Optimizing Traffic Signal Timings for Mega Events*

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Abstract

Most approaches for optimizing traffic signal timings deal with the daily traffic. However, there are a few occasional events like football matches or concerts of musicians that lead to exceptional traffic situations. Still, such events occur more or less regularly and place and time are known in advance. Hence, it is possible to anticipate such events with special signal timings. In this paper, we present an extension of a cyclically time-expanded network flow model and a corresponding mixed-integer linear programming formulation for simultaneously optimizing traffic signal timings and traffic assignment for such events. Besides the mathematical analysis of this approach, we demonstrate its capabilities by computing signal timings for a real world scenario.

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1 Traffic signal settings for major events

Some major sporting events or gigs of celebrated musicians attract tens to hundreds of thousands of supporters and fans. A venue in a downtown area is also a major challenge for the transportation infrastructure. In inner-city traffic, intersections are the main bottleneck, since crossing traffic has to obey traffic signals and the right of way, reducing the available capacity significantly. Traffic signals either operate in a pretimed manner or they rely on sensor data to switch adaptively. Anyway, common standard strategies for operating traffic signals do not suffice to cope with the extreme traffic situations due to such mega events since the traffic volume on some roads may exceed the normal traffic volumes many times.

In this paper we present an approach to optimize traffic signal timings for such mega events in advance. That is, we compute optimal signal timings for inner-city traffic, where a subset of commodities has very high demand and a common origin or a common destination, respectively. The main objective is a fast reduction of the additional demand without disrupting the normal traffic.

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1.1 Literature Overview

In practice, there exist various approaches to optimize traffic signal timings. Roughly one can distinguish two main approaches. In pretimed signal settings, green and red phases follow a fixed schedule and repeat periodically. Actuated timings use sensor data to react on actual traffic.

The Traffic Signal Timing Manual [11] suggests to use pretimed signals when traffic demands and patterns do not vary widely, when crossing roads carry a similar traffic load, and when short distances between intersections allow a coordination of consecutive traffic signals. Several approaches have been developed to optimize such coordinations. First results date back to 1964, when Morgan and Little [16] presented a graphical method for maximizing the bandwidth of a signalized road. This approach was later extended using mixed integer programming [15]. First results for road networks were obtained by Allsop [1]. Shortly after, Robertson [17] presented his theoretical work on offset optimization based on a simplified simulation model and genetic programming which lead to the development of TRANSYT. These early approaches did not consider route choice, but Allsop and Charlesworth [2] demonstrated that coordination and assignment do interact. Since then, besides several heuristic algorithms, only a few models using exact mathematical programming techniques for optimizing coordination and assignment have been reported, e.g., [3, 14, 19, 20]. Recently, we presented a cyclically time-expanded network flow model to address this task [8, 9, 10].

Actuated signal timings may perform well where detection is provided in locations without nearby signals, rural areas or intersections of two arterials where traffic patterns vary widely [11]. Depending on the used sensors, e.g., only stop-line detection or upstream-downstream detectors and communication between signals, actuated or adaptive systems can be subdivided into further categories. The most widely deployed adaptive system is SCOOT (Split Cycle Offset Optimisation Technique, developed in the United Kingdom), but there are several other systems in use like RHODES (Real Time Hierarchical Optimized Distributed Effective System, using peer-to-peer-communication) or SYLVIA+ (widely deployed in Germany). Recently, a new approach was presented by Lämmer [12]. The author states stability as one of the main problems of all adaptive systems and he suggests an underlying pretimed signal coordination to stabilize the system. Further, he states that an adaptive system has to be run below saturated traffic demand to prevent degeneracy [13]. As a disadvantage, all adaptive systems need accurate detection systems, hence, initial and maintenance costs are higher than that of other control types.

Transportation is considered to be one of the critical factors for the success of sporting events like the Olympic Games in Rio de Janeiro in 2016 [18]. However, traffic signals in particular are rarely studied in this context. The Traffic Signal Timing Manual ([11], see Section 9.5) highlights the importance of adjusted timings for such situations and it suggests to increase green times of (manually identified) arterial roads or corridors. Yet, automated routines how to identify such arterials and how to set timings, also with respect to the normal traffic, are not specified.

In contrast, most urban administrations seem to simply believe that actuated signal control will suffice to cope with the additional traffic. Such adaptive signal timings are used quite often, but to the best of our knowledge, there is no scientific justification and the prerequisites contradict the proposals of the Traffic Signal Timing Manual. Unfortunately, such high traffic volumes may lead to permanent requests for green on all incoming roads of an intersection since all sensors are permanently triggered. In such situations, most actuated signal timings behave like pretimed signals, but coordination between consecutive intersections is not present. Hence, a coordinated pretimed signal control anticipating the upcoming traffic peak may perform much better than the standard traffic signal timings for the daily traffic or actuated signal timings.

1.2 Our contribution

In this paper, we discuss how mega-events or evacuation scenarios can be integrated in traffic signal control such that the extreme traffic volume is resolved in an optimized way. We consider route choice simultaneously, i.e., roads and arterials to be used by the visitors of such an event are not fixed in advance. Instead, we use an integrated approach to optimize signal settings and traffic assignment simultaneously. Parameters under consideration are offsets, split times and phase orders of the signals in the road network. Our approach uses mixed integer linear programming techniques, hence, it also provides dual bounds on the obtained solutions. We present numerical results for a real world scenario, namely a football match in the city of Cottbus, Germany.

