The Dirac-Motzkin Problem on Ordinary Lines and the Orchard Problem*

Ben J. Green

Mathematical Institute, University of Oxford
Oxford, UK
ben.green@maths.ox.ac.uk

Abstract
Suppose you have $n$ points in the plane, not all on a line. A famous theorem of Sylvester-Gallai asserts that there is at least one ordinary line, that is to say a line passing through precisely two of the $n$ points. But how many ordinary lines must there be? It turns out that the answer is at least $n/2$ (if $n$ is even) and roughly $3n/4$ (if $n$ is odd), provided that $n$ is sufficiently large. This resolves a conjecture of Dirac and Motzkin from the 1950s. We will also discuss the classical orchard problem, which asks how to arrange $n$ trees so that there are as many triples of colinear trees as possible, but no four in a line. This is joint work with Terence Tao and reports on the results of [1].

1998 ACM Subject Classification G.2 Discrete Mathematics

Keywords and phrases combinatorial geometry, incidences

Digital Object Identifier 10.4230/LIPIcs.SOCG.2015.405

Category Invited Talk

References

* This work was partially supported by ERC Starting Grant number 279438, Approximate algebraic structure and applications.