Degrees of Belief

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1. Three Types of Degree of Belief or Evidential Support.

Inquiring and deliberating agents discriminate between conjectures with respect to the degrees of belief or disbelief. The conjectures are potential answers to a question under investigation identified by the inquirer and judged to be *serious possibilities* consistent with the inquirer's current state of full belief.

I shall not attempt a comprehensive survey of all the diverse notions of degree of confidence, degree of belief or the like that have been proposed for drawing fine grained distinctions between seriously possible conjectures. I shall focus on just three:

(1) *Degrees of Credal Probability*:
(2) *Degrees of Belief in the Maximizing Sense*.
(3) *Degrees of Belief in the Satisficing Sense*.

It is commonly held that a rational agent ought to accord degrees of belief to seriously possible conjectures that are supported by the agent's state of full belief. If we are to take this commonplace on board, its force must be carefully understood.

If inquirer X is in state of full belief K at time t, X is committed at t to distinguishing between seriously possible doxastic propositions consistent with K accessible to X and doxastic propositions accessible to X that X is committed to ruling out as not seriously possible. According to X at t, X's state of full belief supports the seriously possible propositions to varying degrees. That is to say, at t X is committed to a standard of evidential support relative to K as well as a standard for assigning degrees of belief relative to K. The commonplace requires as a condition of rational coherence that X should believe the seriously possible propositions to degrees that are equal to their degrees of evidential support.

Insofar as we should take this commonplace as gospel while acknowledging the distinction between three senses of belief to a degree, we should acknowledge three distinct notions of evidential support: support for degrees of credal probability, for degrees of belief in the maximizing sense and for degrees of belief in the satisficing sense.

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Should we endorse the commonplace? I shall return this question later.

2. Credal Probability, Confirmational Commitments and States of Full Belief.

Credal or subjective or personal probability has an undoubtedly important significance for practical deliberation. It is used to determine the expected values of options in decision-making. To be sure, this expectation-determining function cannot always insure that agents are in a position to maximize expected utility. Credal probabilities (and, indeed, utilities as well) may well go indeterminate so that even when choosing between a finite set of options, there may be no option that maximizes expected utility. But indeterminate states of credal probability judgment can always be represented by *sets* of credal probability functions so that one can consider a set of expected utility functions for the available options and recommend restricting choice to the E-admissible options. These are the options that are best according to at least one permissible expected utility for realizing the goals of the decision-maker (i.e., at least one permissible probability-utility of payoffs pair). Hence, even in the case of indeterminacy, the expectation-determining role of credal states remains important.

Until the last half of the 20th century, those who considered credal probability thought X's state of credal probability at a given time ought to be determined by X's state of full belief in accordance with a methodological rule that specified for each potential state of full belief K in a domain of (conceptually) accessible potential states of full belief Φ the appropriate credal state *B* that X should have. Such a rule can be represented by a function $C: \Phi \rightarrow \Pi$ where Π is the domain of credal states and Φ is the domain of accessible states of full belief.[1]

[1] A conceptual framework is a set Φ of accessible potential states of full belief constituting a Boolean algebra closed under meets of subsets of Φ of any cardinality less than or equal to that of Φ. (Levi, 1991.) The accessible potential states in the framework may also be called *doxastic propositions*. A framework Φ may have maximally consistent potential states according to Φ. But these atoms can always be refined further thereby enlarging the framework. Hence, these atoms do not qualify as total theories of possible worlds. The framework deployed in characterizing changes in states of full belief and their justification depends on the range of problems and issues being covered. K_T is the weakest element of framework Φ and $K_\perp$ the strongest. The potential states accessible according to a framework Φ should be distinguished from the set Ψ of accessible potential states that are also relevant answers to some set of questions. Let LK be some element of Φ that is stronger than K_T and let the *basic partition* U_{LK} be a set of accessible potential states such that LK has as a consequence that exactly one element of U_{LK} is true and each element of U_{LK} is consistent with LK. The set Ψ of relevant potential answers consists of all Boolean combinations of U_{LK} (including meets of cardinality up to the cardinality of Ψ and all accessible states equivalent given LK to such Boolean combinations.) We suppose that the inquirer's current state of full belief K is an expansion of LK belonging to Φ. Insofar as the inquirer is concerned with the question characterized by U_{LK}, attention is restricted to consequences of K in Ψ. K divides the relevant potential states of full belief or potential answers in Ψ into serious possibilities and impossibilities. The *ultimate partition* U_K relative to K is the subset

According to this view (with which I am sympathetic), a rational inquirer takes credal probability to be a function of his or her state of full belief. In a generous sense of "evidence", inquirer X's state of full belief constitutes X's state of evidence. In that sense, credal probability is a function of evidence.

For the most part, authors who thought this way maintained that credal states are representable by real valued conditional probability functions. Thus, for Carnap (1962a, 1962b), the appropriate rule is represented by a confirmation function or, more accurately a credibility function. We may recast Carnap's approach so that it can be seen as a special case of the approach I favor. If K is X's current state of full belief, the degree of confirmation or credibility in Carnap's sense accorded a hypothesis h by K should recommend a real valued credal probability function to be X's credal state at that time.[2] Authors like J.M.Keynes (1921) and H.E. Kyburg (1961, 1974) dissented from the demand that the function *C* deliver a numerically determinate credal state. But they did maintain that the credal state should be a function of the state of full belief or evidence.

