multiplier lg until the last member of the objective function (7) is equal to zero. If a last member of the objective function (7) is non-equal to zero then we obtain some solution. The value of the obtained solution provides the lower bound (12) of the problem (1–6) which is written in a relation:

$$OF_{RP} = \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J} c_{ij} z_{ij} + lg(\sum_{i\in I} y_i - p) \tag{12}$$

The relation (13) provides the value of the obtained feasible solution of the problem (1–6):

$$OF_{NP} = \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J} c_{ij} z_{ij} \tag{13}$$

We obtain some feasible solution, but a value of the optimal solution is between the values of the non-relaxation problem solving OF_{NP} (13) and the LP-relaxation solving OF_{RP} (12). If our approach does not obtain the optimal solution then we improve the near-to-optimal solution by the heuristic, which will be presented in the next chapter.

3.1 An exchange 1–1 heuristic with the reallocation of locations

An exchange 1–1 heuristic with the reallocation of locations works on the assumption that we have the best feasible and infeasible solution. The best feasible solution is the solution where number of locations is smaller and the nearest to p locations. The solution obtained by the algorithm pMBBDual is the best feasible solution. The best infeasible solution is the solution where number of locations is bigger and the nearest to p locations. The exchange 1–1 heuristic with the reallocation of locations has 3 phases:

Phase 1 – A reallocation of locations
A phase of the reallocation consists in creating of sets I_S and I_D from the locations in the best feasible and infeasible solution. The set I_S consists of the same locations in both solutions and the set I_D consists of the different locations in both solutions.

Phase 2 – An addition of locations to p
A phase of the addition consists in a separation of the set I_D to the two subsets I_{DI} and I_{DN}. The locations from I_{DI} are included in the set I_S and they create initial solution which will be improved. The set I_{DI} consists of the included locations. The set I_{DN} consists of the locations which are not included in the solution.

Phase 3 – A searching of the suitable exchange and its realization
A phase 3 searches the combination of locations which improves the actual solution. We exchange only one location from the set I_{DI} and the location from the set I_{DN}. If we obtain an exchange which improve the actual solution then we realize the exchange of the locations and update sets of I_{DI} and I_{DN}. The improving exchange can be obtained by the strategy first admissible or best admissible.

The exchange 1–1 heuristic ends when we do not find an improving exchange.

4 Numerical experiments

All numerical experiments mentioned in this paper were performed on a PC equipped with Intel(R) Core(TM) i7 Q720 1.6 GHz processor, 8 GB RAM. The tested benchmarks consist of the Slovak cities and villages in Slovak road network. The benchmarks in the Table 1 create the cost matrix consisted of only a distance between the places. The benchmarks in the Table 2 create the cost matrix consisted of a distance between the sites and the demands of the villages or the cities. We compare the solution obtained by our approach and the optimal solution obtained by the universal IP solver XPRESS-IVE in the computational time and the quality of solutions. The quality of solutions indicates the value of the objective function OF and the GAP_{ES} and the GAP_{LB}. The GAP_{ES} represents the difference between the values of the best found solution OF_{V1} and the exact OF_{ES} one expressed in the percentage of the exact solution OF_{ES} as follows:

$$GAP_{ES} = \frac{OF_{V1} - OF_{ES}}{OF_{ES}} * 100 \quad (14)$$

The GAP_{LB} corresponds to the difference between the value of the best found solution OF_{V1} and the lower bound OF_{RP} expressed in the percentage of the lower bound OF_{RP} as follows:

$$GAP_{LB} = \frac{OF_{V1} - OF_{RP}}{OF_{RP}} * 100 \quad (15)$$

4.1 Experiments 1

The experiments in the Table 1 was realized on the benchmarks from the set of all 315 places in the district of Žilina (Figure 1).

In the Table 1 a column p gives the maximal number of the facility locations, columns $t(s)$ give the computational time in seconds, columns NoF give number of the facility locations for the individual solution methods. A column OF_{ES} gives the value of the optimal solution, a column OF_{NP} gives the value of solving the non-relaxed problem (1–6), a column OF_{RP} gives the value of the solution LP-relaxation – the lower bound of the optimal solution, a column OF_{V1} gives the value of the final solution of the exchange 1–1 heuristic with the reallocation of locations. A column GAP_{EX} gives the representation in the percentage obtained by the relation (14) and a column GAP_{LB} gives the approximate representation in the percentage obtained by the relation (15). The time $t(s)$ in V1 gives the total time of the algorithm pMBBDual and the improving heuristic.

The experiments in the Table 1 shows that the algorithm pMBBDual gets better time in compared to the universal solver XPRESS-IVE, but the obtained solution by the algorithm pMBBDual can be optimal or near-to optimal. The obtained near-to optimal solution can be improved using the improving heuristic. Using of the improving heuristic to the obtained solution shows us the possibility of obtaining the optimal solution or better near-to-optimal solution at the expense of increasing the computational time. The computational time of the algorithm pMBBDual with the improving heuristic is better than the time obtained by the universal solver XPRESS-IVE. The GAP_{ES} shows that the difference in percentage is not worse than 1 %. Based on the experiments in the Table 1 the improving heuristic can provide the very near-to-optimal solution. The universal IP solvers are limited for solving the large problems. If we cannot obtain the value of the optimal solution with XPRESS-IVE, we use the lower bound OF_{RP} for the comparison of solutions. The GAP_{LB} is approximate representation in the percentage because the value of optimal solution do not need to equal the lower bound. The distortion is demonstrated in the Table 1 for the value p equals 210, where GAP_{LB} is 10,94 % and GAP_{ES} is only 0,47 %.

