Report from Dagstuhl Seminar 14342

Equilibrium Computation

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Abstract

This report documents the program and outcomes of Dagstuhl Seminar 14342 "Equilibrium Computation". The seminar was at the leading edge of current topics related to equilibrium computation for games and markets. We summarize these topics, give the talk abstracts, and give brief summaries of the problems that were discussed in the open problem sessions.

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1 Executive Summary

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The aim of this seminar was to study research issues related to algorithms and complexity for computation of equilibria in games and markets. The majority of participants were academics from computer science departments; some were from other disciplines; and several participants were from the corporate research departments of eBay, IBM, and Microsoft. All participants have strong interdisciplinary interests that typically span Economics, Game Theory, and Theoretical Computer Science.

The seminar started with a session of lightening talks, in which participants had two minutes and one slide to introduce themselves. This session was extremely well received, and it was worth the effort to ensure that everyone submitted a slide in advance. It is an effective and efficient way for everyone to get to know a little bit about each other, and thus to have things to talk about outside of talks right from the start of the seminar.

Three tutorials were given on topics chosen by the organizers. Bernhard von Stengel gave a tutorial on complementary pivoting algorithms for the Linear Complementarity Problem (LCP). The tutorial focussed on geometric aspects of LCPs and complementary pivoting algorithms, and in particular Lemke's algorithm. The LCP captures many game and market

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problems, and it came up throughout the seminar, most directly in the final talk by Adler on reductions to bimatrix games from `PPAD` Lemke-verified LCPs.

Complementary pivoting algorithms inspired the complexity class `PPAD`, which, together with `FIXP`, capture the problems of finding fixed points and equilibria of games and markets. The second tutorial, given by Kousha Etessami, was about the complexity of equilibria and fixed points. It covered `PPAD` (= linear-`FIXP`), `FIXP`, and `FIXP-a`, and discussed some associated open problems. Related contributed talks included the following. Etessami, in a separate talk, showed that the complexity of computing a (perfect) equilibrium for an n-player extensive form game of perfect recall is hard for `FIXP-a`. Gairing showed that the problem of finding an equilibrium of a weighted congestion game is `FIXP`-hard. Garg presented several results on market equilibria, including the result that it is `FIXP`-hard to compute an equilibrium of an Arrow-Debreu exchange market with Leontief utility functions. Chen presented a `PPAD`-hardness result for the problem of finding an approximate equilibrium in an anonymous game with seven actions per player. Mehta showed that it is `PPAD`-hard to find an equilibrium of a rank-3 bimatrix game. Paparas presented `PPAD`-hardness results for several market settings with non-monotone utilities. The number of talks related to these complexity classes shows their ongoing importance for the field of equilibrium computation.

The third tutorial was on game dynamics and was given by Sergiu Hart. He showed that "uncoupledness" severely limits the possibilities to converge to Nash equilibria, but on the other hand, there are simple adaptive heuristics, such as "regret matching", that lead to correlated equilibria. At the end of his tutorial, Hart also presented an exponential lower bound on the query complexity of correlated equilibria. In a closely related contributed talk, Goldberg gave bounds for the query complexity of approximate equilibria of various types, including for the relatively new concept of ϵ-well-supported correlated equilibrium.

A large number of contributed talks presented algorithms for computing equilibria of games and markets. On market equilibria we had the following algorithmic talks: Cole presented an asynchronous gradient descent method that implements asynchronous tâtonnement; Mehlhorn presented a combinatorial polynomial-time algorithm for the linear exchange model; Vazirani introduced Leontief-Free Utility Functions and presented a complementary pivoting algorithm for computing an equilibrium in markets with these utilities; and Vegh presented new convex programmes for linear Arrow-Debreu markets. On other game models, we had the following algorithmic talks: Cummings presented an efficient differentially private algorithm for computing an equilibrium in aggregative games; Savani presented a gradient descent algorithm for finding an approximate equilibrium of a polymatrix game; and Skopalik presented algorithms for finding approximate pure equilibria of congestion games.

There were other contributed talks on a range of topics: Harks talked about resource competition on integral polymatroids; Hoefer talked about decentralized secretary algorithms; Jain presented an analysis of several business models and pricing schemes; and Schäfer presented results about coordination games on graphs.