I contend that the confirmational commitment, like the state of full belief, should be revisable when there is good reason. This is an important departure from the received view.[3] Moreover, the state of full belief can change independently of the confirmational commitment and vice versa. But X's credal state cannot coherently change without either the confirmational commitment or state of full belief changing.[4]

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of the basic partition U_{LK} whose members are consistent with K. The set Δ of relevant potential expansions of the current state of full belief K is a subalgebra of Ψ. The remainder of the set Ψ is the set of potential contractions of K. In this discussion we will be focusing on potential expansions of K that are relevant potential answers and, hence, on the set Δ. But credal probability is normally considered to range over all elements of Φ and confirmational commitments (to be explained later) are also understood to be functions from elements of Φ to states of credal probability judgment. When Φ is finite, the potential states may be represented by sets of sentences in a first order language closed under logical consequence. Representation in the infinite case calls for technical qualifications of secondary interest here. For more on the formal apparatus I deploy see Levi, 2004 ch.1 appendix and chapter 2.1-2.3. See also Levi, 1991.

[2] Carnap (1962b and 1971) distinguished between confirmation functions that are characterized in purely logical and mathematical terms and credibility functions that are rules adopted by agents for determining what degrees of credence (credal probability) ought to be relative to different bodies of evidence. My notion of a confirmational commitment is closer to the second idea than to the notion of a mathematical function. In his (1952 and 1962a), this contrast is not made.

[3] The second part of Carnap's (1952) does recognize the revisability of confirmation functions or inductive methods and considers how to rationalize such changes. But this does not seem to be Carnap's position elsewhere.

[4]R.C. Jeffrey (1965) seems to allow changes in credal state without changes in state of full belief and without even recognizing confirmational commitments. Either these changes reflect changes in state of full belief in a larger algebra of

A confirmational commitment ought to satisfy the following constraints (Levi, 1974, 1980):

Confirmational Consistency:

(a) If $K = K_{\perp}$, $C(K) = \varnothing$.

(b) If $K \neq \varnothing$, $C(K)$ is a nonempty subset of Π

Confirmational Coherence:

C(K) is a set *B* of finitely additive and normalized conditional probability measures relative to K. Every member of *B* is a permissible conditional credal probability function according to *B* relative to K

Definition:

Let Γ be the set of seriously possible accessible doxastic propositions relative to K. *Q*(x/y) is a finitely additive and normalized conditional probability relative to K if and only if x and y are accessible doxastic propositions in Φ, y is also in Γ and the following conditions are satisfied.

(1) $Q(x/y) \geq 0$.

(2) If $K^{+}_{x} = K^{+}_{x'}$ and $K^{+}_{y} = K^{+}_{y'}$, $Q(x/y) = Q(x'/y')$.

(3) If K^{+}_{x} has the complement of $x \wedge x'$ as a consequence, $Q(x \vee x'/y) = Q(x/y) + Q(x'/y)$. (*finite additivity*)

(4) If K^{+}_{y} has x as a consequence, $Q(x/y) = 1$.(*normalization*)

(5) $Q(x \wedge x'/y) = Q(x/x' \wedge y)Q(x'/y)$. (*multiplication axiom*)

Confirmational Convexity:

B_x is the set of permissible conditional probability functions according to *C*(K) = B relative to K when each function is restricted by holding specifying a particular y in *Q*(x/y) to form $Q_y(x)$. If $Q_y(x)$ and $Q'_y(x)$ are in B_y, so is every function of the form $\alpha Q(x) + (1-\alpha)Q'(x)$.

Confirmational Conditionalization:

If $C(K^{+}_{y}) = B^{*}$ is consistent, then it is the conditionalization of *C*(K) = B. That is to say every permissible function $Q^{y}(x/z) = Q(x/z \wedge y)$ in B is identical with some *Q*(x/z) in B* and every permissible *Q*(x/z) in B* is identical with some permissible $Q^{y}(x/z)$ in B.

Confirmational conditionalization, like all the other constraints cited above, are conditions imposed on the confirmational commitment endorsed by an inquirer at a given time or in a given context. By itself it implies nothing about how credal states ought to be changed over time.

Given confirmational conditionalization, it is possible to characterize a confirmational commitment by first specifying the credal state relative to K_T. Every other consistent state K of

doxastic propositions or changes in confirmational commitment without change in full belief, changes in both or changes that the inquiring agent cannot rationalize. In the latter case, each credal state may be coherent and rational but the agent cannot justify the change from one to the other either prospectively or retrospectively. The suggestion that the agent may still be responsive to external inputs can be checked by some other agent but cannot be ascertained by the agent when reflecting on his or her changes in view especially if this must be done without any change in state of full belief.

full belief is an expansion of K_T. Confirmational conditionalization then implies that *C*(K) is the conditionalization of *C*(K_T).[5]

According to authors like Keynes, Jeffreys and Carnap, there should be a standard confirmational commitment to which all rational agents ought to be committed. This standard was hopefully to be secured by principles of a probability logic.[6] On this view, counter to the position I have taken, confirmational commitments are immune to modification. The *probability logic* would define a *logical probability* or confirmational commitment.

But according to the proposal made here, a logical confirmational commitment *LC* should be the largest set of conditional probability functions that probability logic allows being the set of permissible probabilities relative to K_T. This logical probability fails to rule out as impermissible any conditional credal probability relative to K_T that satisfies the constraints of probability logic. But unless probability logic rules out all but one probability function as a member of *C*(K_T) there are alternative confirmational commitments that are subsets of *LC*(K_T) and, hence, stronger than *LC*. In that case, probability logic alone cannot secure a standard confirmational commitment.

