# Continuous Optimization: The "Right" Language for Graph Algorithms?*

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## Abstract

Traditionally, we view graphs as purely combinatorial objects and tend to design our graph algorithms to be combinatorial as well. In fact, in the context of algorithms, "combinatorial" became a synonym of "fast".

Recent work, however, shows that a number of such "inherently combinatorial" graph problems can be solved much faster using methods that are very "non-combinatorial". Specifically, by approaching these problems with tools and notions borrowed from linear algebra and, more broadly, from continuous optimization. A notable examples here are the recent lines of work on the maximum flow problem [5, 1, 4, 6, 9, 3, 8, 7, 2], the bipartite matching problem [6, 7, 2], and the shortest path problem in graphs with negative-length arcs [2].

This raises an intriguing question: Is continuous optimization a more suitable and principled optics for fast graph algorithms than the classic combinatorial view? In this talk, I will discuss this question as well as the developments that motivated it.

**1998 ACM Subject Classification** F.2 Analysis of Algorithms

**Keywords and phrases** maximum flow problem, bipartite matchings, interior-point methods, gradient decent method, Laplacian linear systems

**Digital Object Identifier** 10.4230/LIPIcs.FSTTCS.2016.4

**Category** Invited Talk

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* This work was partially supported by NSF grant CCF-1553428, and Sloan Research Fellowship.





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