Apart from the topics of the tutorials, all other talk topics were chosen by the presenters, not by the organizers. Generally talks were informal, and were very interactive, often with lengthy discussions taking place during them. All talks were well received. Open problems were discussed in two sessions, the first during a normal seminar room session, and the second with cheese and wine in the evening. Below we give abstracts for the talks and brief summaries of the open problems that were discussed.

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Open Problems

3 Overview of Talks

3.1 A direct reduction of PPAD Lemke-verified linear complementarity problems to bimatrix games

Ilan Adler (University of California – Berkeley, US)

Main reference I. Adler, S. Verma, "A direct reduction of PPAD Lemke-verified linear complementarity problems to bimatrix games," arXiv:1302.0067v1 [cs.CC], 2013.
URL http://arxiv.org/abs/1302.0067v1

The linear complementarity problem, LCP(q, M), is defined as follows. For given M, q find z such that $q + Mz \geq 0, z >= 0, z(q + Mz) = 0$, or certify that there is no such z. It is well known that the problem of finding a Nash equilibrium for a bimatrix game (2-NASH) can be formulated as a linear complementarity problem (LCP). In addition, 2-NASH is known to be complete in the complexity class `PPAD` (Polynomial-time Parity Argument Directed). However, the ingeniously constructed reduction (which is designed for any `PPAD` problem) is very complicated, so while of great theoretical significance, it is not practical for actually solving an LCP via 2-NASH, and it may not provide the potential insight that can be gained from studying the game obtained from a problem formulated as an LCP (e.g. market equilibrium). The main goal of this paper is the construction of a simple explicit reduction of any LCP(q, M) that can be verified as belonging to `PPAD` via the graph induced by the generic Lemke algorithm with some positive covering vector d, to a symmetric 2-NASH. In particular, any endpoint of this graph (with the exception of the initial point of the algorithm) corresponds to either a solution or to a so-called secondary ray. Thus, an LCP problem is verified as belonging to `PPAD` if any secondary ray can be used to construct, in polynomial time, a certificate that there is no solution to the problem. We achieve our goal by showing that for any M, q and a positive d satisfying a certain nondegeneracy assumption with respect to M, we can simply and directly construct a symmetric 2-NASH whose Nash equilibria correspond one-to-one to the end points of the graph induced by LCP(q, M) and the Lemke algorithm with a covering vector d. We note that for a given M the reduction works for all positive d with the exception of a subset of measure 0.

3.2 Complexity of Nash Equilibria in Anonymous Games

Xi Chen (Columbia University – New York, US)

We show that finding a $(\frac{1}{2}^n)$-approximate Nash equilibrium in an anonymous game with seven actions is `PPAD`-complete.

3.3 From Asynchronous Gradient Descent to Asynchronous Tatonnement

Richard Cole (New York University, US)

Gradient descent is an important class of iterative algorithms for minimizing convex functions. Classically, gradient descent is a sequential and synchronous process. Distributed and asynchronous variants of gradient descent have been studied since the 1980s, and they have been experiencing a resurgence due to demand from large-scale machine learning problems running on multi-core processors. We provide a version of asynchronous gradient descent (AGD) in which communication between the processors is minimal and there is little synchronization overhead. We also propose a new timing model for its analysis. With this model, we give the first amortized analysis of AGD on convex functions. The amortization allows for bad updates (updates that increase the value of the convex function); in contrast, most prior work makes the strong assumption that every update must be significantly improving. Typically, the step sizes used in AGD are smaller than those used in its synchronous counterpart. We provide a method to determine the step sizes in AGD based on the Hessian entries of the convex function. In certain circumstances, the resulting step sizes are a constant fraction of those used in the corresponding synchronous algorithm, enabling the overall performance of AGD to improve linearly with the number of processors. Our amortized analysis of AGD can be applied to show that tatonnement, a simple distributed price update dynamic, converges toward the market equilibrium in a number of economic markets. We use the Ongoing market model due to Cole and Fleischer [STOC'08], a fairly recent market model that supports distributed and asynchronous price updates. We show that asynchronous tatonnement converges toward the market equilibrium in Ongoing Fisher markets in which the buyers have CES utility functions; our analysis of AGD can be applied to the market problem due to the fact that tatonnement is equivalent to gradient descent for this class of markets [Cheung, Cole, Devanur STOC'13].