Probability logic, so I shall assume, includes the constraints I have thus far imposed. But there might be additional ones. If they could be plausibly restrictive enough, one might take *LC*(K_T) to be a singleton. *Confirmational Uniqueness* would be secured as a consequence of probability logic and with it the identification of the logical probability *LC* as the standard confirmational commitment.

But even the most ardent enthusiasts for logical probability seem to have conceded that no acceptable probability logic implies confirmational uniqueness. Nonetheless, many authors

[5] In Levi, 1980a, ch.16, I argue that H.E. Kyburg, R.A. Fisher and A.P.Dempster are committed to rejecting confirmational conditionalization and, hence, would not accept this alternative characterization of confirmational commitments. In this discussion, I restrict attention to those who agree that confirmational commitments ought to be Bayesian – i.e, endorse confirmational conditionalization. In Levi (1974), I characterized confirmational commitments the second way indicated here so that confirmational conditionalization was presupposed. In 1980a, I adopted the first method so that Kyburg, et al. could be represented as exploiting confirmational commitments even though they did not endorse confirmational conditionalization.

[6] Carnap called the logic an “inductive logic” and the probability a “logical” or “inductive” probability.

have endorsed probability logics that impose no further constraints on confirmational commitments than those I have listed - except for one.[7]

Confirmational Uniqueness.
C(K) = B is a singleton.

Notice that it is one thing to construct a probability logic that entails confirmational uniqueness because it entails a specific singleton for $C(K_T)$ which then is identical with $LC(K_T)$. It is quite another to abandon that project and yet insist on confirmational uniqueness. In the latter case, there is no confirmational commitment that qualifies as *the* logical confirmational commitment. De Finetti (1972), Savage (1954) and other personalist or subjectivist Bayesians seem committed to some such view as this.

On this subjectivist view, what becomes of the notion of a standard confirmational commitment? There cannot be, of course, a single standard confirmational commitment secured by probability logic. The personalists have tended, however, to think that each rational agent adopts his or her own permanent confirmational commitment. As the later and more personalist inclined Carnap suggested, each inquirer adopts his or her own "credibility function" that once adopted is retained without modification (save perhaps in the case of some conceptual or arational upheaval).

Once this view is in place, changes in credal state are uniquely determined by changes in state of full belief as it would be for necessitarians who insist on adopting *LC* (on the assumption that it exists) as the standard. And changes in credal state due to expansion of the state of full belief can be characterized by what I have called *Temporal Credal Conditionalization* (Levi, 1974, 1980) that is often called "conditionalization" or "Bayesian updating".. Changes in credal state due to contraction are characterized by *Inverse Temporal Credal Conditionalization*.

Many subjectivists seem to endorse *Confirmational Tenacity* according to which inquirer X should keep X's confirmational commitment fixed even though probability logic does not mandate that everyone adopt X's confirmational commitment and even though Y adopts a

[7] T.Seidenfeld, among others have raised questions, about confirmational convexity. I endorse confirmational convexity for reasons that cannot be elaborated here without taking us too far a field.

different confirmational commitment that is also rationally coherent. X and Y are obliged to keep the faith with their respective confirmational commitments under all circumstances.

An alternative form of subjectivism is discomfited by this. R.C. Jeffrey (1965) proposed a point of view that can be represented as recommending that an inquirer X should change X's confirmational commitment in response to sensory stimulation in a manner that can be represented by conformity to a certain "rigidity condition" that yields a change in credal state often called "Jeffrey Conditionalization". Since Jeffrey thought that inquirers should never change states of full belief, there was no reason to deploy confirmational commitments. On his view, there is a fixed state of full belief and credal states in constant flux. Jeffrey's view so understood abandons the idea that X's commitment to a credal state at a given time is a function of X's state of full belief or evidence.[8]

Yet, an account of inquiry aimed at justifying the modification of states of full belief should be accompanied with an account of efforts to justify changes in confirmational commitment. Just as X stands in no need to justify X's current full beliefs unless X has good reasons to call them into doubt, X does not need to justify continuing to deploy the same confirmational commitment unless it is called into doubt. Confirmational tenacity is untenable. But confirmational inertia seems to make good sense.

An account of when confirmational commitments are justifiably changed is equivalent to an account of when an inquirer is justified in changing the "prior" probabilities used in Bayesian updating. Like Jeffrey's view, such an account would allow for violations of temporal credal conditionalization without abandoning the synchronic condition of confirmational conditionalization. Unlike Jeffrey's view, the one I favor makes essential use of the presupposition that the concern of the inquirer is to provide a basis for inductive expansion. And it provides for indeterminacy in credal probability judgment in contravention of confirmational

[8] As the total knowledge or total evidence requirement demands. Jeffrey would deny that his view is insensitive to evidence understood as that stream of sensory inputs impinging on the agent. Since, however, the agent cannot recognize the inputs, the agent cannot engage in critical control of these inputs to secure conformity to any regimen of belief change. We can imagine an agent or robot who conformed to Jeffrey's requirements in virtue of training or design. It is difficult to understand how a reflective inquirer X could exploit Jeffrey's recommendations to identify the changes in credal state X should institute unless X changed X's state of full belief as well.

uniqueness. I shall return to the topic of revising confirmational commitments briefly at the end of the paper.

3. Inductive Expansion

Both in science and everyday life, there is interest in identifying both the best supported of rival answers to a given question and the answer all of whose consequences are supported to a sufficiently high a degree. The question arises: How are notions of evidential support in the maximizing and in the satisficing sense related to each other and to the notion of evidential support in the expectation-determining sense associated with credal probability?