This paper is organized as follows. In Section 2, we present the basic model for optimizing traffic signal timings and traffic assignment simultaneously. The modifications that are necessary for coping with peak commodities are presented in Section 3. Afterwards, we study the real-world scenario in Section 4. Finally, we close with a discussion of our results.

2 Basic model

Signal coordination requires the optimization of a large number of parameters. In the simplest approach, we have to choose the beginning and the end of green times for each turning direction at each signalized intersection of a road network with respect to several constraints. Computing route choice at the same time adds time-dependent flow variables and corresponding constraints for each arc and each commodity.

Recently, we presented a cyclically time-expanded network flow model [8, 10] for optimizing signal coordination and traffic assignment simultaneously, which reduces the number of variables significantly. In the following, we present the main ideas of this mixed-integer linear programming model. However, it is not directly suitable for computing traffic signal timings for situations with high peaks in traffic volume. Thus, we also describe an extension of the model in the subsequent section.

2.1 Basic Model

Time-expanded networks have been used to study dynamic network flows since Ford and Fulkerson introduced them in their seminal work about flows more than 60 years ago [4, 5]. However, the time horizon determines the size of the networks and corresponding approaches lead almost directly to models of pseudo-polynomial size.

But inner-city traffic with traffic signals has a very high periodicity. Let Γ be the least common multiple of all cycle times of signals in the road network, we can define a cyclically time-expanded network.

▶ **Definition 1** (Cyclically time-expanded network). Let $G = (V, A, u)$ be a network with capacities $u : A \to \mathbb{N}$ and non-negative integral transit times t_e for each $e \in A$. For a given number k of time steps of length $t = \frac{\Gamma}{k}$, the corresponding *cyclically time-expanded network* $G^k = (V^k, A^k, u^k)$ is constructed as follows.

- For each node $v \in V$, we create k copies $v_0, v_1, \ldots, v_{k-1}$, thus $V^k = \{v_t | v \in V, t \in \{0, \ldots, k-1\}\}$.

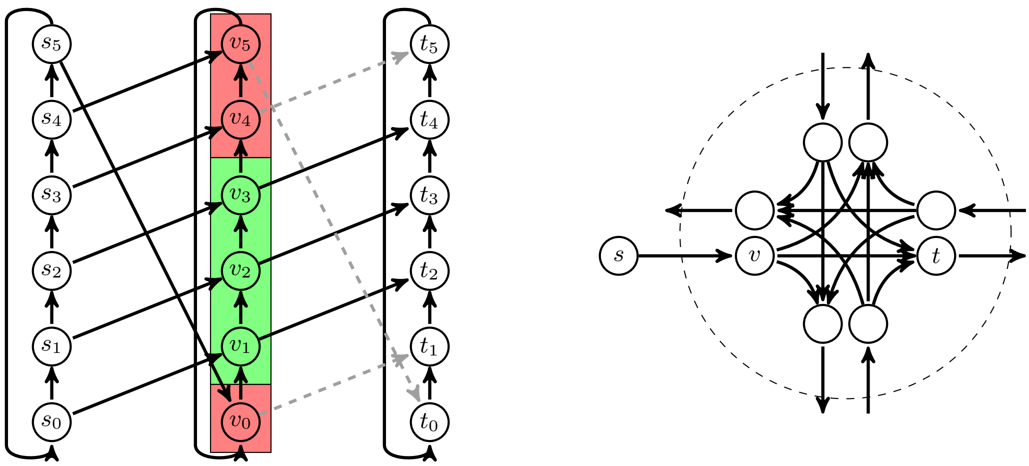


Figure 1 A very simple example of a cyclically time-expanded network with three consecutive nodes s, v, and t and $k = 6$ time steps. A traffic signal is installed at node v, a sample timing and corresponding capacities (dashed: $b_i = 0$, standard: $b_i = 1$) of the outgoing links are shown. On the right-hand side, an intersection of the unexpanded model with separate arcs for each turning direction is shown.

- For each link $e = (v, w) \in A$, we create k copies $e_0, e_1, \ldots, e_{k-1}$ in A^k where arc e_t connects node v_t to node $w_{(t+\lceil \frac{t_e k}{\Gamma} \rfloor) \bmod k}$. These arcs are called *transit arcs* and e_t has capacity $u(e_t) := \frac{u(e)}{k}$ and cost t_e.
- Additionally, we add *waiting arcs* from v_t to v_{t+1} $\forall v \in V$ and $\forall t \in \{0, \ldots, k-2\}$ and from v_{k-1} to v_0 $\forall v \in V$ with cost $\frac{\Gamma}{k}$ and infinite capacity to A^k.

Throughout this paper, we choose $k = \Gamma$, i.e., a time step is exactly one second long. In our numerical experiments, this choice seems to be the best trade-off between accuracy and calculation time. With larger steps of 2, 3, or even 5 seconds, the model is solved much faster but solutions get worse. Especially steps of length 5 and larger may cause conflicts, since minimum green times, clearance times, et cetera have to be rounded up to multiples of the step length. Thus, one may not even find a feasible signal setting for a single intersection that meets all the constraints. In contrast, shorter steps less than a second have no significant impact on the objective value.