3.4 Privacy and Truthful Equilibrium Selection for Aggregative Games

Rachel Cummings (Northwestern University – Evanston, US)

Joint work of Cummings, Rachel; Kearns, Michael; Roth, Aaron; Wu, Zhiwei Steven
Main reference R. Cummings, M. Kearns, A. Roth, Z. S. Wu, "Privacy and Truthful Equilibrium Selection for Aggregative Games," arXiv:1407.7740v2 [cs.DS], 2014.
URL http://arxiv.org/abs/1407.7740v2

We study a very general class of games – multi-dimensional aggregative games – which in particular generalize both anonymous games and weighted congestion games. For any such game that is also large (meaning that the influence that any single player's action has on the utility of others is diminishing with the number of players in the game), we solve the equilibrium selection problem in a strong sense. In particular, we give an efficient weak mediator: an algorithm or mechanism which has only the power to listen to reported types and provide non-binding suggested actions, such that (a) it is an asymptotic Nash equilibrium for every player to truthfully report their type to the mediator, and then follow its suggested action; and (b) that when players do so, they end up coordinating on a particular asymptotic pure strategy Nash equilibrium of the induced complete information game. In fact, truthful reporting is an ex-post Nash equilibrium of the mediated game, so our solution applies even in settings of incomplete information, and even when player types are arbitrary or worst-case (i.e. not drawn from a common prior). We achieve this by giving an efficient differentially private algorithm for computing a Nash equilibrium in such games. The rates of convergence to equilibrium in all of our results are inverse polynomial in the number of players n. We also give similar results for a related class of one- dimensional games with weaker conditions on the aggregation function, and apply our main results to a multi-dimensional market game. Our results can be viewed as giving, for a rich class of games, a more robust version of the Revelation Principle, in that we work with weaker informational assumptions (no common prior), yet provide a stronger solution concept (Nash versus Bayes Nash equilibrium). Previously, similar results were only known for the special case of unweighted congestion games. In the process, we derive several algorithmic results that are of independent interest, and that further the connections between tools in differential privacy and truthfulness in game-theoretic settings. We give the first algorithm for efficiently computing Nash equilibria in aggregative games of constant dimension $d > 1$. We also give the first method for solving a particular class of linear programs under the constraint of joint differential privacy.

3.5 The complexity of computing a (perfect) equilibrium for an n-player extensive form game of perfect recall

Kousha Etessami (University of Edinburgh, GB)

We study the complexity of computing or approximating an equilibrium for a given finite n-player extensive form game of perfect recall (EFGPR), where $n \geq 3$. Our results apply not only to Nash equilibrium (NE), but also to various important refinements of NE for EFGPRs, including: subgame-perfect equilibrium in behavior strategies, sequential equilibrium (SE), and extensive form trembling-hand perfect equilibrium (PE). Of these, the most refined notion is PE. By a classic result of Selten, a PE exists for any EFGPR. We show that, for all these notions of equilibrium, approximating an equilibrium for a given EFGPR, to within a given desired precision, is `FIXP-a`-complete. We also show that computing a "delta-almost subgame-perfect equilibrium" in behavior strategies for a given EFGPR and given $\delta > 0$, is `PPAD`-complete. In doing so, we also define the more refined notion of a "δ-almost ϵ-perfect" equilibrium, and show that computing one is `PPAD`-complete. Thus, approximating one such (delta-almost) equilibrium for n-player EFGPRs, $n \geq 3$, is `P`-time equivalent to approximating a (δ-almost) NE for a normal form game (NFG) with 3 or more players. NFGs are trivially encodable as EFGPRs without blowup in size. Thus our results extend the celebrated complexity results for NFGs to the considerably more general setting of EFGPRs. For 2-player EFGPRs, analogous complexity results follow from the algorithms of Koller, Megiddo, and von Stengel (1996), and von Stengel, van den Elzen, and Talman (2002). However, prior to the present paper, no analogous results were known for EFGPRs with 3 or more players.

3.6 Tutorial: Complexity of Equilibria and Fixed Points

Kousha Etessami (University of Edinburgh, GB)

This tutorial will discuss the complexity of equilibria and fixed points. It focusses on the complexity classes `FIXP`, `FIXP-a`, and linear-`FIXP` (= `PPAD`), and some associated open problems.

3.7 Weighted Congestion Games are FIXP-hard

Martin Gairing (University of Liverpool, GB)

In this talk, I will discuss the complexity class `FIXP` and show that weighted congestion games are `FIXP`-hard. The proof builds on a recent result of Milchtaich (https://faculty.biu.ac.il/~milchti/papers/representation.pdf).

