Demand models for the static retail price optimization problem – A Revenue Management perspective

Timo P. Kunz and Sven F. Crone

Department of Management Science, Lancaster University
Lancaster LA1 4YX, United Kingdom
`{t.p.kunz, s.crone}@lancaster.ac.uk`

Abstract

Revenue Management (RM) has been successfully applied to many industries and to various problem settings. While this is well reflected in research, RM literature is almost entirely focused on the dynamic pricing problem where a perishable product is priced over a finite selling horizon. In retail however, the static case, in which products are continuously replenished and therefore virtually imperishable is equally relevant and features a unique set of industry-specific problem properties. Different aspects of this problem have been discussed in isolation in various fields. The relevant contributions remain therefore scattered throughout Operations Research, Econometrics, and foremost Marketing and Retailing while a holistic discussion is virtually non-existent. We argue that RM with its interdisciplinary, practical, and systemic approach would provide the ideal framework to connect relevant research across fields and to narrow the gap between theory and practice. We present a review of the static retail pricing problem from an RM perspective in which we focus on the demand model as the core of the retail RM system and highlight its links to the data and the optimization model. We then define five criteria that we consider critical for the applicability of the demand model in the retail RM context. We discuss the relevant models in the light of these criteria and review literature that has connected different aspects of the problem. We identify several avenues for future research to illustrate the vast potential of discussing the static retail pricing problem in the RM context.

1998 ACM Subject Classification G.1.10 Applications

Keywords and phrases Revenue Management, Pricing, Retail, Demand Modeling

Digital Object Identifier 10.4230/OASIcs.SCOR.2014.101

1 Introduction

The practical value of Revenue Management (RM) and its success throughout many industries is well documented. RM can also be lauded for its achievements as an academic discipline as it has been tremendously successful in bridging the gap between theory and practice and connecting industry and academia. Even more importantly, it has established itself as a discipline in its own right that encourages a broader view on problems usually only studied in isolation, bringing together aspects from various fields ranging from forecasting, econometrics and mathematical programming, to computing, strategic management, operations management, and behavioural marketing.

While RM takes different perspectives (e. g. quantity vs. price based or industry specific), the commonality typically is the scope of dynamically pricing perishable products over a finite selling horizon. However, in most retail formats, the static problem of pricing products that are continuously replenished and virtually non-perishable is fundamental and constitutes

4th Student Conference on Operational Research (SCOR'14).
Editors: Pedro Crespo Del Granado, Martim Joyce-Moniz, and Stefan Ravizza; pp. 101–125
OpenAccess Series in Informatics
OASICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

a problem in its own right. This has been insufficiently acknowledged in RM literature where the dynamic problem is ubiquitous and the static case, as it presents itself in the retail scenario, is only sparsely recognized: While most RM textbooks do not address the problem at all (e. g. [98], [63]), Talluri et al. [109] very briefly discuss the special characteristics of the grocery pricing case and mention the determination of "baseline prices". In their survey on emerging trends in retail pricing practice, Levy et al. [72] acknowledge "two disparate pricing problems: Fashion and staple merchandise" and discuss the static pricing problem for the latter.

We argue that studying static retail pricing under an interdisciplinary proposition offers interesting opportunities in research and practice alike, as well as for the convergence of the two, and promises to be equally rewarding as it has been for the dynamic problem. While there is a considerable body of surveys for the dynamic scenario where many also address the dynamic retail case ([13], [27], [45], [39], [122], [83]), a review that frames the static retail price optimization problem in this way does not exist.

We want to advance the discussion by filling this gap and discussing the demand models used for the static retail price optimization problem in an RM context. In this spirit, we take an interdisciplinary view, highlighting the interplay of the optimization system components – in particular the data, demand, and optimization model – and the questions emerging from the interactions of these components. We define five criteria from a practical point of view that we consider critical for the relevance of the demand model for real life applications and discuss existing demand models under this premise. We argue that it is beneficial to include the static problem in the RM discourse and develop directions for future research.

We proceed as follows: In Section 2, we give a broad overview of where the static pricing problem has received attention and discuss contributions that take a systemic view in an RM sense. In Section 3, we highlight the characteristics of the problem that originate from the specific properties of the retail environment and define five criteria that we consider essential for applicability. Subsequently, in Section 4, we review demand models which we organise in absolute and relative models. In Section 5, we discuss these models with respect to the criteria defined earlier. We conclude by developing suggestions for future research in Section 6.

2 Literature on static retail pricing

2.1 The scope of Revenue Management literature

RM as a field is designed to be interdisciplinary. This is illustrated by the standard text books of the field that cover diverse aspects of the RM process, such as demand model estimation, optimization, forecasting, behavioural aspects, or implementation (e. g. [109], [98], [63]). Talluri et al. [109, p.5] argue in their text that the scientific advances in Economics, Statistics, Operations Research (OR) and Information Technology are driving the RM approach. Interdisciplinarity is also a cornerstone in the declared scope of the journals dedicated to the field (e. g. Journal of Revenue and Pricing Management, International Journal of Revenue Management) that also emphasize applicability and stress their ambitions to bring practice and academia together. Dedicated RM articles covering questions around the 'traditional', dynamic RM problem also exist side by side in the literature that discusses the theoretical groundwork for these contributions, most notably in Operations Research and Management Science where RM holds a strong footprint.

The static retail pricing problem is virtually not included in this discussion. Questions around retail pricing and the optimal price are discussed within Marketing and Retailing instead. While the relevant publications (e. g. Journal of Retailing, Marketing Science) regularly feature contributions that are useful in the context, RM as a discipline is here practically non existent. Relevant articles on pricing topics, a very popular research stream, are usually discussed under terms such as 'product line pricing', 'pricing and promotion', or 'optimal pricing'. The difference in naming is of course not an issue, however RM has gone a long way in facilitating an interdisciplinary and practical discussion as described above which currently does not exist for the static problem.

The above can be further illustrated by the challenges and suggestions for future research presented in dedicated reviews of the different fields. In RM literature, the research proposed seems to span a wide variety of themes that include aspects not directly associated with typical OR interests and often driven by very practical concerns: reoccurring themes are processing and operationalisation of data, often with links to forecasting and predictive accuracy, concerns of decision support systems, including implementation, and the (probably more typical) overarching question of how to increase the accuracy, efficiency and general benefit of the system and its components (e. g. [27], [73], [39]). While the avenues for future research suggested in similar contributions in the Marketing and Retailing literature point to equally important questions, a systemic view is rarely considered. The focus seems more rooted in the respective discipline and often centered around issues of customer behaviour, such as the acceptance of and the reaction to (optimal) prices and promotions, price implementation from an organisational perspective, and effectiveness from a behavioural point of view (e. g. [72], [47]).

2.2 Retail pricing literature in the Revenue Management context

Research that examines the static pricing problem in this way has been undertaken. However, contributions are scattered throughout disciplines and hence do not form a coherent discussion. In the following we want to give a brief overview where the relevant studies can be found and link to questions and articles for the three connecting fields demand modeling, optimization, and decision support systems. Rather than providing an exhaustive review, we want to highlight a few selected studies that can provide a starting point for a more comprehensive review.

The largest body of literature relevant for the problem can be found under the term 'product line pricing'. It is a rather wide field, mainly motivated by demand interdependencies, that includes the small scale manufacturer's as well as the large scale retail problem (reviews in [49], [84], [99]). However, in Marketing and the affiliated Retail literature the discussion of product line pricing has been accelerated by the adoption of the Category Management idea. The primary focus of these studies is usually price elasticities and their purpose is often purely descriptive (e. g. [24], [18], [102], [14], [55]). In a time where the only normative guidance offered was in the form of pricing heuristics, Urban [113] presented one of the first works to use econometric instruments to explicitly address product line pricing decisions with notable contributors to follow in the decades to come (e. g. [100], [26], [127], [119], [85], [87]). In a parallel research stream, similar models have been been developed and used for the promotion problem (e. g. [111], [15]).

First, we want to highlight research at the intersection of data and demand modeling. As this is obviously not a specialty of Marketing, few descriptive and almost no normative contributions focusing on such questions can be found. The influence of data conditions (e. g. seasonality, stationarity, intermittency) on the problem at hand, a topic foremost treated in the forecasting literature, has not been evaluated yet. Some papers have addressed questions of data processing (e. g. data pruning, data aggregation, data cleansing) in a descriptive way (e. g. [4], [19], [126]). Questions around data types and data availability have been discussed similarly (e. g. [40], [54]). Directly intertwined with the above is the development of corresponding estimation methods, a topic usually primarily treated in the corresponding Econometrics literature. Most visible in our context have been contributions based on Bayesian methods (e. g. [58], [101]).

Second, unlike for the dynamic case, there is no natural link to OR literature where the intricacies of the resulting optimization problem are discussed. The literature dedicated to special questions arising from that context appears especially sparse. The only contribution explicitly dedicated to the problem that has come to our attention is Subramanian et al. [108] who present a linearization for an MNL based retail price optimization model. In some instances, the static pricing problem can also be found treated jointly with questions from inventory management or assortment planning (review e. g. [65]). Here, the literature covering aspects of optimization seems significantly more developed.