Commodities are expanded in the same way. Let $\Phi \subset V \times V \times \mathbb{R}^+$ be the set of commodities with $\varphi = (s, t, d) \in \Phi$ being a triple of a source (or origin) s, a sink (or destination) t, and demand d. Here, d is scaled to Γ, that is, d denotes the amount of flow starting during one cycle. For simplicity, we assume that traffic demand is uniformly distributed over all copies of the original source. That is, the net outflow of flow of commodity φ of each s_i, $i \in \{0, \ldots, k-1\}$, is $\frac{d}{k}$. However, flow may also directly use a waiting arc from s_i to s_{i+1}. Each commodity may leave the network at an arbitrary copy t_i of the original sink t, i.e., we do not fix the net inflow of each t_i, but the total inflow is d. Flow is now a standard multi-commodity network flow $f_\varphi : A^k \to \mathbb{R}_{\geq 0}$ $\forall \varphi \in \Phi$ in this network, obeying to flow conservation at each node for each commodity separately and obeying the capacities in total, i.e., $\sum_{\varphi \in \Phi} f_\varphi(e_t) \leq u(e_t)$ for all $e_t \in A^k$.

Traffic signals can now be modeled via variable capacities. For this purpose, each turning direction at an intersection (cf. Figure 1) is represented by an arc which is expanded into a set of k arcs as described in Definition 1. The capacity of these arcs is multiplied with binary decision variables. Let e^1 and e^2 be two such arcs with capacities u_1 and u_2, then

$e_0^1, \ldots, e_{k-1}^1$ and $e_0^2, \ldots, e_{k-1}^2$ are the arcs in the cyclically time-expanded network with capacities $\frac{u_1}{k}$ and $\frac{u_2}{k}$, and let $b_0^1, \ldots, b_{k-1}^1 \in \{0,1\}$ and $b_0^2, \ldots, b_{k-1}^2 \in \{0,1\}$ be the binary decision variables. A signal for e^i can now be realized by setting the capacity of arc e_j^i to $b_j^i \frac{u_i}{k}$ for all $j \in \{0, \ldots, k-1\}$. Green at the same time for both signals can now be prohibited by $b_j^1 + b_j^2 \leq 1 \; \forall j \in \{0, \ldots, k-1\}$.

Yet, this is not a realistic timing and we have to link the binary variables, e.g., for achieving only one period of green during one cycle. We introduce binary decision variables $B_j^{i,on}$ and $B_j^{i,off}$ for each turning direction i and each time step j. Now, the (in-)equalities $b_{j-1}^i \geq b_j^i - B_j^{i,on}$ and $\sum_{j=0}^{k-1} B_j^{i,on} = 1$ guarantee only one switch to green per cycle at time step $\hat{\jmath}$ with $B_{\hat{\jmath}}^{i,on} = 1$. Similarly, the signal switches to red at time step $\hat{\jmath}$ with $B_{\hat{\jmath}}^{i,off} = 1$. Several other requirements can now be modeled via linear constraints. For example a minimum green time of x time steps is realized by $\sum_{j=0}^{k-1} b_j^i \geq x$. For a more detailed description, we refer to [10]. Now, this model allows the simultaneous optimization of traffic assignment (multi-commodity flow) and traffic signal timings (coordination).

The main advantage of this approach is a rather low number of variables and a complete linear model which allows the use of exact mathematical programming techniques and the use of solvers like CPLEX or GUROBI, i.e., we can prove optimality of a solution or we can at least provide dual bounds. Although the model is a linear one, we have shown that it provides very realistic travel times and link performance functions, i.e., the raise in the travel times is non-linear in relation to an increasing traffic demand [10].

As a disadvantage, the rolling horizon limits the total capacity. Whereas we can send any desired amount of flow in a standard time-expanded network whenever the time horizon is chosen large enough, flows in the cyclically time-expanded network may turn out to be infeasible due to exceeded capacities.

2.2 Queues and Overload

Before we focus on very high demand and traffic load in the network, let us shortly describe how queues and spillback are handled in the cyclically time-expanded model. When a signal is red, i.e., the capacity of arc $e_i = (v_i, w_j)$ is set to zero, incoming flow to v_i may either use the waiting arc (v_i, v_{i+1}) or any other outgoing arc if there is one. Thus, instead of setting the capacity of a waiting arc to infinity, a finite capacity can limit the amount of flow waiting at a certain node. This may be interpreted as a limited queue length and parameters should be chosen appropriately. Consequently, when the capacity on the waiting arc is reached, flow already has to wait, i.e., use the waiting arcs, at the previous node. Thus, also spillback may occur in the cyclically time expanded network.

Still, the total amount of flow that can be sent from a source s to a sink t is obviously limited. In other words, we cannot store arbitrary amounts of flow at a node and we also cannot store flow for more than one cycle at each node even when the waiting arcs have infinite capacity. Eventually, all flow has to reach the sink. In contrast, considering peak traffic after a mega event, already the first signalized intersection can be the main bottleneck and most road users will not pass it during the first cycle.

3 Coping with congested roads

In this section, we consider additional event commodities $\Theta \subset V \times V \times \mathbb{R}^+$. As a first difference, the demand of these commodities is not scaled to a time unit and flow does not start uniformly distributed over time. Instead, the entire demand d of $\vartheta = (s, t, d)$ enters the cyclically time-expanded network at the first copy s_0. Due to the limited outflow of s_0, the flow of this commodity will use several consecutive waiting arcs (s_i, s_{i+1}). Given a feasible flow, the *draining time* of commodity ϑ is $\Gamma\frac{i}{k}$ with $i = \min\{j \geq 0 : f_\vartheta((s_j, s_{j+1})) = 0\}$. In other words, it is the time until the last flow unit has left the source.