3.8 Leontief Exchange Markets Can Solve Multivariate Polynomial Equations, Yielding FIXP and ETR Hardness

Jugal Garg (MPI für Informatik – Saarbrücken, DE)

We show `FIXP`-hardness of computing equilibria in Arrow-Debreu exchange markets under Leontief utility functions, and Arrow-Debreu markets under linear utility functions and Leontief production sets, thereby settling the open question of [Vazirani-Yannakakis'11]. In both cases, as required under `FIXP`, the set of instances mapped onto will admit equilibria, i.e., will be "yes" instances. If all instances are under consideration, then in both cases we prove that the problem of deciding if a given instance admits an equilibrium is `ETR`-complete, where `ETR` is the class defined by the Existential Theory of Reals. The main technical part of our result is the following reduction: Given a set S of simultaneous multivariate polynomial equations in which the variables are constrained to be in a closed bounded region in the positive orthant, we construct a Leontief exchange market M which has one good corresponding to each variable in S. We prove that the equilibria of M, when projected onto prices of these latter goods, are in one-to-one correspondence with the set of solutions of the polynomials.

3.9 Bounds for the query complexity of approximate equilibria

Paul W. Goldberg (University of Oxford, GB)

Joint work of Goldberg, Paul W.; Roth, Aaron
Main reference P. W. Goldberg, A. Roth, "Bounds for the query complexity of approximate equilibria," in Proc. of the 2014 ACM Conf. on Economics and Computation (EC'14), pp. 639–656, ACM, 2014.
URL http://dx.doi.org/10.1145/2600057.2602845

We analyze the number of payoff queries needed to compute approximate equilibria of multi-player games. We find that query complexity is an effective tool for distinguishing the computational difficulty of alternative solution concepts, and we develop new techniques for upper- and lower bounding the query complexity. For binary-choice games, we show logarithmic upper and lower bounds on the query complexity of approximate correlated equilibrium. For well-supported approximate correlated equilibrium (a restriction where a player's behavior must always be approximately optimal, in the worst case over draws from the distribution) we show a linear lower bound, thus separating the query complexity of well supported approximate correlated equilibrium from the standard notion of approximate correlated equilibrium. Finally, we give a query-efficient reduction from the problem of computing an approximate well-supported Nash equilibrium to the problem of verifying a well supported Nash equilibrium, where the additional query overhead is proportional to the description length of the game. This gives a polynomial-query algorithm for computing well supported approximate Nash equilibria (and hence correlated equilibria) in concisely represented games. We identify a class of games (which includes congestion games) in which the reduction can be made not only query efficient, but also computationally efficient.

3.10 Resource Competition on Integral Polymatroids

Tobias Harks (Maastricht University, NL)

Joint work of Harks, Tobias; Klimm, Max; Peis, Britta
Main reference T. Harks, M. Klimm, P. Peis, "Resource Competition on Integral Polymatroids," arXiv:1407.7650v1 [cs.GT], 2014.
URL http://arxiv.org/abs/1407.7650v1

We derive a new existence result for integer-splittable congestion games on integral polymatroids.

3.11 The Query Complexity of Correlated Equilibria

Sergiu Hart (The Hebrew Univ. of Jerusalem, IL)

Joint work of Hart, Sergiu; Nisan, Noam
Main reference S. Hart, N. Nisan, "The Query Complexity of Correlated Equilibria," arXiv:1305.4874v1 [cs.GT], 2013.
URL http://arxiv.org/abs/1305.4874v1

We consider the complexity of finding a Correlated Equilibrium in an n-player game in a model that allows the algorithm to make queries for players' utilities at pure strategy profiles.

Many randomized regret-matching dynamics are known to yield an approximate correlated equilibrium quickly: in time that is polynomial in the number of players, n, the number of strategies of each player, m, and the approximation error, $1/\epsilon$;. Here we show that both randomization and approximation are necessary: no efficient deterministic algorithm can reach even an approximate equilibrium and no efficient randomized algorithm can reach an exact equilibrium.

3.12 Tutorial: Game Dynamics

Sergiu Hart (The Hebrew Univ. of Jerusalem, IL)

An overview of work on dynamical systems in multi-player environments. On the one hand, the natural informational restriction that each participant does not know the payoff functions of the other participants – "uncoupledness" – severely limits the possibilities to converge to Nash equilibria. On the other hand, there are simple adaptive heuristics – such as "regret matching" – that lead in the long run to correlated equilibria, a concept that embodies full rationality.