Third, some studies that focus on operationalisation of the models from a Decision Support System view or in the form of practice reports exist. Naturally, commercial providers of optimization systems do not bring their proprietary models into the public domain. Exceptions are some applications with commercial implementations that have been driven by academia and are hence published. Most notable are a price optimization system for a DIY retailer ([94], [95], [93]), and a system for treating the very unique problem of an automotive aftermarket retailer [78]. Some systems are documented that cover promotion planning but are insightful for the static problem as well (e. g. [12], [38], [105]). Further, some contributions highlight aspects of the architecture of such a system (e. g. [41]) or address questions around their implementation (e. g. [86]).

3 Framing static retail pricing in the Revenue Management context

Many models rest on assumptions or simplifications that can not be upheld when considering the reality of the retail environment. While studying these models undoubtedly holds academic merit, in this review we want to focus on, and accentuate applicability. It is therefore essential to consider the defining characteristics of the static retail problem which are decisive for the practical suitability of the model. In agreement with Kopalle [66] who names factors to consider when optimizing retail merchandising decisions and Levy et al. [72] who determine aspects to consider for determining optimal prices in retail, we focus entirely on the interaction between data, demand, and optimization model in the RM system and define five requirements that a demand model needs to satisfy in order to be applicable in a real life retail environment:

1. Inclusion of cross price effects
2. Reliance on operational (store-level) data only
3. Suitability for industry size problems
4. Potential to accommodate typical retail data conditions
5. Sensible solutions when used for optimization

(1) Inclusion of cross price effects: The demand model can be considered to be the core of the price optimization system [118]. Within the system, it acts as the middle piece between data and optimization model and is also the centre of our analysis. As the first requirement, we want to discuss a characteristic that is demand model intrinsic: The interdependency between products is a defining property of the retail assortment and a key-differentiator to the equivalent manufacturer problem. The consideration of cross price effects in the demand model formulation is therefore fundamental. If this is relaxed, which effectively implies ignoring the assortment context and optimizing the price of each product in isolation, the problem is greatly reduced but clearly leads to inferior results (e. g. [100], [46]). Even though we acknowledge that systems that only rely on an abstract or indirect consideration of cross price effects have also proven to be useful (e. g. [95], [93]), we formulate as our first requirement that the demand model needs to include cross price effects.

(2) Reliance on operational (store-level) data only: We want to highlight the sources of retail data. Two forms of data are commonly discussed: Store-level data, that usually contain sales and marketing mix information per item and store, most often aggregated to a weekly level, and panel data, which are on a household level and therefore tend to include additional information such as demographics. While both have individual advantages and disadvantages and often have complementary uses, there is no consensus in research whether either will actually deliver superior results or affect model accuracy ([110], [1], [51], [5]). However, for applied purposes, store level data can be considered far more relevant as they are readily available as a byproduct of the payment process and therefore inexpensive to acquire. Data that contain additional details such as in-store product location, store layout, or manufacturer marketing efforts can normally not be reliably extracted from operative systems and, even though potentially usable in experimental settings, are largely unsuited for a productive RM system with the capability to address managerial issues on a realistic scale. Further, the cost and effort to collect panel data make them prohibitive for operational use. Even though a lot of retailers have customer card schemes that allow them to capture similar, panel like data, due to the self selective nature of these schemes that usually only appeal to a specific group of customers, the data generated are not representative and only partially useful for our purposes. Therefore, as a second requirement, it needs be possible to estimate the demand model given store-level scanner data that can be captured from operational processes.

(3) Suitability for industry size problems: We want to discuss the challenge of estimating the demand model given the empirical dimensions of the retail problem described above. The data reality of retail is usually defined by its high volume as can be seen by the dimensions of data sets such as the Dominick's database [61] or the IRI marketing data set [22]. It involves thousands of products in an assortment in which even the smallest category has more than a hundred individual products, thousands of transactions and often a great number of stores. Levy et al. [72] even describe the sheer size of the problem as "daunting".

Therefore, as a third requirement, it needs to be possible to estimate the model on a realistic, industry size scale without reducing or oversimplifying the data, in a way that the capability of the system to determine prices on the level of the individual product remains uncompromised.

(4) Potential to accommodate typical retail data conditions: We take a closer look at conditions and properties of such data in the form that they are actually found in real data scenarios as it will further complicate the estimation problem described above. Additional challenges for the estimation of the demand model originate from underlying data conditions: Examples are dynamic variation of the choice set over time due to changing assortment through product introduction and discontinuation, strong seasonality, demand intermittency for slow moving items, or seemingly intermittent demand due to stock out situations. It is unreasonable to expect to effortlessly accommodate every possible data condition in a demand model, yet it is essential that the formulation of the model is flexible enough that it can be adjusted accordingly or that it allows for estimation techniques fit for the data conditions encountered. We purposely choose a broad formulation for our fourth requirement and state that the demand model needs to allow estimation under diverse data conditions.

(5) Sensible solutions when used for optimization: Finally, we want to take a closer look at the interaction between the demand model and the optimization model. As mentioned above, the presence of cross price effects is a defining characteristic of the retail assortment. This has implications for the determination of optimal prices: commonly, products in a category are substitutes rather than complements and their cross price elasticities are therefore positive. Due to this property, popular demand model forms such as linear or log-linear formulations, or choice based market share models imply that maximum profit is obtained by raising the price of one product to infinity while decreasing it for the remaining products (detailed discussion in [3]). Obviously, this is not only from a structural, but also from a practical perspective unacceptable. A different source for logical inconsistencies is that, depending on the model configuration, negative values for demand or for prices lead to implausible price sets. In this regard, Zenor [127] describes the tradeoff between logical consistency in the domain of prediction and logical consistency in the domain of optimization for different demand model configurations. As our fifth and last requirement we therefore state that given the demand model, the formulation of our optimization system should account for the above and must not describe an unbounded solution space nor yield negative prices or extreme price sets as optimal solutions.

It is worth to reflect on what is meant by 'extreme price sets': Even though profit maximization is the undisputed long run goal of every retail operation, as a sole objective, it would be misguiding for the short- and mid-term. Retailers are very keen to balance profitability with other objectives such as revenue, unit sales, or market share as well as softer objectives such as price image or assortment attractiveness. This highlights that operational retail pricing is actually a multi-objective problem in which extreme, profit optimal solutions that might involve pricing a product out of the market or losing large amounts of sales or revenue are normally unacceptable. Nonetheless, if the model formulation prevents extreme prices as described above, the price sets produced tend to be useful even under single objective optimization.

For the sake of completeness, we also want to comment on the tractability of the model as optimization literature normally puts emphasis on the topic. Due to the practical approach of this paper, we do not consider this to be a critical characteristic as increased computing power and the existing numerical methods make it nearly irrelevant for practical purposes. This seems counterintuitive at first as we discussed the large scale of the retail problem earlier. Natter et al. [95] even describe in a Marketing Science practice prize report how their industry application effectually works with full enumeration.

4 Demand models for retail pricing

4.1 Overview

In the following we want to review the causal models at the center of the price optimization system which are usually discussed in literature as demand or sales response models. Since we are concerned with the static pricing case, the optimal price set is virtually independent of the demand level. Unless it is treated jointly with questions from inventory management or allocation, non-causal models find little attention in the relevant literature. We therefore exclude time-series models as well as stochastic models from our review; while these are essential for the dynamic pricing problem they are of negligible importance for our purposes. Even though some of the models used for purely descriptive purposes are of little use for the normative case, others can be very beneficial and are therefore included. Further, most of the models mentioned in the promotional literature are very close to the models needed for our purposes and will therefore be considered.

Various useful taxonomies can be found in literature that organise along dimensions such as dynamic effects, uncertainty handling, individual versus aggregated models, and the model level [74], or intended use of the model (descriptive, predictive, normative), behavioural detail, and consideration of competition [71]. For our purposes, we adapt a frequently used taxonomy and organise the models in two categories: absolute and relative demand models. An absolute demand model, also known as a product-level model, is any model that specifies the price-demand relationship focusing on the individual unit, such as stock keeping unit (SKU) or product or an aggregation thereof such as brand or category, as the level of analysis without (or only secondary) regard to the market environment. The dependent variable is usually an absolute performance figure such as revenue or unit sales. A relative demand model is any model where the share of sales of a unit such as SKU or brand is modeled in relation to a group or aggregate such as category. The dependent variable for our purposes is typically market share.