4.2 Experiments 2

Experiments in the Table 2 was realized on the benchmarks from the set of the customers consisting of all cities and villages and the set of the candidates consisting of the 1000 biggest villages and cities in Slovak Republic (Figure 2). In the Figure 2 red points give the candidates and all points give customers.

In the Table 2 a column p gives the maximal number of the facility locations, columns $t(s)$ give the computational time for the solution methods in seconds, a column NoF gives number of the facility locations obtained by the algorithm pMBBDual without the heuristic. A column OF_{NP} gives the value of solving the non-relaxation problem (1–6), a column OF_{RP} gives the value of the solution of the LP-relaxation – the lower bound of the optimal solution, a column OF_{V1} gives the value of the final solution of the exchange 1–1 heuristic with the reallocation of locations. A column GAP_{LB} gives the approximate representation in the percentage obtained by the relation (15). The time $t(s)$ in V1 gives total time of the algorithm pMBBDual and the improving heuristic.

The experiments in the Table 2 shows that we obtain the optimal solution by the algorithm pMBBDual with the improving heuristic for all selected value of p. The value of the optimal solution is equal to the lower bound of the solution. If we compare the computational times

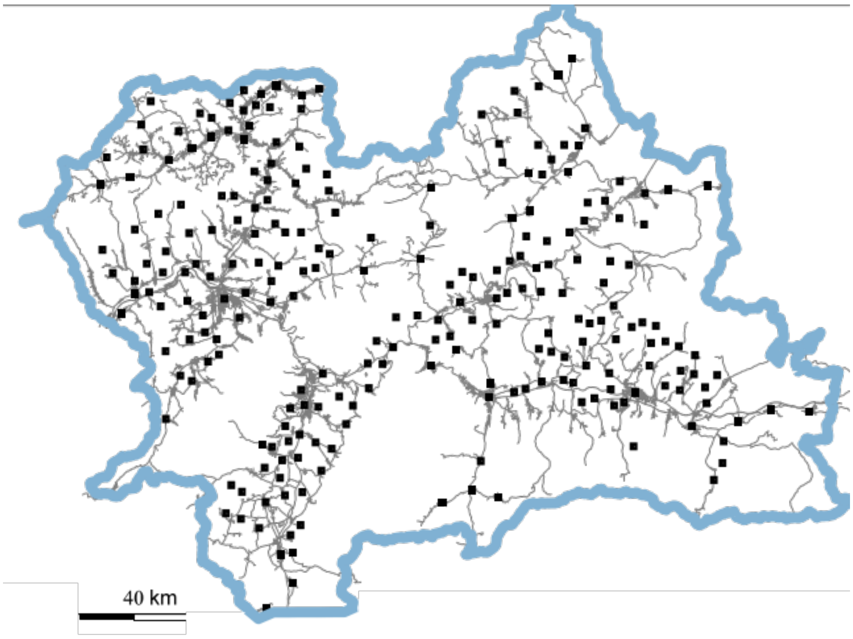

Figure 1 Deployment of cities and villages in district of ŽIlina.

Table 1 Comparison of solutions XPRESS-IVE and pMBBDual without and with heuristic.

	XPRESS-IVE			pMBBDUal				V1			
p	$t(s)$	OF_{ES}	NoF	$t(s)$	OF_{NP}	NoF	OF_{RP}	$t(s)$	OF_{V1}	GAP_{ES}	GAP_{LB}
15	14,13	2803	15	2,52	2803	15	2803	–	–	–	–
30	18,74	1832	30	1,78	1832	30	1832	–	–	–	–
45	13,65	1400	45	2,15	1420	44	1400	2,60	1401	0,07	0,07
60	16,92	1138	60	2,65	1183	57	1138	3,14	1138	0,00	0,00
75	13,6	944	75	2,11	977	72	944	2,20	945	0,11	0,11
90	13,53	801	90	1,88	828	87	801	1,91	802	0,12	0,12
105	13,69	686	105	1,64	707	102	686	2,70	690	0,58	0,58
120	13,69	595	120	2,01	637	113	595	4,31	598	0,50	0,50
135	13,5	516	135	1,88	637	113	505	2,82	517	0,19	2,38
150	13,59	441	150	1,85	506	137	441	4,26	444	0,68	0,68
165	13,72	372	165	1,98	506	137	366	4,04	374	0,54	2,19
180	14,12	312	180	1,85	340	173	312	13,32	313	0,32	0,32
195	14,45	257	195	1,88	340	173	252	10,17	257	0,00	1,98
210	13,96	212	210	1,85	340	173	192	5,86	213	0,47	10,94
225	14,4	167	225	1,88	206	212	167	3,00	167	0,00	0,00
240	14,2	123	240	1,61	206	212	122	3,04	123	0,00	0,82
255	14,29	93	255	1,78	107	248	93	2,00	93	0,00	0,00
270	13,77	63	270	1,74	107	248	63	1,98	63	0,00	0,00
285	13,71	33	285	1,58	107	248	33	2,01	33	0,00	0,00
300	13,74	14	300	1,58	23	291	14	1,85	14	0,00	0,00