The inquirer X answers the question under investigation by expanding X's initial state of full belief. When X uses the evidence available to X (i.e., the initial state of full belief K) to justify adopting a given answer, X deliberately expands the initial state K. I claim that probability as well as maximizing and satisficing evidential support are relevant to justifying ampliative or inductive expansions of states of full belief or states of absolute certainty. They are relevant, however, in distinct ways.

There are several ways of legitimately and non degenerately expanding a state of full belief – that is to say of adding information to a state of full belief that is not already entailed by it. Here are two.

Routine expansion using a program for responding to sensory input or to the testimony of others by adding propositions to one's full beliefs is legitimate when the agent X is certain in advance that the program is sufficiently reliable.[9] In routine expansion, X makes no decision concerning which of rival hypotheses to add to X's state of full belief or absolute certainty. The only decision X might make (if X makes any[10]) is to follow a program for belief acquisition.

[9] Assessments of the reliability of programs for routine expansion are not assessments of evidential support. The latter evaluate how well hypotheses held in suspense are supported by the information in X's current state of full belief. Assessments of reliability are not assessments of how well conjectures are supported by the evidence but rather determinations of what the chances are of avoiding error in using a program for routine expansion. Such determinations are implications of X's state of full belief concerning the objective chances of certain processes.

[10] In many and, perhaps, most contexts, the inquirer already takes for granted that a program for routine expansion is reliable. Perhaps, the inquirer acquired the conviction either explicitly or tacitly by nature or nurture. A program is chosen only in contexts where the inquirer is in doubt as to which program among a roster of alternatives to use and comes to the conclusion that one is to be implemented over alternatives.

Ampliative or *inductive expansion*, as found in theory choice, estimation of a parameter, judgment as to the presence of a correlation, curve fitting and the like, is also a form of belief acquisition.

In such cases, the inquirer has identified a set Δ of doxastic propositions as potential answers responsive to the question under investigation. The members of Δ represent cognitive options from which the inquirer is to choose in a way that should promote the goals of his or her inquiry.

Δ may be characterized as the set of Boolean combinations of an *ultimate partition* U_K of doxastic propositions (potential states of full belief) such that K entails the truth of exactly one member of U_K and each element of U_K is consistent with K.[11] In this discussion, attention will be restricted to finite ultimate partitions. Every potential expansion of K relevant to the question under investigation may then be represented as follows: Identify a subset R of U_K and its complement R^c in U_K (the set theoretic complement in U_K of R). Let h be the join of the members of R^c. The expansion K^+_h of K is the meet of K and h. If R is empty and $R^c = U_K$ the expansion is K itself. If $R = U_K$ and R^c is empty, the expansion is the inconsistent belief state $K_\perp$. Expansion by adding an element of U_K is expansion by adding a maximally specific (with respect to the question under investigation) and consistent potential answer to K. Other consistent potential answers involve some sort of partial suspension of judgment between members of U_K.

Expansion by adding an element of Δ to K is justified provided that it is shown to be the best or a best answer given the goals of the inquiry. A best answer is one that carries maximum evidential support in the maximizing sense. In inductive expansion, the inquirer seeks to adopt an answer that maximizes inductive expansion in this sense.

In adopting an expansion by adding h to K, the inquirer commits him or her self to judging true not only the truth of the new state of full belief but also all other potential states of full belief or doxastic propositions that are consequences in Δ (and indeed of Φ) of that state of full belief.

[11] So Δ is a subalgebra of the algebra of accessible potential states Φ.

Changes in states of full belief are represented as changes in such states represented *collectively*.

Now instead of representing the inductive expansion as the best supported of the available potential expansions in the maximizing sense, one might represent it as the expansion all of whose consequences are supported in the satisficing sense to a degree at or above a given threshold. Whereas support in the maximizing sense applies to the expansion represented *collectively*, support in the satisficing sense applies to the expansion taken *distributively*.

The first question that arises is whether there are maximizing and satisficing measures that recommend the same set of doxastic propositions as the inductive expansion of K.

4. **Probability as evidential support.**

The answer is clearly "Yes!" One way to do this is to take probability to be both the maximizing and satisficing index of evidential support. Regardless of the probability distribution over U_K relative to K, choosing the potential expansion maximizing probability is choosing the weakest potential expansion – to wit, refusing to reject any elements of U_K. R is empty and $R^c = U_K$. Moreover, if the threshold for adding a doxastic proposition is set at probability 1, the threshold for rejecting an element of U_K is 0 so that once more R is empty. Probability can, indeed, be a maximizing and a satisficing measure of degree of belief provided one refuses to recognize the legitimacy of any genuinely ampliative expansion.[12]

These considerations demonstrate that anyone who thinks of probability as a maximizing index of evidential support or degree of belief is an anti inductivist. And anti inductivists can have no use for measures of evidential support for the purpose of evaluating expansion strategies relative to an initial state of full belief in order to decide how to change that state of full belief. To construe degrees of belief in the maximizing sense as degrees of probability is to reduce the conception of degree of belief in the maximizing sense to uselessness. Assessing hypotheses relative to K with respect to probability is highly relevant to the evaluation of options in a decision

[12] Strictly speaking the maximum probability here should be probability in the sense of "absolute certainty" where there is no serious possibility that the proposition bearing probability 1 is false. If h is "almost certain" so that it might be false even though it carries probability 1, the probability is not a maximum. The threshold for satisficing should be understood as absolute certainty as well. I ignore these niceties in the text.

problem with respect to expected value. But the *expectation determining* function of credal probability is distinct from its alleged function as an assessment of evidential support. A way of assessing hypotheses with respect to how well they are supported by K is relevant in a context where the inquirer is still in suspense as to which member of U_K is true and is seeking to decide on the basis of his or her evidence K which members to rule out and which to continue to recognize as possibly true. That is to say, the inquirer is deliberating concerning how to expand inductively. Anti inductivists do not engage in such activity.