4.2 Absolute demand models

4.2.1 Model configurations

In their simplest configuration, demand models are strictly linear in the form of the standard regression model as described in (1), where Q is the dependent, absolute performance variable, $X_1, X_2, \ldots, X_n$ are covariates, $a_0, a_1, \ldots, a_n$ are linear parameters, and ε_i is a normally distributed error term. Additional flexibility can be achieved with a nonlinear configuration, yet it usually comes at the cost of increased complexity. A whole range of models offers this flexibility but can be transformed to a linear formulation: The multiplicative model (also known in Economics as Cobb-Douglas Response Function) (2) or its single variable equivalent known as the power function (3). Another important linearizable configuration is the exponential model (4). All of the above can be linearized by logarithmic transformation. The result can be described as a more general form of the strictly linear model discussed above that can be stated as described in (5) where g represents a transforming function of the variables. This kind of configuration is commonly known as the double-log (also linear-in-logs or log-linear, or, if not all covariates are in logarithmic form, as mixed-log) model (6). The semi-logarithmic model is the equivalent with the response variable in its non-logarithmic form. For the sake of completeness, we want to mention models that are non-linear and can not as easily be linearized and are hence intrinsically non-linear (examples in [74, p.76]). While most of the relative sales models presented in Section 4.3 use a nonlinear form, the use of inherently nonlinear absolute models for the purpose of price optimization purposes is rare.

$$\textit{Strictly linear}: \ Q = a_0 + a_1X_1 + ... + a_nX_n + \varepsilon_i \quad (1)$$
$$\textit{Multiplicative}: \ Q = a_0X_1^{a_1}X_2^{a_2}...X_n^{a_n}\varepsilon_i \quad (2)$$
$$\textit{Power}: \ Q = a_0X_1^{a_1}\varepsilon_i \quad (3)$$
$$\textit{Exponential}: \ Q = a_0e^{a_1X_1+a_2X_2+...+a_nX_n+\varepsilon_i} \quad (4)$$
$$\textit{General}: \ g_0(Q) = a_0 + a_1g_1(X_1) + a_2g_2(X_2) + ... + a_ng_n(X_n) + \varepsilon_i \quad (5)$$
$$\textit{Double-log}: \ ln(Q) = a_0 + a_1ln(X_1) + a_2ln(X_2) + ... + a_nln(X_n) + \varepsilon_i \quad (6)$$

4.2.2 Properties of model configurations

A key advantage of linear models is their ease of use and interpretation. Even though linear models can be criticized for being overly simplistic, they tend to provide good local approximations of more complicated formulations. For price optimization purposes, where an optimal solution that is acceptable from a practical point of view must usually be in close proximity of the current price, profit, and revenue combination, this is not a critical limitation. Further, a linear or linearized model can draw from a broad and well developed range of estimation instruments known from regression analysis. This tool set offers well explored solutions for many data properties or challenges seen in retail data and usually allows estimation based on store-level scanner data.

Returns to scale of the variables as determined by the model configuration and the corresponding implications for the (price) elasticities are of great importance for the optimization system. While the strictly linear model is characterized by constant returns to scale, and the exponential model by increasing returns to scale, the multiplicative model offers additional flexibility and can accommodate increasing as well as decreasing returns to scale. The latter appears particularly useful when sales are expected to go towards 0 for large values of price and price decreases are assumed to feature increasing returns to scale. An advantage of the semi-logarithmic configuration is the intuitive interpretation of the dynamics of the resulting model: constant percentage changes in one of the independent variables lead to constant absolute changes of the dependent variable, which in terms of sales resonates with the idea of a saturation limit. In terms of elasticities, this translates to $\epsilon_{X\,i} = \frac{a_iX_i}{a_0+a_1X_1+...+a_nX_n}$ for the strictly linear formulation while in the double-log form, elasticities are constant and hence the parameter can be interpreted directly as the elasticity so that $\epsilon_{X\,i} = a_i$. For the semi-log model, the elasticities $\epsilon_{X\,i} = \frac{a_i}{a_0+a_1ln(X_1)+...+a_nln(X_n)}$ mirror the diminishing returns to scale imposed by the log formulation of the parameters that translate into an absolute change of the response variable.

The capability of the variables to assume negative values is a property that is problematic due to the logical inconsistency it implies. This is possible in a strictly linear configuration and will lead to implausible price sets or predictions. Non-linearity in the relevant parameters such as it is the case in a log-log model, and (with minor restrictions) the semi-log model will prevent this.

Interaction between variables can be included in the model formulation and the various model forms offer different degrees of flexibility to do so. This significantly complicates the model even with a small number of variables. As mentioned previously, one of the most critical aspects in a retail pricing context is the consideration of cross price effects. While this is easily feasible for all model forms discussed, the inclusion of additional parameters for competing products will quickly lead to a degrees of freedom problem as we will illustrate in Section 5.

4.2.3 Instances in literature

In terms of covariates, the most simplistic models found in literature take one of the formulations named above and only include own price and the prices of competing products (e. g. [12], [121], [7], [100]).

While promotional effects tend to be a critical influence on retail sales, their consideration in a demand model is not as straightforward as price since promotional efforts can take various formats. Many models consider dummy variables indicating activities normally referred to as display, deal, advertising or feature (e. g. [58], [85], [35], [123]) or several of these (e. g. [129], [85]). In rare cases, interactions between two ([31], [69], [90], [103], [114], [124]) or even three ([96], [68]) of these promotional effects are considered. The inclusion of cross promotion effects between products is quiet rare and only sometimes considered for instances with a small number of products (e. g. [90]). In specialised models, the inclusion of promotions is more elaborate and includes influences such as a maximum deal discount for competing brands [14] or a price-cut ratio [103]. In this context, an area of special interest that has been extensively researched is sales effects of promotions before or after the promotional activity. Blattberg et al. define a deal decay variable that indicates the current week of an n-week multiweek deal [16]. More commonly, pre- or post-promotions lag effects are considered in various ways (e. g. [14], [77] ,[90], [114], [116]).

One of the most proliferated, specialized models for promotions is the SCAN*PRO model [124]. It uses a multiplicative formulation to model weekly store-level brand sales including variables for relative price, dummy variables for feature, display, or both, and indicators for week and store. It has been extensively applied in practice and research alike and has been adapted and expanded in numerous ways (e. g. [31], [35], [43], [67], [114], [115]).

Beyond the effects of promotional activities, there are temporal factors which can have a large impact on retail sales, foremost seasonality. The most proliferated approach is the inclusion of dummy variables that capture seasonality at a suitable level such as one variable indicating season or off-season [16], quarterly variables for each season (e. g. [58], [123]), or more granular solutions such as month and day (e. g. [7], [46]). In more sophisticated formulations, trigonometric terms are included [95]. Depending on the nature of the product, temperature can serve as a proxy for season or weather in general can be included in the formulation (e. g. [129]).

4.3 Relative demand models

4.3.1 Theoretical background

Relative sales models are often discussed as brand share, or market share models, even though the models are more versatile than this. While it is feasible to use any of the absolute model formulations discussed above to model market share instead of an absolute performance measure, the main structural shortcoming to this approach is a lack of logical consistency: It is desirable that market share is bound between 0 and 1 and that the sum of all shares will add up to 1 (e. g. [53, p.121] or [71, p.171]). In the following, we will discuss alternative approaches that naturally assure this integrity. We want to briefly highlight two essential concepts that motivate the better part of the models presented and that will serve as a basis for our discussion.

Traditionally, market share is considered to be an aggregated quantity of a product's or brand's individual sales in relation to the overall category sales. Bell et al. [10] stated in their Market Share Theorem that the market share S of a product i equals its attraction A

relative to the sum of the attractions of all products such that $S_i = \frac{A_i}{\sum_{j=1}^{i} A_j}$. The important difference here is that the market share of an individual brand or product does not only depend on its own variables (e. g. price or marketing effort) but is also directly influenced by the share of the competing brands or products and hence their attraction. While this idea can be motivated in different ways, it has become a key differentiator to conventional modeling concepts of brand share.

Another critical concept that motivates an entire class of models is to consider individual buying decisions of the consumers and derive the product's market share from its probability of being purchased. Models that build on this concept are generally referred to as (Discrete) Choice Models or Random Utility Maximization models. Here, customer's purchasing decisions among different alternatives are modeled rather than the sales of an individual product. The model is a straight forward extension of the logistic regression. At its centre is the decomposition of the utility of an option (i.e. a product) as the sum of a deterministic component u_i and a random component ε_i, so that the total utility can be expressed as $U_i = u_i + \varepsilon_i$. While the deterministic component expresses a utility that is perceived identically by all buyers, the random component represents customer heterogeneity. Therefore, realized utility between two buyers may be different even though the expected utility is the same. This can be interpreted as the heterogeneity of preferences across customers or as the unobservable factors affecting the utility of the product for the individual.

4.3.2 Model configurations

The most relevant models to operationalise market share draw from the ideas above. In terms of functional form, there are many different ways to formulate attraction A. A prominent example here is the Multiplicative Competitive Interaction (MCI) model, popularized by Cooper et al. [33], which formulates attraction as described in (7). Popular extension of this model explicitly include cross effects and are known as the Differential-Effects (or Extended) MCI and the fully extended MCI model [25].

A more contemporary approach draws from choice theory. The most widely used choice model is also the most proliferated relative sales model: The Mutinomial Logit (MNL) model, as formulated in (8). The random component follows a Gumbel distribution, so that its cumulative distribution is $F(\varepsilon_i) = e^{-e^{-\varepsilon_i}}$, and assumes independence of the errors. This error formulation is the key differentiator to the less frequently used Probit model that relies on a normal distribution of the error term and allows covariance between error terms to be non zero. A popular variation of the above is the Nested Logit Model which allows choices to be partitioned into subsets. Further, the popular Mixed Logit model relaxes some crucial restrictions by combining aspects of Probit and MNL. Various extensions of these ideas have been proposed.

$$MCI: \; A_i = exp(a_0) X_1^{a_1} ... X_n^{a_n} \varepsilon_i \tag{7}$$

$$MNL: \; A_i = exp(a_0 + a_1 X_1 + ... + a_n X_n + \varepsilon_i) \tag{8}$$

4.3.3 Properties of model configurations

In terms of returns to scale, the formulations are similar to their absolute counterparts: Since choice models have an exponential form, they do not allow for decreasing returns to scale. The multiplicative form of the MCI offers more flexibility in this regard.