Unfortunately, the demand of the event commodities will be too high to be drained during one cycle and the flow will turn out to be infeasible in many cases. Of course, it is possible to extend the cyclic expansion to multiple cycles, i.e., the cyclic time span is $\alpha\Gamma$ for some integer α and αk time steps. However, this foils several advantages of the cyclic model like the rather low number of binary variables. Increasing α, the computation time increased more than linearly in our numerical experiments. The main reason, however, is the lack of a good guess for α. It is not known a priori how much time is needed to drain the whole demand of the event commodities. Hence, there is also no reasonable choice for α which also depends on signal settings. Nevertheless, we will use this approach to compute travel times in a second step once the best choice for α is known and signal timings are optimized.

Before we can do this, we need another method to optimize signal timings in a first step. To cope with high traffic volumes and bottlenecks due to traffic signals, we introduce overload edges in the following subsection. Since these overload edges are suitable for optimizing the signal timings, but they do not provide the correct travel times, total travel time and final route choice are computed in the second step as mentioned above.

3.1 Implementing overload edges

The intended use of overload edges is to provide additional network capacity such that a feasible flow can be found. Furthermore, the remaining flow which is not using these overload edges should reproduce an realistic traffic load in the network which is important for calculating signal timings.

Here, we make the following assumptions. Firstly, overload edges start at the nodes where a signal reduces the total capacity of the outgoing edges, since these are the main bottlenecks. Secondly, overload edges have to remove traffic from the original network. If overload edges bring flow back into the network somewhere else, it loads edges that would not have been loaded in the original network, since the flow would have been locked in front of the bottleneck. Thirdly, the costs have to be chosen carefully. If costs are too low, flow directly uses overload edges. If costs are too high, flow takes unrealistic long detours in the network to avoid these expensive edges. Furthermore, the costs have to reflect the remaining distance to the sink. Flow should not use the first available overload edge. Instead, it should stay in the original network until the actual bottleneck is reached.

For this purpose, we define overload edges as follows.

▶ **Definition 2.** Let $\vartheta = (s, t, d)$ be an event commodity from s to t and let $S \subseteq V$ be the set of nodes where signals are present. The set of overload edges of commodity ϑ is $A_\vartheta = \{(v, t) : v \in S \cup \{s\}\}$. The cost of an overload edge from $v \in S$ to t is given by $t(v, t) + n\Gamma$ where $t(v, t)$ denotes the travel time of a shortest path from v to t and n is the number of signalized nodes $w \in S$ on this path. Overload edges have infinite capacity.

Thus, overload edges connect each node with a signal directly to the sink and they are exclusive for each commodity. The shortest paths in G can easily be computed by a reverse Dijkstra's algorithm starting at the sink of the event commodity. Here, the cycle time is assigned to each traffic light edge as cost. Thus, also the travel time on overload edges is obtained easily.

Since we are interested in solutions where the normal traffic is not blocked by the event traffic, we do not introduce overload edges for the standard commodities. Flow of the standard commodities is not allowed to use any overload edge. Overload edges are cyclically time-expanded like any other edge yielding the subset $A_\vartheta^k \subseteq A^k$ of overload edges for each event commodity ϑ. Due to the overload edge from s to t for each event commodity, we can state the following result.

▶ **Lemma 3.** *Each instance of network flow in the cyclically time expanded network with standard commodities Φ, event commodities Θ, and overload edges for each $\vartheta \in \Theta$ is feasible if it is feasible for the standard commodities Φ.*

The choice of travel times on overload edges also guarantees the following property.

▶ **Theorem 4.** *Let v_i-t_j be a path in G^k, such that the underlying v-t-path is a shortest path in G. If the v_i-t_j-path has a residual capacity greater than zero for a cost minimal multi-commodity flow $f_\vartheta : A^k \to \mathbb{R}_{\geq 0}$, $\vartheta \in \Theta$, then $f_\vartheta((v_q, t_r)) = 0$ $\forall \vartheta \in \Theta$ with sink $t^\vartheta = t$ and for all overload edges (v_q, t_r) with $q, r \in \{0, \ldots, k-1\}$.*

Proof. The v_i-t_j-path has cost lower than $t_{(v,t)} + n\Gamma$, since it may use at most $k-1$ waiting arcs at each of the n signals of length $\frac{\Gamma}{k}$. Thus, if there would be flow on the overload edge (v_q, t_r) in the optimal solution, the value could be improved by rerouting flow from (v_q, t_r) to the v_i-t_j-path. ◀

In other words, flow remains on reasonable paths in the standard cyclically time-expanded network as long as possible before switching to an overload edge.

3.2 Optimization procedure

Optimization of signal timings and traffic assignment is carried out in two steps. After adding overload edges for all event commodities $\vartheta = (s_\vartheta, t_\vartheta, d_\vartheta)$, the cyclically time-expanded network G^k is created as described in Definition 1 with rolling time horizon Γ. Traffic signals and corresponding constraints are added. The resulting mixed integer linear program is solved with a standard solver. This yields a simultaneous optimization of traffic signal timings and traffic assignment in the cyclically time-expanded network with overload edges.