Perhaps probability is a satisficing index of evidential support that licenses inductive expansions distributively by recommending the addition to the state of full belief K of all joins of subsets of U_K with probability greater than some threshold less than 1. If U_K contains n elements, the threshold must be greater than (n-1)/n in order to avoid the addition of a set of doxastic propositions a set at least one of whose members must be false. And, with the threshold at that level, the set of doxastic propositions added to K will not, in general, be a set closed under the consequence relation. Hence, it will not be the set of consequences of the meet of K with a doxastic proposition. It will not be an inductive expansion. Using probability as a satisficing index of evidential support is also an anti inductivist view. Using probability in this way is as subversive of its own applicability as is using probability as evidential support in the satisficing sense.

As noted previously, there were and continue to be many authors attracted to a high probability criterion of "acceptance" or "belief" in some sense or other who are unabashed anti inductivists. They suggest that the "qualitative" notion of belief is a convenient summary of information conveyed by an index of evidential support where the latter is degree of credal probability. Instead of saying that "X believes that *h*" one might say "X judges it highly probable that *h*." On this view, degree of belief does not have to be closed under consequence since it is not a satisficing measure useful for inductive expansion.

Whatever the merits of this proposal might be, it does not capture the idea that X may be said to believe that *h* truly or erroneously. When X judges it probable that *h*, X's judgment is neither true nor false. If belief that *h* is the judgment that the probability that *h*, we cannot say that

X's belief that *h* is true or that it is false. It is a mere doxastic epiphenomenon. It would be best to do without it.

I do not mean to suggest that full belief is the only qualitative notion of belief of importance. X may be said to believe that *h* if and only if X believes that *h* to a positive degree (in the satisficing sense). In another sense, X believes that *h* if and only if X believes that *h* to sufficiently high degree in a satisficing sense to warrant coming to full belief that *h*. In this second sense, the high degree of belief relative to K is regarded as sufficiently high to justify changing one's mind and coming to full belief that *h*. So belief that *h* in the sense in which it is a codeword for sufficiently high degree of belief relative to K is significant because it warrants a change in state of full belief by inductive expansion. Recognizing such warrants is a hallmark of inductivism. Anti inductivists who do not acknowledge genuinely ampliative inductive expansion have tended to engage in a kind of conceptual bad music of the sort that the arch anti inductivist, Rudolf Carnap, charged Heidegger with composing. They seek to exploit the ideas of inductivism while denying its legitimacy. To do so, they engage in conceptual manipulation that is profoundly confusing.

5. Evidential support in the maximizing sense.

In choosing among answers to a question, we are often advised to choose the one that is best supported by the evidence. If the choice of a potential answer is a decision problem, the best answer is the one that optimally promotes the goals of the inquiry. The goals of inquiry may be as diverse as the inquiries themselves. But they may share certain features in common. I assume that the goals ought to share in common that they are multidimensional. On the one hand, an inquirer ought to seek to avoid error in inductive expansion. On the other hand, an inquirer ought to seek to obtain information of value relevant to answer the question under investigation.

If an inquirer were concerned solely to avoid error, a best answer to the question under investigation would be one that minimizes probability that the inductive expansion will import false belief – i.e., that maximizes the probability that the inductive expansion is true. In that case, probability would be evidential support in the maximizing sense. If the considerations adduced against doing so are well taken, then we also have a good reason to reject the view that avoiding error is the sole aim common to all properly chosen inductive expansions.

To avoid the result, account should be taken of other risk-of-error inducing desiderata that should be balanced with avoidance of error. In extremely idealized circumstances, the trade off can be represented as a utility function for the possible outcomes of expanding K according to one of the "cognitive options". Each of these options can be evaluated with respect to expected utility. The inquirer ought then to restrict his choice of an expansion strategy to one that maximizes expected utility.

In this way, the measure of expected (epistemic) utility is the measure of evidential support in the maximizing sense.

5. Information and Informational Value

Before characterizing expected epistemic utility further, attention should be paid to the desideratum other than avoidance of error that provides a legitimate incentive to risk error. We may look at the desideratum in one of two ways:

1. The desideratum may be recommended as the sole desideratum that ought to be pursued in making the choice. Explanatory value, simplicity, easiness and many other such notions have been touted as such desiderata.

2. The desideratum may be urged to be one of two desiderata – the other being avoidance of error – that should be balanced against one another. This is the view I shall eventually defend.

Advocates of these views are not always clear as to the scope of their recommendation. Is the evaluation with respect to one of these desiderata to apply exclusively to the elements of U_K or should it be extended to the entire algebra Δ of potential answers?

If the former reading is correct, then no answer is given to the question of how to decide when two or more elements of U_K are optimal with respect to the recommended desideratum.

One cannot recommend suspension of judgment as a tiebreaker in such cases because there is no basis for suggesting that suspension of judgment between all optimal elements of U_K is optimal. In order for tie breaking in this fashion to be rationally admissible, surely this condition ought to be met.

According to view (2), all elements of the algebra Δ generated by U_K are capable of being evaluated with respect to avoidance of error. If there is to be a trade off between avoidance or error and the new desideratum, the evaluation with respect to the second desideratum should be extended to elements of Δ as well.