A general disadvantage of relative sales models is that parameter estimates are generally harder to interpret. Also elasticities can not be used as commonly defined since the dependent variable is a share value between 0 and 1. A popular alternative are Quasi-Elasticities which relate a change in market share to a change in price $e_i = X\frac{\partial s}{\partial X}$, so that for a simple effect configuration we can formulate the elasticities as $e_i = \beta_l(1-m)$ for the multiplicative case and $e_i = \beta_l(1-m)x_{lj}$ for the MNL. It is clear that the elasticity in both formulations is dependent on the product's market share. A desirable property in this context is that the market share elasticity approaches 0 as share goes to 1 since any additional price decrease will only yield a small percentage increase in market share. This property holds for all model formulations presented above. Further, the formulation visualises that, as a fundamental difference between the two formulations, the elasticity in the MNL depends on the current price level.

While the vast majority of absolute models discussed were linear or linearizable, all relative models named here are inherently non-linear and generally not linearizable with standard transformation strategies. Approaches for linearization exist but are not without drawbacks (e. g. [92], [71, p.176], [33, p.144]). Hence Maximum Likelihood based estimation methods are normally needed. The complexity that comes with these methods is the reason why these models were available but were not as popular as linear or linearizable models. Only the advent of simulation-based estimation methods and the increase in computing power have facilitated their application and proliferation.

In the context of estimation, it is crucial to discuss the data used to calibrate these models. Due to the origin of choice models, the data traditionally and most commonly used are panel data on the individual or household level. Guadagni et al. [48] pioneered the use of scanner panel data for the estimation of an MNL and paved the way for this convenient and reliable data collection mechanism that many studies have followed. As we pointed out in Section 3, we deem it essential for practical price optimization purposes that the demand model can be calibrated on store level data. While this is the standard for the absolute models presented above, the models reviewed here can often not be operationalised in this way. The implications for the price optimization problem will be discussed in Section 5.

4.3.4 Instances in literature

The covariates normally included in the formulation of these relative models are similar to those considered in the absolute models introduced previously. In the same manner, the most simple models often only include price (e. g. [78], [62], [46]), or additionally a promotion component (e. g. [29]). The attraction formulation of this model class ensure that interaction effects between products are implicitly considered. However, the level of heterogeneity of cross price effects various considerably. Bultez et al. established the now popular distinction into simple effects model which only allows one parameter a_n per variable n which is equal across products, differential effects model, which accommodates individual parameters per product a_{ni}, and cross-effects model which allows for all possible cross effects between products a_{nij} [25]. Further, since most of the models found in literature are laid out for household level data, household specific variables such as brand loyalty (e. g. [48]) or also size loyalty (e. g. [52], [24]) are often included. Many studies also use choice models to model 'category incidence', meaning whether or not a product in the category was purchased. While this of course deviates from our definition of relative sales models, it is directly intertwined with the discussion at hand. The models used in this sense often feature covariates such as household inventory (e. g. [119], [24]), or rate of category consumption (e. g. [24]).

When it comes to the configuration of the models, the MCI model has been extensively used in earlier work (e. g. [25], [26], [113]) but has lost its popularity to choice models which, apart from being more versatile and in general feature advantageous properties, can rely on a sound theoretical motivation. Within the class of choice models, the footprint of the Probit model for the problem at hand is comparably small due its complexity when it comes to analytical expressions for its choice probabilities, as well as for the general challenges for its estimation and evaluation. However, we want to highlight the special importance of nested formulations for our problem: One of the most discussed properties of choice models is Independence of Irrelevant Alternatives (IIA), which in our case suggests that if an option is split in equal alternatives that are equivalent for the customer such as the same product with a new UPC or a different color, the combined market share of the split products will increase while the share of the competing products will decrease. Obviously, this conflicts with empirical evidence and managerial wisdom. Many approaches have been proposed to relax this property and most rely on organising the choice alternatives in hierarchies or trees. The Nested Logit model is one way to relax the IIA property and has been used successfully in many situations (e. g. [23], [111]). Moreover, the hierarchical structure imposed here can also be considered consistent with behavioural aspects of the choice process as it is believed that the purchasing decision is organised in hierarchical stages [42].

4.4 Other model forms and dimensions

Beyond the models that are classified and discussed above, there are many other model forms, dimensions, and modeling approaches that can be interesting in the pricing context. In the following, we want to provide some exemplary references for a few that we consider especially relevant.

Reference price: Behavioural aspects can greatly affect sales response and especially the reception of promotional stimuli. It is worth to consider aspects of prospect theory, mental accounting, and, for our purposes most relevant, reference price (review in [82]) in RM systems [118]. Kalyanaram et al. [60] identify empirical generalizations from reference price research while Briesch et al. [20] provide a comparative study of several models. Natter et al. [95] include a reference price component in the demand model of their price optimization system.

Game theory: Competitive effects are intensely studied in the retail context. To analyse the influence of competitive dynamics using the tools of game theory has proved to be very rewarding. In the retail scenario, relevant influences are for one customers that act strategically to promotions and markdowns which can have an impact on their sales response to undiscounted items. For another, manufacturers can react to prices set by the retailer by adjusting purchasing conditions and trade marketing payments for that retailer which can directly affect cost and profit. Further, competing retailers can strategically respond to price changes by adjusting their prices which again can have an impact on sales response. Moorthy [88], [89] provides a general overview of models. Even though game theoretical models are expected to have an increasing importance in the price optimization context in the future (e. g. [118]), an example in literature of their explicit use for normative static retail pricing has not come to our attention.

Hedonic pricing: A very traditional approach to pricing is the idea of hedonic pricing where the price of an item is determined as a combination of its attributes. The approach has a long history in economics and has been enjoying popularity for pricing real estate, while generating only little interest in retail (e. g. [79]). However, an interesting approach that is useful for our purposes draws from the same idea: As the level of analysis typically is either SKU, brand, or category, it can be advantageous to build models that use product characteristics as the basis of analysis. A small number of studies have made use of this concept in the context of retail demand modeling (e. g. [40], [42], [54], [117]).

Models from Economics: Demand modeling is a traditional field of interest of Economics. We want to highlight some models that are rooted in economic theory, and that focus on economic influences such as disposable income, or a firm's advertising expenditure (e. g. [104], [70]). Some of these models are well known and have been used and re-interpreted for decades such as the Translog Model [32], the Generalized Leontief model [37], and most prominently the Almost Ideal Demand System (AIDS) [36], and the Rotterdam model ([9], [112]). These models have been extended, extensively compared (e. g. [6], [8]), and have inspired and influenced other models (e. g. [14]). There are also contributions that adapt and re-interpret them for our purposes (e. g. [34], [3]).

Semi- and non-parametric models: We have focused on parametric models, but semi- and non-parametric approaches exist that allow more flexibility in response modeling. Models based on non-parametric or semi-parametric regression techniques (e. g. [59], [107]), Neural Networks (e. g. [56]), or Support Vector Machines (e. g. [80], [81]) have been used to model promotion and sales response in retail and can also serve as a basis for price optimization.

4.5 Discussion

As we have seen, there is a vast number of models as well as a plethora of modeling options available and naturally a single superior model for our purposes does not exist. Many of the studies using the models proposed have a primarily descriptive scope and do not share our normative focus. Further, the form of the model ultimately used is highly dependent on the aim of the study and the data available. Nonetheless, researchers obviously try to construct the model that yields the best performance, but even the criteria to assess this performance are disputable and can rely on various quantification concepts of model fit and forecasting accuracy. This is further complicated by the researcher's discretion in modeling and data processing.

We want to discuss functional form first. The question what is popular in literature is easier to assess than what is appropriate for our purposes. Due to its limited flexibility, there is a general criticism in literature for the strictly linear model (e. g. [75], [58]). Depending on era and scope, all other linearizable absolute configurations have been popular choices for modeling retail demand. Wildt [123] states that the most often found additive models are log-log and semi-log. Bitran et al. [13] describe the exponential configuration as commonly used to model demand in retail. Steiner et al. [107] state that the multiplicative, semi- and double-log models are the most popular choices to include nonlinearity in price response models. In a 1988 meta study by Tellis [110] of 424 models from 42 studies between 1960 and 1985, the two most common functional forms were additive and multiplicative models (including exponential and semi-log models). While attraction style formulation certainly has been used for decades, the rise in popularity of choice modeling facilitated by better estimation techniques has fairly recently shifted the field, so that choice models and foremost the MNL have become ubiquitous. Gupta [51] notes that regression models are often used to analyze store level data while the MNL is the preferred model for brand choice.

Many studies address which functional forms deliver the best results in their particular scenario (e. g. [119], [58]), or even focus on the comparison of model formulations under different objectives and applying different criteria (e. g. [44], [21], [57]). Most interesting for our problem are contributions that evaluate functional forms of models according to their capabilities of determining price elasticities: Bolton [17] studies differences between a linear, multiplicative and exponential formulation in own and cross price elasticity estimates. She concludes that results are very similar, yet overstatement is lowest for the multiplicative form while there can be significant differences across stores. Tellis [110] can not find statistical significant influence of the functional form on price elasticities. A direct comparison with relative models is difficult due to the definition of price elasticity outlined earlier. Hanssens et al. [53, p. 242] provide an overview of comparative studies that analyse functional forms of market share models.