Afterwards, we determine the ratio between flow in the network and flow on the overload edges for each commodity. We set

$$\alpha = \left\lceil \max_{\vartheta \in \Theta} \frac{d_\vartheta}{d_\vartheta - \sum_{e \in A_\vartheta^k} f_\vartheta(e)} \right\rceil .$$

In other words, at least a fraction of $\frac{1}{\alpha}$ of each commodity reaches the sink without using overload edges. Thus, if the network is completely blocked by the standard commodities and the event commodities only use overload edges, no feasible solution exists and we set $\alpha = \infty$. For $\alpha < \infty$, we cyclically time-expand the original network *without overload edges* with time span $\alpha\Gamma$ and αk time steps in the second phase of the optimization procedure, obtaining a network $G^{\alpha k}$. Again, event commodities $\vartheta \in \Theta$ start at the first copy of the original source, but standard commodities $(s_\varphi, t_\varphi, d_\varphi) \in \Phi$ start uniformly distributed over the whole time span $\alpha\Gamma$ with $\frac{d_\varphi}{k}$ flow units entering the network at every time step. Furthermore, we canonically transfer the signal timings obtained for G^k to $G^{\alpha k}$, that is, we repeat the sequences α times. This yields a network with α-times the capacity for the event commodities, hence, we can conclude the following theorem.

▶ **Theorem 5.** *If $\alpha < \infty$, there exists a feasible multi-commodity flow for commodities Φ and event commodities Θ in the cyclically time-expanded network $G^{\alpha k}$ (without overload edges).*

Since all binary variables are fixed in the corresponding programming formulation obtained from G^k, we now have a standard multi-commodity flow problem in $G^{\alpha k}$ to solve. This yields an optimized assignment in $G^{\alpha k}$ without using any overload edges.

Please note that α can be infinite, when all the flow of one event commodity only uses overload edges. In this case, the network does not provide enough capacity, i.e., we cannot find a reasonable solution to drain the event commodities without blocking the standard commodities. Moreover, the optimal assignment still depends on α. Considering the projection of the flow from G^k into G, i.e., computing the underlying static traffic assignment by ignoring the time component, there may be differences between the projections of G^k with overload edges and $G^{\alpha k}$ as well as between the projections of $G^{\alpha k}$ and $G^{(\alpha+1)k}$. Due to space limitations, we have to omit an example.

3.3 Additional modeling issues

If several overload commodities start at the same source s, we force the optimization to distribute the flow over the overload edges (s, t_ϑ) according to the fraction of the demand of the commodity to the total demand at this source. Let d^* be the total demand of all commodities originating from s and let f^* be the total flow over overload edges from source s, then the flow $f_\vartheta((s, t_\vartheta))$ on overload edge (s, t_ϑ) of commodity $\vartheta = (s, t_\vartheta, d_\vartheta)$ has to fulfill $f_\vartheta((s, t_\vartheta)) = \frac{d_\vartheta}{d^*} f^*$. The main reason for this condition is a uniform shift of flow to overload edges. In reality, we assume the traffic of the different commodities to be uniformly distributed in the queue, hence, there is no preference who has to wait. Without this constraint, the optimization will prefer sending traffic with a greater gap between overload cost and normal cost to the normal network and commodities with a smaller gap will disproportionately often use the overload edges. Note that this additional constraint may imply that Theorem 4 holds only for at least one commodity, but it no longer holds for all commodities.

The normal traffic of the standard commodities Φ can be considered as in the standard model, i.e., they are subject to rerouting. However, some road users may be aware of the mega event, others are not. We use the following approach to integrate this base traffic volume in a more realistic way. Firstly, traffic assignment of the normal commodities Φ is optimized for the standard signal timings, that is, there is no mega event and we compute a base case. Secondly, each commodity $\varphi \in \Phi$ is split into two commodities. The first one (the unaware road users) uses the same relative path decomposition as the original commodity in the base case, that is, their routes are fixed. In contrast, the assignment of the second commodity (the knowing) is completely re-optimized together with the new signal timings and the event commodities Θ. The percentage of the split has to be given as a predefined parameter.

4 Numerical results

We will now present a numerical case study of our traffic signal optimization scheme for mega events. To this end, let us have a look on the city of Cottbus, Germany, and a match of the local association football team on a Saturday afternoon. Cottbus is a city with about 100,000 inhabitants and we consider the whole inner-city with about thirty signalized