Potential answers that are more specific, rule out more elements of U_K, relieve doubt more or carry more *information* relative to K partially orders the elements of Δ with respect to the information carried or with respect to the strength or specificity of the answer. *h* is stronger than *g* given K if and only if *g* is a consequence of $K \wedge h$ but h is not a consequence of $K \wedge g$.

According to this partial ordering, the elements of U_K are non comparable. But the elements of U_K may be completely ordered with respect to simplicity, explanatory power or the like. Although joins of subsets of U_K are sometimes compared in such an ordering, there is by no means complete unanimity as to whether weaker hypotheses are more or less explanatory than the alternatives that entail them or whether suspending judgment between two or more elements of U_K is simpler or more complex than the alternatives themselves.

We do not need to settle such questions. We are looking for a weak ordering of the elements of Δ or, even better a real valued utility function that is faithful to that ordering. To achieve this, the evaluation of the elements of U_K is used to develop a measure $Cont(x) = 1 - M(x)$ defined over the elements of U_K. This function represents the assessment of simplicity, explanatory power or whatever is taken to be the concern of the inquirer. This assessment is extended to the entire domain Δ by taking *M* to be a probability measure. The extension respects the evaluation of the elements of Δ as carrying more or less information. But the appeal to the evaluation of the elements of U_K with respect to the interest in simplicity, explanatory power or whatever yields an integrated single weak ordering or, indeed, into a single quantitative measure.

I call the resulting weak ordering of or measure for Δ an assessment of *informational value*.[13] My preferred way of doing this (see footnote 3) is to represent the weak ordering by a measure of *undamped informational value Cont*(h) = 1-*M*(h) where *M* is an unconditional probability over Δ.

In contrast to the views of Popper, Carnap and Bar Hillel, the probability *M* that defines the measure *Cont* representing undamped informational value is not in general the same probability measure representing the inquirer's credal probability judgments used in determining expected values. (Levi, "Information and Inference", 1967). The credal probability *Q* is expectation-determining and in this sense characterizes degrees of belief. The *M*-function does not represent degrees of belief. It is *informational-value* determining.

On the account I have just given, no commitment has been made as to what aspects of the strongest K-consistent potential answers in U_K are evaluated by the *Cont*-function. It could be explanatory power in some sense or other, simplicity in some sense or other, congeniality with some theological, ideological or political agenda. An inquirer may be conflicted between several different desiderata of this kind. If the inquirer is able to do so to his or her satisfaction, the conflict may be resolved by adopting a utility function that is a weighted average of the competing desiderata. If no such resolution is endorsed, the inquirer should remain in doubt as to which potential resolution (as represented by a weighted average) to adopt. Hence, his assessments of the informational value of elements of U_K will be represented by a set of *Cont*-functions. [I require that the set be convex but I shall not elaborate on this point here.] Each such *permissible Cont*-function can be extended to a function over Δ in the manner indicated before.

Let us now return to the consideration of thesis (1). According to (1), informational value is the sole desideratum. This leads to the dubious recommendation that inquirers contradict

[3] A few assumptions are needed to make this work. *Weak Positive Monotonicity* states that rejecting an element of the ultimate partition never decreases the informational value of a potential answer. *Constant Marginal Increment in Informational Value of Rejection* states that the increment in informational value afforded by rejecting element x of U_K is the same regardless of which and how many other elements of U_K are rejected. if h is more informative than g, h carries at least as much informational value as g. If we assume that any numerical evaluation of informational value is normalizable so that it can be restricted to values between 0 and 1, a representation of informational value is unique up to a positive affine transformation of *Cont*(h) = 1 –*M*(h) where *M* is an unconditional probability defined for elements of Δ. See "Information and Inference" from 1967 reissued in Levi (1983) ch.5 and Levi, (2004 3.2).

themselves for, in that way, informational value will be maximized regardless of what *Cont*-functions or *M*-functions are permissible.

I conjecture that those advocates of “inference to the best explanation” and cognate views mean to disregard expansion into inconsistency for some reason or other. Doing so lands them in more hot water. They must then choose an element of U_K. There can be no room for suspense between two or more elements of U_K carrying maximum informational value (explanatory value or what have you). The damped informational value must be less than the damped informational values carried by the elements of U_K. So for this view to work, there must be a unique element of U_K carrying maximum informational value.

Fans of so called “inference to the best explanation” often bolster their view by shifting in the direction of thesis (2). Simplicity (or explanatory power or whatever) is taken to be *sigillum vera*. On the assumption that the simplicity ordering ranks elements of U_K, this view implies that for x and y that are elements of U_K, $Q(x) \geq Q(y)$ if and only if $Cont(x) \geq Cont(y)$ if and only if $M(y) \geq M(x)$. Thus, maximizing informational value among the elements of U_K is maximizing probability among them so that someone seeking a simple hypothesis could also be concerned with acquiring true beliefs (avoiding error). This is not quite what thesis (2) asserts as we shall see but it is a gesture in that direction that is often taken as enough to secure fealty to the idea that we seek to avoid error in fixing belief.

It is but a gesture and not a very convincing one.

For starters, why should anyone think that the ranking of U_K with respect to simplicity or whatever agrees with the ranking of U_K with respect to probability of truth?

If probability is taken to be statistical probability or chance, the truth or falsity of the claim depends upon nature acquiescing to our assessments of simplicity being correlated with probability. I suppose there may be cases where this is true but they will be matched by cases where the claim is false.