We can summarize that the two groups presented above do not only show some fundamental differences in the formulation of their analytical objective but also differ in properties that are likely to affect the optimization system. While in the more traditional approach of modeling sales in absolute terms we can rely on easy accessible instruments for estimation, the modern, and theoretically more appealing concept of modeling relative market share comes with increased complexity in model formulation and estimation. We further see that the choice models introduced and foremost the MNL are ubiquitous due to a solid theoretical base. The MCI model as an alternative relative formulation has virtually vanished from literature and can be disregarded going forward. We now want to discuss the modeling concepts reviewed in light of the five criteria for applicability defined earlier in Section 3.

5 Demand models in the Revenue Management context

In Section 3, our first criterion from a data perspective was the **reliance on operational (store-level) data only**. While this is common for absolute models, we see that the majority of choice models in literature rely on panel data. Apart from the obvious challenge that these models often include covariates that can not be operationalised with aggregate data, other challenges are more severe. To discuss this further, we want to briefly revisit the original theoretical motivation of choice modeling: With its focus on the decision of an individual as probability of choice given the attributes of an individual, the application to model market share given the attributes of a product seems to depart considerably from the original intention. Therefore two major re-interpretations are necessary. First, we need to rely on the characteristics of the choice alternatives rather than the attributes of the individuals. The theoretical groundwork for this is provided with the conditional logit model. Second, we need to re-interpret choice probabilities as market shares which from a theoretical point of view is the bigger problem. Studies that rely on household level data do not need to make this assumption as choice probability of the household within the category can be modeled, which is not possible when using store level data. Choice probabilities are then often simply re-interpreted as market shares: e. g. Guadagni [48] models choice behaviour for a coffee category based on panel scanner data and derives market share by assuming that “for a given population the average probability of choosing an alternative is the expected share of choices for that alternative”.

However, to make such an assumption based on store level data is not without problems: Hardie et al. [54] name as main concerns the extreme assumptions about buying behaviour, the unrealistic assumption of an underlying multinomial process, and the problems with the random error component when using maximum likelihood estimation. A considerable body of literature exists though that contrasts and attempts to consolidate store-level and household-level estimation (e. g. [5], [51], [62], [128]). From a practical point of view, often little or no consequences are found when moving from household level to aggregated data

(e. g. [5], [2]). The discussion is directly intertwined with the question whether the inclusion of customer heterogeneity is beneficial and many studies present approaches to recover said heterogeneity from aggregated data (e. g. [28], [11], [62]).

In this context, a different, for its theoretical groundwork highly interesting research stream is a fairly recent development in Econometrics. To model a ratio, Papke et al. [97] introduced a generalized linear model named the fractional logit model that relies on a binomial distribution and a logit link function. This idea has been extended to the multinomial case (e. g. [64], [91], [106], [125]) and provides a different approach to the problem at hand. An (explicit) application to the retail price optimization problem has not come to our attention yet.

In the following we would like to discuss two criteria in conjunction. For the demand model we argued that the **inclusion of cross price effects** is imperative as intra assortment product dependency is one of the defining properties of the retail problem. From a data perspective, we also required the **suitability for industry size problems**. While even the estimation of a simplistic model can already be considered a challenge if done on a realistic scale, the inclusion of cross price effects considerably complicates this problem.

We want to briefly illustrate the above. If we were to ignore any cross price effects in a demand model formulation, we could either include a single, unified price parameter or an individual price parameter for each of N products. The obvious advantage of an attraction based over an absolute formulation is that even with such a simplistic model, interdependencies between products are implicitly captured. In this context, Mantrala et al. [78] point out the inferiority of an absolute log-log model for their three product scenario, because it would involve nine price parameters in comparison to one for the MNL. However, also in choice models, it can be advantageous to explicitly include cross effects in the model. Carpenter at al. [26] argue that an explicit inclusion can help dealing with the IIA property. If cross price effects are explicitly included, the number of parameters increases rapidly in any model configuration: In the context of Cooper's MCI mode, we already briefly discussed the different levels of cross price effects potentially considered. The total number of parameters needed depends on the exact model formulation but if we only consider a product specific, (intersect or attraction like) parameter and the price parameters, there would be $N+1$ parameters to estimate for the simple effects model, and $N+N$ parameters if we include product-specific price parameters like it is the case in a differential effects model. Accounting for cross price effects in the spirit of a fully extended model, we already need to determine $N+N+0.5\,N\,(N-1)$ parameters if we consider symmetric cross price elasticities, or even $N+N+N\,(N-1)=N+N^2$ parameters if we include asymmetric cross price effects.

Since even a smaller category has 50 active SKUs at any given time, this would already sum up to 2550 parameters to be estimated. With the number of parameters increasing quadratically with N, and considering the large product counts of a realistic retail environment, a degrees of freedom problem is easily visible. Several studies consider different levels of cross effects in their model construction (e. g. [46], [100]). The estimatability when done with the standard, well known methods, depends on the level of parameters included in the model formulation and the size of the problem as determined by the data available. It is obvious that in most applied situations, normative models can not 'afford' to include effects at this level of granularity without retreating to more advanced estimation techniques. As a result, many academic papers rest on assumptions that can not be upheld when considering the reality of the retail environment. It is common to reduce complexity by only considering 5 or 10, rather than 50 or 100 items per category. Even when working with real data, researchers normally prune or aggregate data and retreat to pricing on brand rather than product level which certainly is not possible for any practical application. The popular practice of focussing on the strongest products in the assortment is especially unfortunate from a practical perspective: Natter et al. [95] describe how their decision support system is especially helpful for low selling products as the retailer uses discretional pricing for the most popular items.

Fortunately, next to the high amount of products, the retail environment typically also features a high volume of transactions, an assortment that spans multiple categories, and often a large amount of stores. This information can be used on different levels to facilitate the estimation process and add stability to parameter estimates. Many contributions make use of this by using pooling (e. g. [31]) or Bayesian methods (e. g. [14], [85]) within the assortment context.

The aspect of multiple stores warrants special attention. On the one hand these data can be used as additional information to strengthen estimates (e. g. [78], [105]). On the other hand, store heterogeneity can be responsible for variance and many studies account for it in their model (e. g. [87], [7]). In regards to pricing, this also implies that store specific pricing can be beneficial (e. g. [78], [87]). A popular approach in theory and practice alike is the creation of clusters for pricing or estimation purposes usually discussed as zone pricing. The determination and utilization of such clusters has attracted much research attention (e. g. [30], [78]).

Within the context of the data model, we also formulated the requirement that the model should have the **potential to accommodate typical retail data conditions**. While this is a field that is intensely cared for in the forecasting literature, it is neglected in a retail pricing context. We will only discuss one example here: Dealing with censored demand is a prominent topic in forecasting literature and has also been popular in RM [50]. Vulcano et al. [120] propose a demand untruncation method based on store level retail data while Kok et al. [65] take a similar approach in an retail assortment planning context. The topic has not been discussed in a normative retail pricing scenario yet.

In terms of our requirements for the optimization model, it is most important that our model produces **sensible solutions when used for optimization**. Unfortunately, all demand models discussed above encounter problems when used in their simple form for optimization purposes as they yield infinite prices [3]. In literature, three different approaches can be found to avoid this issue:

1. A common solution is to impose restrictions on the size of the price changes (e. g. [100]). While retailers usually work with pre-defined maximum price changes, the outcome of the optimization would entirely rely on this arbitrarily defined restriction which is unsatisfying and unacceptable from a practical as well as from a theoretical perspective.
2. A more appealing alternative for attraction-style models is to define an 'external good', which represents the customer's no-buy option or the decision to buy outside of the category (e. g. [78]). While this is a seamless continuation of the theoretical reasoning behind the choice models introduced, the estimation in most empirical situations becomes difficult given our data requirements. Customers shopping but not buying in a category are not captured in scanner data. Moreover, the determination of quantity and marketing-mix of the external good is not obvious.
3. A better approach to the problem is to explicitly include the category purchase incidence into the model. Often, a second choice model is included to determine purchase probability of the category which will nest the brand choice model (e. g. [24], [119]). Again, operationalisation of the model will be difficult given our data requirements as some sort of absolute reference of category size will have to be determined that will not be available in store level data: e.g Mantrala et al. [78] treat the maximum weekly sales of the product

observed in a store as the weekly market potential, while Vilcassim et al. [119] utilise a household's number of visits to the store. A solution that can be estimated without any compromises is to chose an absolute model for category incidence so that category sales is modeled in dependence of the price level of the category (e. g. [3], [46], [108]).

Apart from solving the problem described above, dividing the demand model in this way has some additional advantages. It corresponds well with our behavioural understanding of the demand process as it is assumed that the decision maker first chooses whether or not to shop in a store or a category and then subsequently makes a choice between the available options. Further, Little et al. [76] formulate the idea of a two stage theory of price setting: once the customer is in the store, he will maximize his utility (short-run price response). However, utility level becomes a policy parameter that determines the long run attractiveness of the store. Further, a combination of models allows the modeler to combine advantages of relative models with those of absolute models such as accounting for seasonal or cyclical components in the absolute model without having to adapt the brand choice model. Such a configuration can also help to reduce multicollinearity.

We also mentioned the chance of extreme or negative prices in our requirements. The problem of optimal prices possibly being negative only arises if a linear formulation is used and can be avoided with practically any other formulation allowing non-linear returns of scale. The occurrence of other extreme prices can usually be attributed to unstable or false parameter estimates, usually with incorrect sign and of unreasonable size, due to data issues. Given a meaningful formulation and acceptable fit of the model, the optimal price set usually does not accommodate extreme prices.