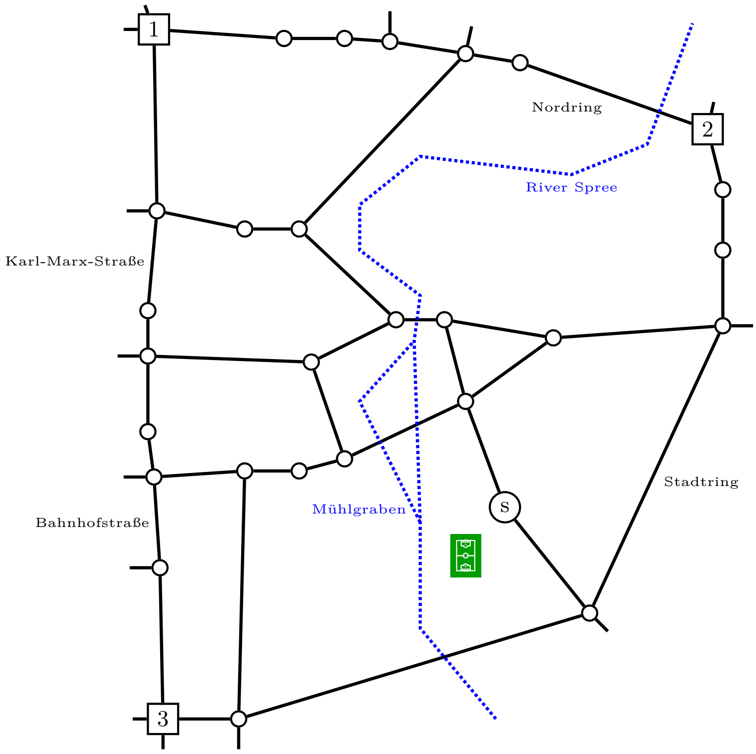


Figure 2 The Cottbus network with source s at the "Stadion der Freundschaft", destinations 1, 2, and 3 of the three football commodities, and about 30 signalized intersections. Minor roads in housing areas et cetera are not shown.

intersections. The road network is presented in Figure 2 and the underlying scenario was already presented in previous work [10].

Figure 2 also shows the location of the football stadium, which holds up to 20,000 spectators. For football matches, many supporters from the surrounding rural area arrive by car. In the following, we assume three *football commodities* (Θ) to leave the stadium after the end of the match, headed towards the three major roads leading out of Cottbus. The demand of each commodity is around 300 cars. Furthermore, we use 17 *standard commodities* (Φ) to represent the normal traffic on a Saturday afternoon. Corresponding to the rather low traffic volume at this time, the total demand of these commodities is scaled to 100 cars per minute. Please note, that 900 cars in total may not sound 'mega' at first glance, but the traffic volume of the event commodities is about 100 times higher than the average demand of the standard commodities. Furthermore, 900 is a realistic number of cars, since most cars are used by three or four persons, local spectators arrive by foot, and supporters of the visiting team usually arrive by train. The common cycle time of the network is $\Gamma = 90s$. Accordingly, we use $k = 90$ time steps of one second for the cyclic time-expansion.

Using CPLEX 12.6, we are now going to compute and compare the following scenarios. Firstly, we determine the *base case*. This is an optimal signal timing computed for the 17 standard commodities, that is, no football match is happening. We will test this fixed signal timings for the three football commodities and for all 20 commodities together. Thus, we estimate the draining time using overload edges. Then, we calculate average travel times and the draining time in a sufficient cyclically time-expanded network as described in the previous section. In both cases, we apply complete re-routing of all standard commodities,

Table 1 Average travel times and draining times for combinations of signal settings and demands. Travel times are given in seconds, draining time is given in multiples of the cycle time Γ. Travel times are computed in $G^{27\Gamma}$, $G^{6\Gamma}$, and $G^{10\Gamma}$, respectively.

signal setting	average travel times for				draining time
	standard	football	total		
			standard	football	
base	225.46	253.33	225.86	256.88	26.31
football offset	288.22	156.53	290.66	159.86	26.31
total offset	225.71	225.73	225.87	227.13	26.31
football split	-	149.57	-	-	5.78
total split	242.27	170.52	242.83	174.46	9.62

that is, all road users are aware of the match and the increased traffic volume around the stadium.

Secondly, we re-optimize *offsets* for both cases, the three football commodities alone and all 20 commodities together. This optimization is carried out in the cyclically time-expanded network with one cycle and overload edges. The restriction to offsets reduces the number of binary variables significantly, since a lot of binaries can be fixed. Thus, solutions can be obtained faster and the gap is smaller in most cases. On the other hand, re-optimizing only offsets does not resolve bottlenecks. Thus, one should expect a reduced average travel time, especially for the three football commodities, but the draining time will only be slightly affected. To compare, we also compute the travel times and draining times for all pairs of demands and signal timings, that is, we also compute the consequences when timings and demand do not match. Again, this final travel time calculation is done in the cyclically time-expanded network with multiple cycles.

Thirdly, we also re-optimize *split times* and *phase orders* on all routes used by the football commodities. Doing so for the three football commodities should reduce the draining time significantly. However, when considering only these three commodities, we may create bottlenecks for the 17 standard commodities. In other words, the total demand of these 17 commodities could not be routed with this signal timing due to short green splits. Hence, we also calculate optimal signal timings for all 20 commodities. This should yield a slightly higher draining time, but all standard commodities can reach their destinations.

4.1 The base case

The base case signal timings are computed as described above. Introducing overload edges the estimate for draining all demand is $\alpha = 27$ cycles for the three football commodities as well as for all 20 commodities. The obtained average travel times (in seconds) and draining times (in multiples of Γ) are presented in Table 1 (signal setting *base*).

Please note that the average travel time for the three football commodities in Table 1 is the time spent in the standard network, i.e., it does not include the waiting time at the source s. These costs are taken into account in the draining time. Thus, the total average travel time for the event commodities is around 1400 seconds.

The standard commodities are only slightly decelerated. The increased traffic volume only yields an increase of about 1 percent in travel time. On the other hand, since the progressive signal timings for these commodities are still in effect, a very significant increase would have been surprising.

4.2 Optimizing offsets

Offsets are now re-optimized with help of the cyclically time expanded network model with overload edges. Since the 17 standard commodities in the base case use routes which will be partially used by the three football commodities as well, already the base case should provide a rather good signal timing for the football commodities.