If probability is taken to be personal or credal probability, urging individuals to conform to its dictates seems more akin to fatuous optimism than to reason and, indeed, to *a priori* reason as some have asserted. Charles Peirce pointed out that Mill's Uniformity of Nature principle is false. Nature is uniform in some respects but not in others. To think of simplicity as the mark of truth or likelihood of truth is no better.

In any case, the question arises: what should one do when two or more elements of U_K are both simplest elements of U_K? Suspending judgment between them minimizes risk of error. But it incurs a loss in informational value. An answer grounded on a conception of the aims of inquiry is called for. None is given.

I conclude that the risk inducing desideratum, whatever it is, should not be the sole desideratum being promoted in inductive expansion. I also suggest that postulating principles of reason proclaiming that hypotheses that are attractive according to the desideratum are more likely to be true are, as a general rule, unacceptable.

6.Seeking new, error free and valuable information.

Given K and U_K, the cognitive options are the several expansion strategies. Throughout this discussion, U_K will be assumed to be finite. Infinite cases are discussed in Levi, 2004.

On the assumption that the goal to be attained is to add new information that is (1) error free and (2) is of value, the expansion strategy that recommends suspending judgment between the elements of R^c or equivalently rejecting the elements of R where R is a subset of U_K has a payoff $<T(R^c,t), Cont(R^c)>$ if the true element of U_K is in R^c and $<T(R^c,f), Cont(R^c)>$ if the true element of U_K is in R. $T(R^c,x) = 1$ (0) when x = t (f). The utility of this payoff is $\alpha T(R^c) + (1-\alpha)Cont(R^c)$ where $0 \leq \alpha \leq 1$.[14]

If we also assume that no payoff for any option that imports error is assigned greater utility by the aggregate or compromise utility than the payoff for any option that avoids error, α cannot be less than 0.5. The compromise utility function can be converted into any positive

[14] That a potential compromise between two conflicting desiderata representable as von Neumann Morgenstern utilities over the mixture set of the set of available options ought itself to be a von Neumann Morgenstern utility together with the requirement that all shared (paretian) agreements between the two desiderata should preserved in the potential compromise argues for this weighted average condition.

transformation without altering any conclusions to be recommended. The following is a useful transformation. $T(R^c,x) - qM(R^c)$ where $q = (1-\alpha)/\alpha$ ranges from 0 to 1.

The expected epistemic utility of rejecting the elements of R (i.e., expanding K by adding all the consequences of $K \wedge R^c$) is then

$$Q(R^c) - qM(R^c) = EV(R^c)$$

According to the idea posed previously, $EV(R^c)$ should be the evidential support in the maximizing sense for the potential answer R^c provided by K given the demands for information imposed by U_K and *Cont,* the assessment of risk of error provided by Q, and the degree of boldness exercised by the inquirer in reaching a balance between avoiding error and acquiring new information of value.

When is $EV(R^c)$ a maximum? $EV(R^c) = \sum_{x \in R^c}[Q(x) - qM(x)]$. For finite U_K, the maximum will be reached when every x in R^c is such that $Q(x) - qM(x)$ is non negative and all x in U_K such that $Q(x) - qM(x)$ that are positive are in R^c. If there elements of U_K such that $Q(x) - qM(x) = 0$, several sets R^c can carry maximum EV value. I have long advocated breaking ties for optimality by choosing the largest optimal set. $Q(x) - qM(x) \geq 0$ for all x in this R^c.

Inductive Rejection Rule

Element y in U_K is rejected (is in R) if and only if $Q(y) < qM(y)$.

Inductive Expansion Rule

Expand K by adding R^c obtained by the inductive rejection rule.

Before inductive expansion, X's belief state is K and X is committed to fully believing all consequences of K.

After expansion, X's belief state is $K \wedge R^c$ and X is committed to believing all consequences of this expanded belief state.

7. Is the difference between a posterior and a prior probability a maximizing index of evidential support?

It is clear that this measure of evidential support in the maximizing sense is not probability. But it bears a formal resemblance to one of a family of measures that many Bayesians are prepared to endorse.

To explain let us subscript the current credal probability with the state of full belief K that supports it so that the function is Q_K. Let K* be a state of full belief such that K = K*∧E = K^{*+}_{E} – i.e., K is an expansion of K* by adding E (consistent with K*) and the consequences of doing so. Finally, let Q_K(H/F) = Q_{K*}(H/F∧E). Q_{K*} is the prior probability and Q_K is the posterior probability.[15]

Many Bayesians have adopted Q_K(H) – Q_{K*}(H) as a measure of confirmation or evidential support. When this difference is positive, E positively confirms or supports H (E is positively relevant to H). Similar conditions obtain *mutatis mutandis* for 0 and negative differences.

Formally the Bayesian formula and the EV-function are both differences between two distinct probability functions. The EV-function modifies the difference by the insertion of a "boldness" parameter q; but many of the formal properties of the two functions are similar.

However, not only are the intended interpretations different. So are some of the formal properties. In the EV-function, the second term is the informational value determining probability. It represents the assessment of informational value that takes into account the relative informational values of the elements of U_K and their Boolean combinations.