6 Conclusion

We have shown that the static retail price optimization problem and its dynamic counterpart are discussed very differently: While for the latter, an interdisciplinary and application oriented discussion is entertained within the scope of RM, a similar discourse does currently not exist for the former. Instead, the primary discussion of the problem is taking place in the Marketing and Retailing literature, where a long history of very different, and mostly very well explored, demand modeling approaches are available to form the theoretical centre of the static retail price optimization problem. Naturally, said literature is primarily concerned with descriptive studies foremost focused on price elasticities, not with the normative questions and the implications of the systemic context derived from data, optimization and implementation. The body of literature addressing these questions is currently very small. We have seen that when these models are analysed with an applied, and integrative view, they collide with the practical requirements imposed by the environment of the price optimization system. Accordingly, there is a wide variety of areas that show great potential for future research. We believe that any sensible contribution on the topic that subscribes to such an integrative view in the spirit of RM is worthwhile. However, we want to highlight a few points of interest for which we see the most pressing need for research:

(Realistic) data conditions: We saw that every relevant paper engaging in empirical evaluation does so with an unrealistically small number of brands or products while using models or estimation techniques that are not fit for larger instances. Further, studies explicitly dedicated to analyzing the effects of the typical data conditions regularly studied in time-series analysis do not exist. We can encourage any effort that addresses the above by advancing estimation methods and studying the impact of these conditions on the price optimization system, its results and its effectiveness.

Data type, processing and organisation: The majority of studies rely on panel data even though store level data is much more relevant for retailers. Further, there is large discretion in the data treatment process that precedes any empirical evaluation. No unified data processing standard has been suggested yet. In their survey, Levy et al. [72] point to the connected question: "How do retailers group items into categories? What is the best way to categorize?". This is a promising area for future research, which undoubtedly is also very relevant from an RM perspective if linked back to the implications on the system. We can encourage any effort that helps making models accessible for use with store level data. Further, we consider the questions how data preprocessing methods influence the effectiveness and optimality of the pricing system especially pressing. Research in this area should pave the way towards a unified data processing standard for retail price optimization systems.

Operationalisation of choice models: According to van Ryzin [118], a shift from product centred models to choice models is needed. Choice modeling has already come a long way in the last decades. However, a lot of questions concerning the operationalisation of choice models in the retail environment remain unanswered such as the implications of the theoretical compromises made when estimating given store level data, including the assumption of discrete choice, and the handling of particularly large choice sets. Further, pretty much all efforts concerning this rely on the choice set as the critical base for modeling. Linking the usually data driven choice set determination to behavioural theory remains unexplored in the pricing context. We therefore believe that contributions fostering the theoretical base of choice modeling in a price optimization context will have great impact.

Multi-objectivity of the optimization problem: We want to reiterate that this is an area where literature is especially scarce. As the optimization problem as such is comparably simple and not as versatile as its dynamic equivalent, it might appear not as attractive from a purely academic perspective. However, a very intriguing aspect is the multi-facetted and conflicting objectives of retail pricing which goes beyond the scope of the original RM problem. Next to revenue and profit, retailers are often interested in preserving price image, market share or in keeping a competitive price level. While Levy et al. [72] suggest in their review to consider the question "How do these conflicting goals affect their customers and their profits?" we would also like to point to the potential to answer the questions usually asked in an OR context, including problem reformulation, efficient solving algorithms, or even in connection with robust optimization and risk management.

The innovations of the past two decades have paved the way for the practical implementation of retail price optimization systems. While the traditional, dynamic RM systems have matured, the static problem is trailing behind in theory and practice alike. We believe that a discussion of the problem in RM literature would encourage cross-disciplinary and practical oriented research that would go a long way in improving this situation.

References

1 Makoto Abe and Kirthi Kalyanam. Store Sales and Panel Purchase Data: Are They Compatible? *Mimeo*, 1995.

2 Greg M. Allenby and Peter E. Rossi. There is no aggregation bias: Why macro logit models work. *Journal of Business & Economic Statistics*, 9(1):1–14, 1991.

3 Eric Anderson and Naufel J. Vilcassim. Structural demand models for retailer category pricing. *London Business School Mimeo*, 2001.

4 Rick L. Andrews and Imran S. Currim. An experimental investigation of scanner data preparation strategies for consumer choice models. *International Journal of Research in Marketing*, 22(3):319–331, September 2005.

5 Rick L. Andrews, Imran S. Currim, and Peter S.H. Leeflang. A Comparison of Sales Response Predictions From Demand Models Applied to Store-Level versus Panel Data. *Journal of Business and Economic Statistics*, 29(2):319–326, April 2011.

6 Clifford L.F. Attfield. A Comparison of the Translog and Almost Ideal Demand Models. *Mimeo University of Bristol*, 564(4), 2004.

7 George Baltas. Modelling category demand in retail chains. *Journal of the Operational Research Society*, 56(11):1258–1264, March 2005.

8 William A. Barnett and Ousmane Seck. Rotterdam model versus almost ideal demand system: will the best specification please stand up? *Journal of Applied Econometrics*, 23(6):795–824, 2008.

9 AP P Barten. Evidence on the Slutsky conditions for demand equations. *The Review of Economics and Statistics,*, 49(1):77–84, 1967.

10 David E. Bell, Ralph L. Keeney, and John D.C. Little. A market share theorem. *Journal of Marketing Research*, 12(2):136–141, 1975.

11 David Besanko, Sachin Gupta, and Dipak C. Jain. Logit Demand Estimation Under Competitive Pricing Behavior: An Equilibrium Framework. *Management Science*, 44(11):1533–1547, November 1998.

12 Chitrabhanu Bhattacharya and Leonard M. Lodish. An advertising evaluation system for retailers. *Journal of Retailing and Consumer Services*, 1(2):90–100, 1994.

13 Gabriel R. Bitran and Rene Caldentey. An overview of pricing models for revenue management. *Manufacturing & Service Operations Management*, 5(3):203–229, 2003.

14 Robert C. Blattberg and Edward I. George. Shrinkage estimation of price and promotional elasticities: Seemingly unrelated equations. *Journal of the American Statistical Association*, 86(414):304–315, 1991.

15 Robert C. Blattberg and Scott A. Neslin. Sales promotion: The long and the short of it. *Marketing Letters*, 1(1):81–97, 1989.

16 Robert C. Blattberg and Kenneth J. Wisniewski. Price-induced patterns of competition. *Marketing Science*, 8(4):291–309, 1989.

17 Ruth N. Bolton. The Robustness Of Retail-Level Price Elasticity Estimates. *Journal of Retailing*, 65(2):193–219, 1989.

18 Ruth N. Bolton and Venkatesh Shankar. An empirically derived taxonomy of retailer pricing and promotion strategies. *Journal of Retailing*, 79(4):213–224, January 2003.

19 Richard A. Briesch, William R. Dillon, and Robert C. Blattberg. Treating Zero Brand Sales Observations in Choice Model Estimation: Consequences and Potential Remedies. *Journal of Marketing Research*, 45(5):618–632, October 2008.

20 Richard A. Briesch, Lakshman Krishnamurthi, Tridib Mazumdar, and S.P. Raj. A Comparative Analysis of Reference Price Models. *Journal of Consumer Research*, 24(2):202–214, 1997.

21 Roderick J. Brodie and Cornelis Kluyver. Attraction Versus Linear and Multiplicative Market Share Models: An Empirical Evaluation. *Journal of Marketing Research*, 21(May):194–201, 1984.

22 Bart J. Bronnenberg, Michael W. Kruger, and Carl F. Mela. The IRI marketing data set. *Marketing Science*, 27(4):745–748, 2008.

23 Patrick G. Buckley. Nested multinomial logit analysis of scanner data for a hierarchical choice model. *Journal of Business Research*, 17(2):133–154, 1988.

24 Randolph E. Bucklin and James M. Lattin. A model of product category competition among grocery retailers. *Journal of Retailing*, 68(3):271–294, 1992.

25 Alain V. Bultez and Philippe A. Naert. Consistent sum-constrained models. *Journal of the American Statistical Association*, 70(351):529–535, 1975.

26 Gregory S. Carpenter, Lee G. Cooper, Dominique M. Hanssens, and David F. Midgley. Modeling asymmetric competition. *Marketing Science*, 7(4):393–412, 1988.

27 Wen-Chyuan Chiang, Jason C. H. Chen, and Xiaojing Xu. An overview of research on revenue management: current issues and future research. *International Journal of Revenue Management*, 1(1), 2007.

28 Pradeep K. Chintagunta. Endogeneity and Heterogeneity in a Probit Demand Model: Estimation Using Aggregate Data. *Marketing Science*, 20(4):442–456, 2001.

29 Pradeep K. Chintagunta. Investigating Category Pricing Behavior at a Retail Chain. *Journal of Marketing Research*, 39(2):141–154, May 2002.

30 Pradeep K. Chintagunta, Jean-Pierre Dubé, and Vishal Singh. Balancing profitability and customer welfare in a supermarket chain. *Quantitative Marketing and Economics*, 1(1):111–147, 2003.