Nevertheless, as can be seen in Table 1 (signal settings *football offset* and *total offset*), offset optimization can reduce the average pure travel time of the football commodities by nearly 40 percent. However, when optimization only considers the football commodities (football offset), the travel time of the standard commodities is increased by 30 percent. Considering both football and standard commodities in the optimization (total offset), we can reduce the travel time of the football commodities by 10 percent compared to the base case. Surprisingly, the travel time of the standard commodities in this case is nearly the same as in the base case.

Again, average travel time in Table 1 only accounts for travel and waiting times in the network. The time which is spent before the first link is entered does not add to the average travel time, but it is indirectly accounted for in the draining time. As expected, offset optimization can only improve the travel times, but draining times are unaffected. Thus, the total average travel time for the football commodities is still around 1300 seconds when including the waiting time at the source s.

4.3 Optimizing split times and phase orders

Finally, we also re-optimize split times and phase orders. The results are presented in Table 1 in the rows *football split* and *total split*. As expected, the specifically for the football commodities optimized signal timings (football split) do not provide a feasible solution for the standard commodities, because bottlenecks are created and there is not enough capacity available due to short green phases. Hence, one should always consider all commodities in practice. Yet, focusing only on the football commodities yields a solution, where the additional traffic is drained four times faster compared to the solution with fixed split times. Also the average travel time is improved.

To guarantee feasibility, signal timings are optimized for all commodities in a final run (total split). Of course, the very short draining time cannot be maintained, but the event commodities are still drained three times faster than in the base case solution. Also the average travel time for the football commodities is significantly reduced compared to offset optimization. Surprisingly, this solution also means an increase in travel time of only seven percent for the standard commodities compared to the base case.

Overload edges were introduced, since a good choice for α was not known. After computing traffic assignments in both the network with overload edges and the extended network $G^{\alpha\Gamma}$, we can now compare the travel times in Table 2 to evaluate the accuracy of the model with overload edges. In other words, flows in G^{Γ} and $G^{\alpha\Gamma}$ should behave the same. Since there are only slight differences, the travel times and the traffic assignment in G^{Γ} with overload edges is a reasonable prediction about the situation in a classically time-expanded network without these overload edges.

The resultant traffic assignment for the scenario *total split* is shown in Figure 3 and 4. Here, we sum up all traffic volumes over time, i.e., the figures correspond to a projection of the dynamic traffic into the static network. In this example, there is hardly any difference between the projection from G^{Γ} with overload edges and $G^{\alpha\Gamma}$ without overload edges besides the overload edges themselves. Thus, we only present the figures for G^{Γ}.

Table 2 The cyclically time-expanded network model with overload edges slightly overestimates travel times compared to the expansion with multiple cycles. Here, average travel times in G^Γ refer only to flow in the actual network, travel times on overload edges are disregarded.

	average travel time in seconds in	
	G^Γ with overload edges	$G^{\alpha\Gamma}$ without overload edges
total offset standard	232.56	225.87
total offset football	228.69	227.13
total split standard	244.24	242.83
total split football	181.19	174.46

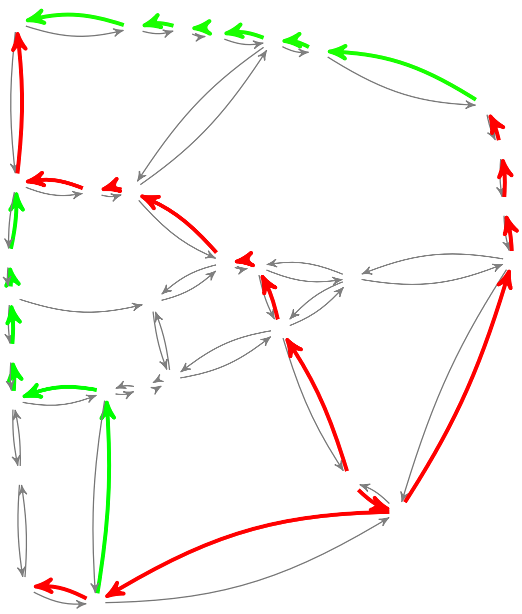


Figure 3 Traffic volumes of the three football commodities in the scenario *total split* (the 17 standard commodities are not shown). Colors encode the traffic density with respect to the total demand of the football commodities (gray: no traffic; green: very low traffic; red: about 20 % of the total demand use this road). The first commodity to t_1 uses three different routes. Moreover, other routes were used in the other scenarios.

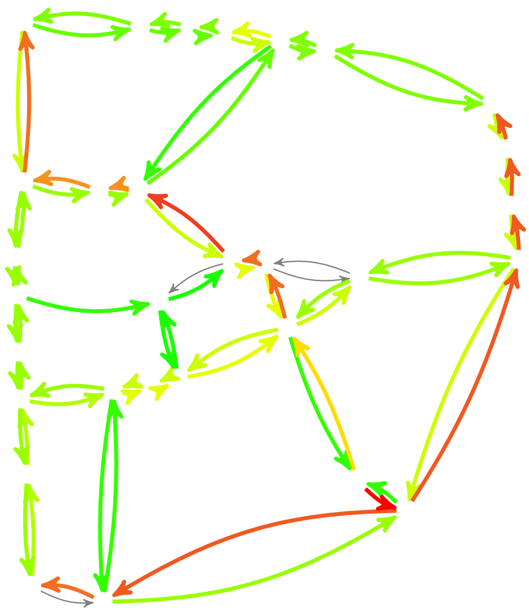


Figure 4 Total traffic volumes of all commodities in the scenario *total split*. Colors now encode traffic density regarding the total demand of all commodities. Yet, the increased traffic volume due to the football commodities is clearly visible.