3.13 Decentral Secretary Algorithms

Martin Hoefer (Universität des Saarlandes, DE)

The secretary model is a popular framework for the analysis of online admission problems beyond the worst case. In many markets, however, decisions about acceptance or rejection of applicants have to be made in a decentralized fashion and under competition. In this paper, we cope with this problem and design algorithms for decentralized secretary problems with competition among firms. In the basic model, there are m firms and each has a job to offer. n applicants arrive iteratively in random order. Upon arrival of an applicant, a value for each job is revealed. Each firm has to decide whether or not to offer its job to the current applicant without knowing the actions or values of other firms. Applicants then decide to accept their most preferred offer. We consider the overall social welfare of the matching, as well as the value of the match for each single firm. We design a decentralized randomized thresholding-based algorithm with ratio $O(\log n)$ that works in a very general sampling model. In addition, it can be used by firms hiring several applicants based on a local matroid. On the other hand, even in the basic model we show a lower bound of $\Omega(\log n/(\log \log n))$ for all thresholding-based algorithms. Moreover, we provide secretary algorithms with constant competitive ratios, e.g., when values of applicants for different firms are stochastically independent. In this case, we can show a constant ratio even when each firm offers several different jobs, and even with respect to its individually optimal assignment. We also analyze several variants with stochastic correlation among applicant values.

3.14 Business Model Analysis

Kamal Jain (eBay Research Labs, US)

The talk presented how to mathematically analyze business models to study some of their advantages and disadvantages.

3.15 A Combinatorial Algorithm for the Linear Exchange Model

Kurt Mehlhorn (MPI für Informatik – Saarbrücken, DE)

Joint work of Ran Duan; Mehlhorn, Kurt
Main reference R. Duan, K. Mehlhorn, "A Combinatorial Polynomial Algorithm for the Linear Arrow-Debreu Market," arXiv:1212.0979v3 [cs.DS], 2014.
URL http://arxiv.org/abs/1212.0979v3

We present the first combinatorial polynomial time algorithm for computing the equilibrium of the Arrow-Debreu market model with linear utilities. Our algorithm views the allocation of money as flows and iteratively improves the balanced flow as in [Devanur et al. 2008] for Fisher's model. We develop new methods to carefully deal with the flows and surpluses during price adjustments. Our algorithm performs $O(n^6 \log(nU))$ maximum flow computations, where n is the number of agents and U is the maximum integer utility. The flows have to be presented as numbers of bitlength $O(n \log(nU))$ to guarantee an exact solution. Previously, [Jain 2007, Ye 2007] have given polynomial time algorithms for this problem, which are based on solving convex programs using the ellipsoid algorithm and the interior-point method, respectively.

3.16 Resolving the Complexity of Constant-Rank Bimatrix Games

Ruta Mehta (Georgia Institute of Technology, US)

The rank of a bimatrix game (A, B) is defined as the rank of $(A+B)$, e.g., rank-0 is zero-sum games. In 2005, Kannan and Theobald asked if there exists a polynomial time algorithm for constant rank games. We answer this question affirmatively for rank-1 games, and negatively for games with rank three or more (unless `PPAD` =`P`); the status of rank-2 games remains unresolved. In the process we obtain a number of other results, including a simpler proof of `PPAD`-hardness for 2-Nash.

3.17 The Complexity of Non-Monotone Markets

Dimitris Paparas (Columbia University – New York, US)

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Joint work of Chen, Xi; Paparas, Dimitris; Yannakakis, Mihalis
Main reference X. Chen, D. Paparas, M. Yannakakis, "The Complexity of Non-Monotone Markets," arXiv:1211.4918v1 [cs.CC], 2012.
URL http://arxiv.org/abs/1211.4918v1

We introduce the notion of non-monotone utilities, which covers a wide variety of utility functions in economic theory. We then prove that it is PPAD-hard to compute an approximate Arrow-Debreu market equilibrium in markets with linear and non- monotone utilities. Building on this result, we settle the long-standing open problem regarding the computation of an approximate Arrow-Debreu market equilibrium in markets with CES utility functions, by proving that it is PPAD-complete when the Constant Elasticity of Substitution parameter ρ is any constant less than -1.