31 Markus Christen, Sachin Gupta, John C. Porter, Richard Staelin, and Dick R. Wittink. Using Market-Level Data to Understand Promotion Effects in a Nonlinear Model. *Journal of Marketing Research*, 34(3):322, August 1997.

32 Laurits R. Christensen and William Greene. Economies of scale in US electric power generation. *The Journal of Political Economy*, 84(4):655–676, 1976.

33 Lee G. Cooper and Masao Nakanishi. *Market-share analysis: evaluating competitive marketing effectiveness*. Springer, 1988.

34 Ronald W. Cotterill, William P. Putsis Jr., and Ravi Dhar. Assessing the competitive interaction between private labels and national brands. *Journal of Business*, 73(1):109, 2000.

35 Peter J. Danaher, Andre Bonfrer, and Sanjay Dhar. The effect of competitive advertising interference on sales for packaged goods. *Journal of Marketing Research*, XLV(April):211–225, 2008.

36 Angus Deaton and John Muellbauer. An almost ideal demand system. *The American Economic Review*, 70(3):312–326, 1980.

37 Erwin Diewert. An application of the Shephard duality theorem: a generalized Leontief production function. *The Journal of Political Economy*, 79(3):481–507, 1971.

38 Suresh Divakar, Brian T. Ratchford, and Venkatesh Shankar. CHAN4CAST: A Multichannel Multiregion Forecasting Model for Consumer Packaged Goods. *Marketing Science*, 24(3):334–350, July 2005.

39 Wedad Elmaghraby and Pinar Keskinocak. Dynamic pricing in the presence of inventory considerations: research overview, current practices, and future directions. *IEEE Engineering Management Review*, 31(4):47–47, 2003.

40 Peter S. Fader and Bruce G. S. Hardie. Modeling Consumer Choice among SKUs. *Journal of Marketing Research*, 33(4):442, November 1996.

41 Juan-Carlos Ferrer and Diego Fuentes. A system design to bridge the gap between the theory and practice of retail revenue management. *International Journal of Revenue Management*, 5(2):261–275, 2011.

42 Eijte W. Foekens, Peter S.H. Leeflang, and Dick R. Wittink. Hierarchical versus other market share models for markets with many items. *International Journal of Research in Marketing*, 14:359–378, 1997.

43 Eijte W. Foekens, Peter S.H. Leeflang, and Dick R. Wittink. Varying parameter models to accommodate dynamic promotion effects. *Journal of Econometrics*, 89(1-2):249–268, 1998.

44 Avijit Ghosh and Robert Shoemaker. A Comparison of Market Share Models and Estimation Procedures. *Journal of Marketing Research*, XXI(May):202–211, 1984.

45 Jochen Goensch, Robert Klein, and Claudius Steinhardt. Dynamic Pricingâ€“State-of-the-Art. *Zeitschrift für Betriebswirtschaft*, (3):1–40, 2009.

46 Óscar González-Benito, María Pilar Martínez-Ruiz, and Alejandro Mollá-Descals. Retail pricing decisions and product category competitive structure. *Decision Support Systems*, 49(1):110–119, April 2010.

47 Dhruv Grewal, Kusum L. Ailawadi, Dinesh Gauri, Kevin Hall, Praveen Kopalle, and Jane R. Robertson. Innovations in Retail Pricing and Promotions. *Journal of Retailing*, 87(1):43–52, July 2011.

48 Peter M. Guadagni and John D.C. Little. A logit model of brand choice calibrated on scanner data. *Marketing Science*, 2(3):203–238, 1983.

49 Joseph Guiltinan. Progress and Challenges in Product Line Pricing. *Journal of Product Innovation Management*, 28:744–756, April 2011.

50 Peng Guo, Baichun Xiao, and Jun Li. Unconstraining Methods in Revenue Management Systems: Research Overview and Prospects. *Advances in Operations Research*, 2012:1–23, 2012.

51 Sachin Gupta, Pradeep K. Chintagunta, Anil Kaul, and Dick R. Wittink. Do Household Scanner Data Provide Representative Inferences from Brand Choices: A Comparison with Store Data. *Journal of Marketing Research*, 33(4):383, November 1996.

52 Sunil Gupta. Impact of Sales Promotions on When, What, and How Much to Buy. *Journal of Marketing Research (JMR)*, 25(4):342–355, 1988.

53 Dominique M. Hanssens, Leonard J. Parsons, and Randall L. Schultz. *Market Response Models: Econometric and Time Series Analysis*. Springer, 2001.

54 Bruce G. S. Hardie, Leonard M. Lodish, Peter S. Fader, Alistair P. Sutcliffe, and William T. Kirk. Attribute-based Market Share Models: Methodological Development and Managerial Applications. *London Business School Mimeo*, 1998.

55 Stephen J. Hoch, Byung-Do Kim, Alan L. Montgomery, and Peter E. Rossi. Determinants of Store-Level Price Elasticity. *Journal of Marketing Research*, 32(1):17, February 1995.

56 Harald Hruschka. Relevance of functional flexibility for heterogeneous sales response models - A comparison of parametric and semi-nonparametric models. *European Journal of Operational Research*, 174(2):1009–1020, October 2006.

57 Dipak C. Jain and Naufel J. Vilcassim. Testing functional forms of market share models using the Box-Cox transformation and the Lagrange multiplier approach. *International Journal of Research in Marketing*, 6:95–107, 1989.

58 Kirthi Kalyanam. Pricing decisions under demand uncertainty: A Bayesian mixture model approach. *Marketing Science*, 15(3):207–221, 1996.

59 Kirthi Kalyanam and Thomas S. Shively. Estimating Irregular Pricing Effects: A Stochastic Spline Regression Approach. *Journal of Marketing Research*, 35(1):16–29, 1998.

60 Gurumurthy Kalyanaram and Russell S. Winer. Empirical generalizations from reference price research. *Marketing Science*, 14(3):161–169, 1995.

61 University of Chicago Booth School of Business Kilts Center for Marketing. Dominick's Data Manual. 2013.

62 Byung-do Kim, Robert C. Blattberg, and Peter E Rossi. Modeling the Distribution of Price Sensitivity and Implications for Optimal Retail Pricing. *Journal of Business*, 13(3):291–303, 1995.

63 Robert Klein. *Revenue Management*. Springer, Berlin, Heidelberg, 1. edition, 2008.

64 Steven F. Koch. Fractional multinomial response models with an application to expenditure shares. *Mimeo University of Pretoria*, 2010-21(October), 2010.

65 Gürhan A. Kök, Marshall L. Fisher, and Ramnath Vaidyanathan. Assortment Planning: Review of Literature and Industry Practice. *Retail Supply Chain Management*, 122:99–153, 2009.

66 Praveen K. Kopalle. Modeling Retail Phenomena. *Journal of Retailing*, 86(2):117–124, June 2010.

67 Praveen K. Kopalle, Carl F. Mela, and Lawrence Marsh. The dynamic effect of discounting on sales: Empirical analysis and normative pricing implications. *Marketing Science*, 18(3):317–332, 1999.

68 V Kumar and Robert P. Leone. Measuring the effect of retail store promotions on brand and store substitution. *Journal of Marketing Research*, 25(2):178–185, 1988.

69 V Kumar and Arun Pereira. Explaining the Variation in Short-Term Sales Response to Retail Price Promotions. *Journal of the Academy of Marketing Science*, 23(3):155–169, June 1995.

70 Jean-Jacques Lambin. Measuring the profitability of advertising: An empirical study. *Journal of Industrial Economics*, 18(2):86–103, November 1969.

71 Peter S.H. Leeflang, Dick R. Wittink, Michel Wedel, and Philippe A. Naert. *Building models for marketing decisions.* Kluwer Academic Publishers, Boston, 2000.

72 Michael R. Levy, Dhruv Grewal, Praveen Kopalle, and James Hess. Emerging trends in retail pricing practice: implications for research. *Journal of Retailing*, 80(3):xiii–xxi, 2004.

73 Warren H. Lieberman. Revenue management trends and opportunities. *Journal of Revenue and Pricing Management*, 3(1):91–99, 2004.

74 Garry L. Lilien and Philip Kotler. *Marketing Decision Making - A Model-Building Approach.* 1983.

75 John D.C. Little. *Models and managers: The concept of a decision calculus.* PhD thesis, 1970.

76 John D.C. Little and Jeremy F. Shapiro. A theory for pricing nonfeatured products in supermarkets. *Journal of Business*, 53(3):199–209, 1980.

77 Sandrine Macé and Scott A. Neslin. The determinants of pre-and postpromotion dips in sales of frequently purchased goods. *Journal of Marketing Research*, 41(3):339–350, 2004.

78 Murali K. Mantrala, P.B. Seetharaman, Rajeeve Kaul, Srinath Gopalakrishna, and Antonie Stam. Optimal Pricing Strategies for an Automotive Aftermarket Retailer. *Journal of Marketing Research*, 43(4):588–604, November 2006.

79 Josué Martínez-Garmendia. Application of hedonic price modeling to consumer packaged goods using store scanner data. *Journal of Business Research*, 63(7):690–696, July 2010.

80 María Pilar Martínez-Ruiz. Evaluating temporary retail price discounts using semiparametric regression. *Journal of Product and Brand Management*, 15(1):73–80, 2006.