Furthermore, we present an example of the signal settings of the intersection at node s. In Table 3, the green intervals of the original coordination and of the signal settings *football split* and *total split* are shown.

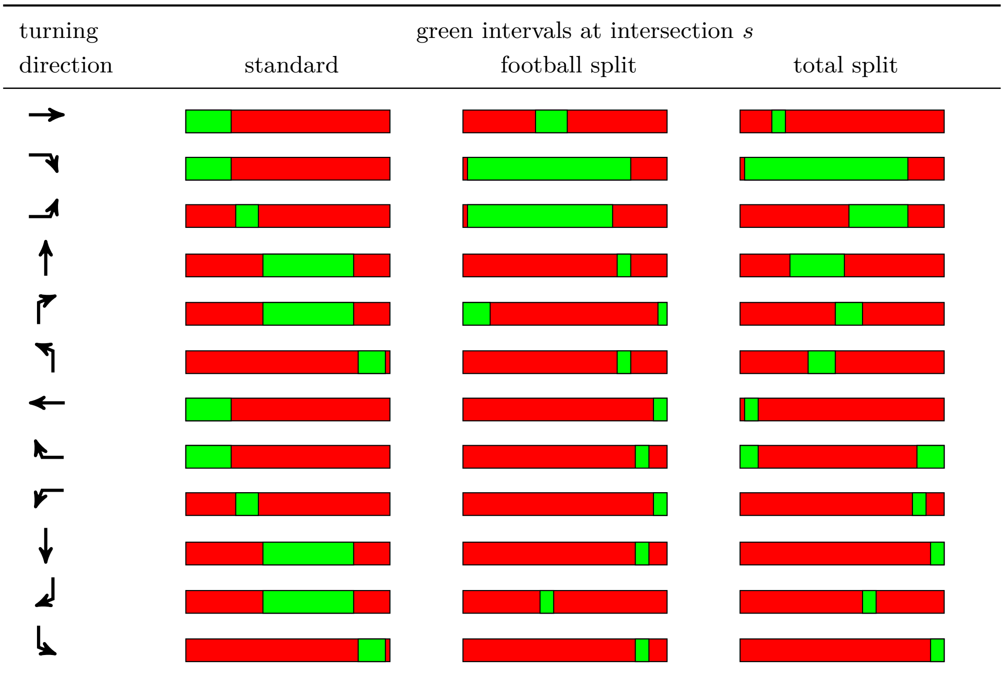
Summarizing, we have found signal timings for this football scenario, which hardly influence the base traffic, but which lead the additional event traffic towards its destinations very efficiently. In practice, one has two switch to these new timings for only about 15 to 20 minutes to drain the stadium. In contrast, leaving the old signal timings in place will stress the road network with additional traffic for more than 45 minutes. Observe that these times nearly linearly increase with the demand of the event commodities. That is, doubling the number of cars will cause about a doubled draining time.

5 Discussion

Concluding, the results of Section 4 provide evidence that optimizing pretimed signals for mega events can significantly improve traffic flow and reduce traffic congestion. With help of the optimized signal timings, not only travel times can be reduced in the presented real-world scenario, but also the duration of this exceptional traffic situation is shortened by around 70 percent. We had to introduce overload edges for two reasons: computing a bound on draining time and creating a work-around for the strict capacity constraints in the cyclically time-expanded network flow model. Comparing the predicted travel times in the network with overload edges to travel times in the network $G^{\alpha\Gamma}$ with increased time horizon, the extension with overload edges seems to be a suitable approach to find very good signal timings for such mega events.

The supposed model can also be used to compute signal timings for evacuation scenarios, e.g., to evacuate cities as fast as possible in case of a nearby forest fire or volcanic activity. For this purpose, the standard commodities Φ can be ignored and timings are just computed for the exceptional commodities as in the scenario *football split*.

Table 3 Green intervals at intersection s for different coordinations. The football commodities arrive from the left and the corresponding splits are increased in both football scenarios. The road to the right leads into a residential area, splits are reduced. Observe that also phases are significantly re-arranged.



Traffic simulation with the multi-agent simulation tool MATSim, developed by TU Berlin and ETH Zurich, also reveals the good performance of our approach for signal optimization, but we have to omit these results here due to space constraints.

In future developments, we will investigate whether it is possible to re-optimize signal timings of only a few intersections near to an exceptional traffic event like an accident in a few seconds to also provide optimized reactions to unexpected events. Furthermore, pedestrians were not considered so far. On the one hand, traffic signal coordination is hardly possible for pedestrians, since the comparatively high ratio of distance and differing speeds causes very high deviations in travel time between consecutive intersections. Without a tight estimation of arrival times at traffic signals, something like a green wave does not exist. On the other hand, pedestrians are indirectly incorporated in minimum green times and clearance times of traffic signals. For example, a minimum green time of traffic light for cars can be at least as high as the clearance time of the parallel pedestrian signal. Since the considered events involve an greatly increased number of pedestrians, parameters like the minimum green times could be included in a simultaneous or parallel optimization process to also improve the convenience for pedestrians.

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