3.18 Computing Approximate Nash Equilibria in Polymatrix Games

Rahul Savani (University of Liverpool, GB)

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Joint work of Deligkas, Argyrios; Fearnley, John; Savani, Rahul; Spirakis, Paul
Main reference A. Deligkas, J. Fearnley, R. Savani, P. Spirakis, "Computing Approximate Nash Equilibria in Polymatrix Games," in Proc. of the 10th Int'l Conf. on Web and Internet Economics (WINE'14), to appear; pre-print available as arXiv:1409.3741v2 [cs.GT].
URL arxiv.org/abs/1409.3741v2

In an ϵ-Nash equilibrium, a player can gain at most ϵ by unilaterally changing his behaviour. For two-player (bimatrix) games with payoffs in $[0,1]$, the best-known ϵ achievable in polynomial time is 0.3393 (Tsaknakis and Spirakis). In general, for n-player games an ϵ-Nash equilibrium can be computed in polynomial time for an ϵ that is an increasing function of n but does not depend on the number of strategies of the players. For three-player and four-player games the corresponding values of ϵ are 0.6022 and 0.7153, respectively. Polymatrix games are a restriction of general n-player games where a player's payoff is the sum of payoffs from a number of bimatrix games. There exists a very small but constant ϵ such that computing an ϵ-Nash equilibrium of a polymatrix game is PPAD-hard. Our main result is that a $(0.5+\delta)$-Nash equilibrium of an n-player polymatrix game can be computed in time polynomial in the input size and $\frac{1}{\delta}$. Inspired by the algorithm of Tsaknakis and Spirakis, our algorithm uses gradient descent on the maximum regret of the players. We also show that this algorithm can be applied to efficiently find a $(0.5+\delta)$-Nash equilibrium in a two-player Bayesian game.

3.19 Coordination Games on Graphs

Guido Schäfer (CWI – Amsterdam, NL)

We introduce natural strategic games on graphs, which capture the idea of coordination in a local setting. We show that these games have an exact potential and have strong equilibria when the graph is a pseudoforest. We also exhibit some other classes of games for which a strong equilibrium exists. However, in general strong equilibria do not need to exist. Further, we study the (strong) price of stability and anarchy. Finally, we consider the problems of computing strong equilibria and of determining whether a joint strategy is a strong equilibrium.

3.20 Approximate pure Nash equilibria

Alexander Skopalik (Universität Paderborn, DE)

Among other solution concepts, the notion of the pure Nash equilibrium plays a central role in Game Theory. Pure Nash equilibria in a game characterize situations with non-cooperative deterministic players in which no player has any incentive to unilaterally deviate from the current situation in order to achieve a higher payoff. Unfortunately, it is well known that there are games that do not have pure Nash equilibria. Furthermore, even in games where the existence of equilibria is guaranteed, their computation can be a computationally hard task. Such negative results significantly question the importance of pure Nash equilibria as solution concepts that characterize the behavior of rational players. Approximate pure Nash equilibria, which characterize situations where no player can significantly improve her payoff by unilaterally deviating from her current strategy, could serve as alternative solution concepts provided that they exist and can be computed efficiently. We discuss recent positive algorithmic and positive existence results for approximate pure Nash equilibria in unweighted and weighted congestion games.

3.21 Leontief-Free Utility Functions

Vijay V. Vazirani (Georgia Tech, US)

Joint work of Garg, Jugal; Mehta, Ruta; Vazirani Vijay V.

Leontief utility functions capture the joint utility of a bundle if the goods in it are complements. We give an analogous notion for the case of substitutable goods. Even though our utility function is non-separable, we show that it always admits an equilibrium using rational numbers and we give a complementary pivot algorithm for finding one.

3.22 Convex programmes for linear Arrow-Debreu markets

László A. Végh (London School of Economics, GB)

We give a new, flow-type convex programme for linear Arrow-Debreu markets, along with a simple proof of the existence and rationality of equilibria and some further properties. We also survey previous convex programs for the problem and investigate connections between them.

3.23 Tutorial: Geometric Views of Linear Complementarity Algorithms and Their Complexity

Bernhard von Stengel (London School of Economics, GB)

The linear complementarity problem (LCP) generalizes linear programming (via the complementary slackness conditions of a pair of optimal primal and dual solutions) and finding Nash equilibria of bimatrix games. It suffices to look only for symmetric equilibria of symmetric games, which simplifies the problem setup. Lemke's classical complementary pivoting algorithm finds a solution to an LCP in many cases.