81 María Pilar Martínez-Ruiz, José Luis Rojo-Álvarez, and Francisco Javier Gimeno-Blanes. Evaluation of Promotional and Cross-Promotional Effects Using Support Vector Machine Semiparametric Regression. *Systems Engineering Procedia*, 1:465–472, January 2011.

82 Tridib Mazumdar, S.P. Raj, and Indrajit Sinha. Reference Price Research: Review and Propositions. *Journal of Marketing*, 69(4):84–102, October 2005.

83 Jeffrey McGill and Garrett J. van Ryzin. Revenue management: Research overview and prospects. *Transportation Science*, 32(2):233–256, 1999.

84 Kent B. Monroe and Albert J. Della Bitta. Models for Pricing Decisions. *Journal of Marketing Research*, 15(3):413–428, 1978.

85 Alan L. Montgomery. Creating micro-marketing pricing strategies using supermarket scanner data. *Marketing Science*, 16(4):315–337, 1997.

86 Alan L. Montgomery. The implementation challenge of pricing decision support systems for retail managers. *Applied Stochastic Models in Business and Industry - Bridging the*

Gap between Academic Research in Marketing and Practitioners' Concerns, 1(4-5):367–378, 2005.

87 Alan L. Montgomery and Peter Rossi. Estimating price elasticities with theory-based priors. *Journal of Marketing Research*, 36(4):413–423, 1999.

88 Sridhar K. Moorthy. Using game theory to model competition. *Journal of Marketing Research*, 22(3):262–282, 1985.

89 Sridhar K. Moorthy. Competitive marketing strategies: Game-theoretic models. In *Handbooks in Operations Research and Management Science*, volume 5, pages 143–190. 1993.

90 Mark M. Moriarty. Retail promotional effects on intra-and interbrand sales performance. *Journal of Retailing*, 61(3):27–47, 1985.

91 John Mullahy. Multivariate fractional regression estimation of econometric share models. *University of Wisconsin-Madison Mimeo*, WP2011/33, 2010.

92 Masao Nakanishi and Lee G. Cooper. Simplified Estimation Procedures for MCI Models. *Marketing Science*, 1(3):314–322, 1982.

93 Martin Natter, Thomas Reutterer, and Andreas Mild. Dynamic Pricing Support Systems for DIY Retailers - A case study from Austria. *Marketing Intelligence Review*, 1(1):17 –23, 2009.

94 Martin Natter, Thomas Reutterer, Andreas Mild, and Alfred Taudes. Ein sortimentsübergreifendes Entscheidungsunterstützungssystem für dynamische Preis- und Werbeplanung im DIY-Handel. *Marketing Zeitschrift für Forschung und Praxis*, 28(4):260–268, 2006.

95 Martin Natter, Thomas Reutterer, Andreas Mild, and Alfred Taudes. Practice Prize Report: An Assortmentwide Decision-Support System for Dynamic Pricing and Promotion Planning in DIY Retailing. *Marketing Science*, 26(4):576–583, July 2007.

96 Purushottam Papatla and Lakshman Krishnamurthi. Measuring the dynamic effects of promotions on brand choice. *Journal of Marketing Research*, 33(1):20–35, 1996.

97 Leslie E. Papke and Jeffrey M. Wooldridge. Econometric Methods for Fractional Response Variables with an Application to 401(k) Plan Participation Rates. *Journal of Applied Econometrics*, 11(February):619–632, 1996.

98 Robert L. Phillips. *Pricing and Revenue Optimization*. Stanford University Press, Stanford, 1. edition, 2005.

99 Vithala R. Rao. Pricing research in marketing: The state of the art. *Journal of Business*, 57(1), 1984.

100 David J. Reibstein and Hubert Gatignon. Optimal Product Line Pricing: The Influence of Elasticities and Cross-Elasticities. *Journal of Marketing Research*, 21(3):259, August 1984.

101 P.B. Seetharaman, Siddhartha Chib, Andrew Ainslie, Peter Boatwright, Tat Chan, Sachin Gupta, Nitin Mehta, Vithala R. Rao, and Andrei Strijnev. Models of Multi-Category Choice Behavior. *Marketing Letters*, 16(3-4):239–254, December 2005.

102 Venkatesh Shankar and Ruth N. Bolton. An Empirical Analysis of Determinants of Retailer Pricing Strategy. *Marketing Science*, 23(1):28–49, January 2004.

103 Venkatesh Shankar and Lakshman Krishnamurthi. Relating price sensitivity to retailer promotional variables and pricing policy: An empirical analysis. *Journal of Retailing*, 72(3):249–272, 1996.

104 Jorge M. Silva-Risso, Randolph E. Bucklin, and Donald G. Morrison. A decision support system for planning manufacturers' sales promotion calendars. *Marketing Science*, 18(3):274–300, 1999.

105 Jorge M. Silva-Risso and Irina Ionova. Practice Prize Winner-A Nested Logit Model of Product and Transaction-Type Choice for Planning Automakers' Pricing and Promotions. *Marketing Science*, 27(4):545–566, 2008.

106 Aruna Sivakumar and Chandra Bhat. Fractional Split-Distribution Model for Statewide Commodity-Flow Analysis. *Transportation Research Record*, 1790(1):80–88, January 2002.

107 Winfried J. Steiner, Andreas Brezger, and Christiane Belitz. Flexible estimation of price response functions using retail scanner data. *Journal of Retailing and Consumer Services*, 14(6):383–393, November 2007.

108 Shivaram Subramanian and Hanif D. Sherali. A fractional programming approach for retail category price optimization. *Journal of Global Optimization*, 48(2):263–277, November 2009.

109 Kalyan T. Talluri and Garrett J. van Ryzin. *The Theory and Practice of Revenue Management*. Springer US, New York, 1. edition, 2005.

110 Gerard J. Tellis. The price elasticity of selective demand: A meta-analysis of econometric models of sales. *Journal of Marketing Research*, 25(4):331–341, 1988.

111 Gerard J. Tellis. Tackling the retailer decision maze: Which brand to discount when and why? *Marketing Science*, 14(3):271–299, 1995.

112 Henri Theil. The information approach to demand analysis. *Econometrica: Journal of the Econometric Society*, 33(1):67–87, 1965.

113 Glen L. Urban. A mathematical modeling approach to product line decisions. *Journal of Marketing Research*, 6(1):40–47, 1969.

114 Harald J. van Heerde, Peter S.H. Leeflang, and Dick R. Wittink. The Estimation of Pre- and Postpromotion Dips with Store-Level Scanner Data. *Journal of Marketing Research*, 37(3):383–395, August 2000.

115 Harald J. van Heerde, Peter S.H. Leeflang, and Dick R. Wittink. How Promotions Work: SCAN*PRO-Based Evolutionary Model Building. *Schmalenbach Business Review*, 54(July):198–220, 2002.

116 Harald J. van Heerde, Peter S.H. Leeflang, and Dick R. Wittink. Decomposing the Sales Promotion Bump with Store Data. *Marketing Science*, 23(3):317–334, June 2004.

117 Erjen van Nierop, Dennis Fok, and Philip Hans Franses. Sales models for many items using attribute data. *ERIM Report Series Research in Management*, (ERS-2002-65-MKT), 2002.

118 Garrett J. van Ryzin. Models of demand. *Journal of Revenue & Pricing Management*, 4(2):204–210, 2005.

119 Naufel J. Vilcassim and Pradeep K. Chintagunta. Investigating retailer product category pricing from household scanner panel data. *Journal of Retailing*, 71(2):103–128, 1995.

120 Gustavo Vulcano, Garrett J. van Ryzin, and Richard M. Ratliff. Estimating Primary Demand for Substitutable Products from Sales Transaction Data. *Operations Research*, 60(2):313–334, May 2012.

121 Rockney G. Walters. Assessing the Impact of Retail Price Promotions on Product Substitution, Complementary Purchase, and Interstore Sales Displacement. *Journal of Marketing*, 55(2):17, April 1991.

122 Lawrence R. Weatherford and Samuel E. Bodily. A taxonomy and research overview of perishable-asset revenue management: yield management, overbooking, and pricing. *Operations Research*, 40(5):831–844, 1992.

123 Albert R. Wildt. Estimating models of seasonal market response using dummy variables. *Journal of Marketing Research*, 14(1):34–41, 1977.

124 Dick R. Wittink, M.J. Addona, W.J. Hawkes, and J.C. Porter. SCAN* PRO: The estimation, validation and use of promotional effects based on scanner data. *Internal Paper, Cornell University*, 1988.

125 Xin Ye and Ram M. Pendyala. A Model of Daily Time Use Allocation Using Fractional Logit Methodology. In *Transportation and Traffic Theory. Flow, Dynamics and Human Interaction. 16th International Symposium on Transportation and Traffic Theory*, 2005.

126 Elaine L. Zanutto and Eric T. Bradlow. Data pruning in consumer choice models. *Quantitative Marketing and Economics*, 4(3):267–287, August 2006.

127 Michael J. Zenor. The profit benefits of category management. *Journal of Marketing Research*, 31(2):202–213, 1994.
128 Michael J. Zenor and Rajendra K. Srivastava. Inferring Market Structure With Aggregate Data: A Latent Segment Logit Approach. *Journal of Marketing Research*, 30(3):369–379, 1993.
129 Giulio Zotteria, Matteo Kalchschmidtb, and Federico Caniatoc. The impact of aggregation level on forecasting performance. *International Journal of Production Economics*, 93-94:479–491, January 2005.