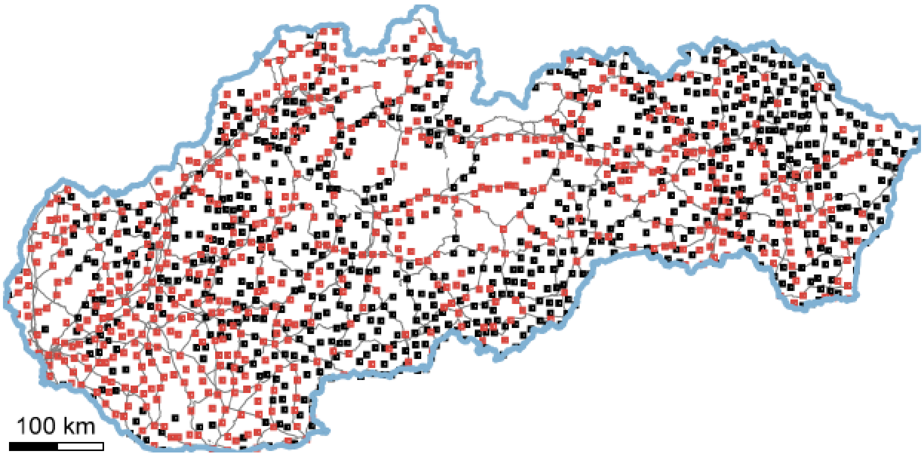

Figure 2 Deployment of cities and villages in Slovak Republic.

Table 2 Results of algorithm pMBBDual without and with the improving heuristic.

	pMBBDUal				V1		
p	$t(s)$	OF_{NP}	NoF	OF_{RP}	$t(s)$	OF_{V1}	$GAP_{LB}(\%)$
50	100,8	535448	50	535448	–	–	–
100	35,4	321869	100	321869	–	–	–
150	65,7	248373	150	248373	–	–	–
200	113,0	207011	200	207011	–	–	–
250	104,5	178645	250	178645	–	–	–
300	132,4	160076	297	159044	143,3	159044	0,00
350	68,2	144076	350	144076	–	–	–
400	68,4	132427	400	132427	–	–	–
450	54,9	123775	449	123615	65,3	123615	0,00
500	71,9	116650	498	116388	72,6	116388	0,00
550	52,3	110558	549	110450	52,7	110450	0,00
600	46,0	105418	600	105418	–	–	–
650	46,1	101209	650	101209	–	–	–
700	41,0	98229	694	97869	41,9	97869	0,00
750	38,6	95308	747	95158	39,4	95158	0,00
800	39,0	93000	796	92832	40,3	92832	0,00
850	38,7	90976	848	90906	40,6	90906	0,00
900	39,1	89648	889	89340	42,1	89340	0,00
950	39,4	88264	945	88164	41,3	88164	0,00
1000	0,3	87427	1000	87427	–	–	–

of the algorithm pMBBDual without and with the improving heuristic then the time of the realization of the improving heuristic is a few seconds.

5 Conclusions

Solving the p-median location problem in the public service system design is NP-hard problem. The optimal solution of the problem can be obtained by the universal IP solvers only for smaller instances of the problem. The universal IP solvers are very time-consuming and often fail when a large instance is solved. Our approach to the problem was based on the Erlenkotter procedure for solving of the uncapacitated facility location problem and on the Lagrangean relaxation of the constraint which limits number of the located center. We designed algorithm pMBBDual which does not provide optimal solution for location problem every time. So we tried to improve the obtained near-to-optimal solution with some heuristic. We designed the exchange 1–1 heuristic with the reallocation of locations. The resulting solution of our approach with the exchange heuristic and the optimal solution obtained by the universal IP solver XPRESS-IVE were compared in the computational time and the quality of solutions. Based on the numerical experiments we review that the solution obtained by the algorithm pMBBDual is possible to improve. We cannot obtain the optimal solution from near-to-optimal solution with the suggested improving heuristic every time. But the improving heuristic can provide the very near-to-optimal solution in many instances of the solved problem.

We improved the obtained solution by one heuristic, but in the future we would like to design the other improving heuristics and choose the best heuristic to our approach. We would like to generalize the Erlenkotter approach, design an algorithm with the Erlenkotter approach for solving the p-median location problem which is not iterative and compare algorithm pMBBDual with the improving heuristic, the algorithm with Erlenkotter approach which is not iterative and the Z-Erlange and branch algorithm [12] in the computational time and the quality of the obtained solution for the large-scale problems.

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