This tutorial reviews complementary pivoting, and two main geometric views of the algorithms by Lemke-Howson and Lemke: the polyhedral view, which describes the nonnegativity (feasilibity) constraints, and the "complementary cones" view, which maintains complementarity. Using complementary cones, Lemke's algorithm is seen as inverting a piecewise linear map along a line segment. One new result is that the Lemke-Howson algorithm is a special case of this description, which allows its implementation as a special case of Lemke's algorithm.

Based on a joint talk with Rahul Savani.

4 Open Problems

Open problems were discussed in two sessions, the first in seminar room, and the second with cheese and wine in the evening. We give brief summaries of the open problems.

4.1 Session 1

- Kamal Jain described an open problem related to a computational variant of Minkowski's Linear Forms Theorem that lies in the complexity class `PPP`.
- Bernhard von Stengel posed the open problem of lower bounding the maximal number of extreme Nash equilibria in 5×5 bimatrix games, where for $k \times k$ games with $k \leq 4$ we know that there can be at most $2^k - 1$ extreme equilibria, and for $k \geq 6$, von Stengel has constructed games with strictly more than $2^k - 1$ extreme equilibria.

4.2 Session 2

Thanks to Yuval Rabani for making the notes on the open problems from the second session.

- Rahul Savani asked: What is the complexity of finding a Nash equilibrium in an asymmetric 2-player network congestion game? Is there a polynomial time algorithm or is the problem PLS-hard?
- Rahul Savani asked about the problem "Network coordination": What is the complexity of finding a mixed Nash equilibrium in a polymatrix game where for every edge of the graph defining the game, the 2-player game has the property that in each pair of payoffs, the two values are equal? The problem is in PLS $\cap$ PPAD.
- Dimitris Paparas asked about classifying utility functions into monotone and non-monotone. In particular, WGS utilities and also CES utilities with $\rho \in [-1, 0)$ are monotone, and CES utilities with $\rho < -1$ and also Leontief utilities are non-monotone. We know that there is a polynomial time algorithm for computing a competitive equilibrium in exchange markets that use specific subsets of monotone utilities. Does this extend to all monotone utilities? We know that there is a class of exchange markets using non-monotone and linear utilities for which the problem of computing a competitive equilibrium is PPAD-hard. Does the result hold if we don't use also linear utilities?
- Bernhard von Stengel posed a problem about Nash codes. We have a noisy channel with k possible inputs and k possible outputs. A stochastic matrix defines the distribution of outputs given an input. The two sides want to convey a specific signal as accurately as possible. A strategy for the sender is a mapping of the possible signals to the possible inputs. A strategy of the receiver is a reconstruction of a signal from the received output (w.l.o.g. this is the best possible such reconstruction given the sender's mapping). Given a matrix, some mappings may lead to a Nash equilibrium and others might not. What is the complexity of deciding whether or not every mapping from signals to inputs leads to a Nash equilibrium?
- Paul Goldberg posed a problem about the communication complexity of finding approximate equilibria.: In a 2-player game, the row and column players get their own payoff matrix (R and C, respectively). Beforehand, they can agree on two functions $f = f(R)$ and $f' = f'(C)$ that are mixed strategies that depends on the partial information they have. Then, an adversary chooses R and C and the game is played by using the strategies $f(R)$ and $f'(C)$. For which values of ϵ can you get an ϵ-Nash equilibrium this way? Known: and upper bound of $3/4$ and a lower bound of 0.501 (i.e., above $1/2$). The upper bound is simply playing with probability $1/2$ a uniform distribution on your strategies and with probability $1/2$ a best response to the other player's uniform distribution on strategies. Perhaps the same question is interesting with respect to an ϵ-correlated equilibrium, using f,f' chosen at random from a joint distribution.
- Ruta Mehta asked about the computational complexity of decision and counting related to k-player games. What is the complexity of various counting problems in (symmetric) k-player games? Various such problems in 2-player games are known to be #P-hard. Also, not all decision problems concerning 2-player Nash equilibrium that are known to be NP-hard, are also known to be ETR-hard in k-player games. (The cases of superset and subset in 3-player games are known to be ETR-complete; what remains open is maximal and minimal supports as far as the list of Gilboa and Zemel goes.)
- Kousha Etessami asked an open problem related to "Solvency Games", which are a special case of one-counter MDPs.

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