Multiple-criteria ranking of a finite set of alternatives using ordinal regression and a set of additive utility functions - a new UTA^{GMS} method

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We are considering a decision situation in which a finite set of alternatives (actions) A is evaluated on a family G of n criteria $g_1, g_2, \cdots, g_n$, where $G = 1, 2, \cdots, n$. We assume, without loss of generality, that the greater $g_i(a)$, the better alternative a on criterion g_i, for all $i \in G$. A decision maker (DM) is willing to rank the alternatives in A from the best to the worst, according to his/her preferences. The ranking can be complete or partial, depending on the preference information supplied by the DM and on the way of exploiting this information. The family of criteria G is supposed to satisfy the following consistency conditions:

- exhaustivity - any two alternatives having the same evaluations on all criteria from G should be considered indifferent,
- monotonicity - when comparing two alternatives, an improvement of one of them on at least one criterion from G should not deteriorate its comparison to the other one,
- non-redundancy - deletion of any criterion from G will contradict one of the two above conditions.

Such a decision problem is called *multiple criteria ranking problem*. It is known that the only information coming out from the formulation of this problem is the dominance ranking. Let us recall that in the dominance ranking, alternative $a \in A$ is preferred to alternative $b \in A$ (denotation aPb) if and only if $g_i(a) \geq g_i(b)$ for all $i \in G$, with at least one strict inequality; moreover, a is indifferent to b (denotation aIb) if and only if $g_i(a) = g_i(b)$ for all $i \in G$; hence, for any two alternatives $a, b \in A$, one of the four situations may arise in the dominance ranking: aPb, bPa, aIb and $a?b$, where the last one means that a and b are incomparable. Usually, the dominance ranking is very poor, i.e. the most frequent situation is $a?b$.

In order to enrich the dominance ranking, multiple criteria decision aiding (MCDA) helps in construction of an aggregation model on the base of preference

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information supplied by the DM. Such an aggregation model is called preference model - it induces a preference structure in set A whose proper exploitation permits to work out a ranking proposed to the DM.

The preference information may be either direct or indirect, depending if it specifies directly values of some parameters used in the preference model (e.g. trade-off weights, aspiration levels, discrimination thresholds, etc.) or if it specifies some examples of holistic judgments from which compatible values of the preference model parameters are induced.

Direct preference information is used in the traditional aggregation paradigm, according to which the aggregation model is first constructed and then applied on set A to rank the alternatives. Indirect preference information is used in the disaggregation (or regression) paradigm, according to which the holistic preferences on a subset of alternatives $A^R \subseteq A$ are known first and then a consistent aggregation model is inferred from this information to be applied on set A in order to rank the alternatives. Presently, MCDA methods based on indirect preference information and the disaggregation paradigm are of increasing interest for they require relatively less cognitive effort from the DM. Indeed, the disaggregation paradigm is consistent with the "posterior rationality" postulated by March (1978) and with the inductive learning used in artificial intelligence approaches (Michalski et al. 1998). Typical applications of this paradigm in MCDA are presented in (Srinivasan and Shocker 1973, Pekelman and Sen 1974, Jacquet-Lagrèze and Siskos 1982, Kiss et al. 1994, Bana e Costa, Vansnick 1994, Mousseau and Slowinski 1998, Greco et al. 1999, 2001, Slowinski et al. 2004).

In this paper, we are considering the aggregation model in form of an additive value function $U(a) = \sum_{i=1}^{n} u_i(a)$, where $u_i(a) \geq 0$, $i = 1, \cdots, n$, are marginal value functions. We are using this aggregation model in the settings of the disaggregation paradigm, as it has been proposed in the UTA method (Jacquet-Lagrèze and Siskos 1982). In fact, our method generalizes the UTA method by using a set of all additive value functions (1) compatible with indirect preference information having the form of a preorder in a subset of alternatives $A^R \subseteq A$. As a result, we will obtain two rankings in the set of alternatives A, such that for any pair of alternatives $a, b \in A$:

- in the first (strong) ranking, a is ranked at least as good as b if and only if, $U(a) \geq U(b)$ for all value functions compatible with the indirect preference information,
- in the second (weak) ranking, a is ranked at least as good as b if and only if, $U(a) \geq U(b)$ for at least one value function compatible with the indirect preference information.

The first (strong) ranking can be considered as robust with respect to the indirect preference information. Such robustness of the strong ranking refers to the fact that any pair of alternatives compares in the same way whatever the additive value function compatible with the indirect preference information. Indeed, when no indirect preference information is given, the strong ranking corresponds to the dominance ranking. Another appeal of such an approach stems from the fact that it gives space for interactivity with the DM. Considering an indirect preference information provided by the DM, the presentation of the strong ranking is a good support for generating reactions from the DM. Namely, (s)he could wish to enrich the ranking or to contradict a part of it. This reaction would be integrated in the indirect preference information in the next iteration.

After an outline of the principle of the ordinal regression via linear programming, as proposed in the original UTA method (see Jacquet-Lagrèze and Siskos 1982), we provide motivations for the new approach. The new UTA method is presented together with its implementation on a PC. An example of application ends the presentation.

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