In the difference between the posterior and prior function, the prior function is defined over a larger domain of possibilities than the posterior function Q_K is. The ultimate partition U_K relative to K could be a subset of some partition U_{K*} relative to K*. Consequently there could be a pair of propositions H and H' whose equivalence is not entailed by K* but is entailed by K. Hence Q_K(H) – Q_{K*}(H) need not equal Q_K(H') – Q_{K*}(H') even though the expansion of K by H and the expansion of K by H' are the same expansion. Using the difference between the posterior and

[15] There are many "prior" states of full belief K* and potential states E that could qualify for use in characterizing the prior probability Q_{K*}. The difficulties raised here apply to all of them so that there is little need to investigate the merits of rival proposals as long as the conditions in the text are satisfied.

the prior assigns at least two distinct degrees of evidential support to the same inductive expansion in such cases. In contexts where the concern is the evaluation of inductive expansion strategies from K, this ambiguity cannot be acceptable. No such ambiguity plagues the EV-function.[16]

To be sure, when an investigator X in state of full belief K* is designing an experiment that may yield by *routine* expansion one of a range of items of information or "data points" including E, X may want to know whether the outcome of the experiment will be probabilistically relevant to elements of an ultimate partition U_{K^*} that includes H. The difference between a posterior and a prior is an index of the probabilistic relevance of E to H.

But appraisals of potential data points with respect to probabilistic relevance are inappropriate as evaluations of the merits of potential inductive expansions aimed at answering a given question. EV-functions are better assessments of evidential support in the maximizing sense.

8.Satisficing Measures of Evidential Support.

The index q = $(1-\alpha)/\alpha$ in the measure of inductive support in the maximizing sense takes values from 0 to 1 due to the restriction of α to values in the interval [0.5,1]. As q increases, the weight attached to *Cont*(*h*) as compared to *T*(*h*,x) increases. When q = 0, *Cont*(*h*) is completely discounted. The inquirer is focused exclusively on avoidance of error. No element *h* of Δ consistent with K is rejected even if *Q*(*h*) = 0.[17] When q = 0.5, the weight attached to *Cont*(*h*) is the maximum allowed. For this reason, the index q may be called the index of boldness.[18]

Consider a state of full belief K, ultimate partition U_K and algebra of potential answers Δ generated by U_K, credal probability function and *Cont* function for Δ. For every consistent *h* in Δ, there is a unique value q(*h*) ($0 \leq$ q(*h*) ≤ 1) for the index of boldness such that when EV is

[16] This ambiguity is related to the multiplicity of factorizations of the difference between a posterior and and prior into an ampliative and explicative component that emerged after Popper and Miller (1983) sought to argue for the impossibility of inductive probability. (Levi, 1984b.

[17] On the other hand, if *h* is inconsistent with K so that *Q*(*h*) = 0 and q = 0, the rule for Ties guarantees that *h* is rejected.

[18] I owe the use of this term to a suggestion made by R.C.Jeffrey. In Levi, 1967a, I used the less felicitous term "degree of caution". q decreases with the increase in caution.

maximized with an index of boldness $q \leq q(h)$ and the Rule for Ties is deployed, h fails to be rejected and for all values of $q > q(h)$ if there are any such values less than or equal to 1, h is rejected. For *h* in Δ inconsistent with K, *h* is rejected for every value of q in the closed interval from 0 and 1, so there is no value of q(*h*) in the permitted range. It is useful, however, to set q(*h*) = 0 even though, in contrast to the case where q(*h*) = 0 but *h* is consistent with K, *h* is rejected when q = 0 as well as when q > 0. q(*h*) is the *degree of unrejectability* of *h* (relative to K, U_K, *Q* and *M*).

If x is an element of U_K, the degree of unrejectability q(x) is easily seen to be equal to the minimum of *Q*(x)/*M*(x) and 1. For values of $q \leq Q(x)/M(x) < 1$, x fails to be rejected. For greater values of q, it is rejected. If $Q(x)/M(x) \geq 1$, x fails to be rejected for every value of q between 0 and 1 and q(x) = 1.

If h in Δ is consistent with K, h is equivalent given K to a disjunction of elements of a subset ||*h*|| of U_K. It is rejected if and only if all elements of ||*h*|| are rejected. Hence, q(*h*) = $\max_{x \in U_K}[q(x)]$. Moreover, there must be at least one element x of U_K for which q(x) = 1 and, hence, for any h, either q(*h*) = 1 or q(~*h*) = 1. As before, in case *h* is inconsistent with K and ||*h*|| is therefore empty, we stipulate that q(*h*) = 0. These are formal properties of possibility measures according to Dubois and Prade (1992).

I have long contended that the degree of *dis*belief or *potential surprise* that *h*, d(*h*), increases with a decrease in its degree of unrejectability and may be conveniently represented by 1 – q(*h*). (Levi, 1967a). If the inquirer fixes a degree of boldness q for purposes of inductive expansion, an element of the ultimate partition is rejected if and only if d(x) = 1-q(x) = 1 – *Q*(x)/*M*(x) is above the threshold 1 – q. All elements in ||*h*|| are rejected just in case the one with the smallest d-value is. Hence, d(*h*) = $\min_{x \in ||h||}[d(x)]$ and *h* will be rejected just in case this minimum is above 1-q.

Let b(*h*) = d(~h).

Once the degree of boldness is specified, a threshold 1-q is determined such that *h* is a consequence of the recommended expansion of K if and only if b(*h*) ≥ 1-q. It is demonstrable that the set of potential states in Δ that meet this requirement constitute the consequences in Δ of the recommended expansion of K. The b-function qualifies as a degree of belief function or evidential support function in the satisficing sense. Fixing the value of the boldness parameter q to be used to determine the recommended expansion is equivalent to specifying the threshold degree of belief such that all and only elements of Δ carrying degree of belief higher than that threshold are consequences of the recommended expansion.

Prior to inductive expansion relative to K, U_K, *Q* and *M*, the b-function (or the d-function, or the q-function) may be used to appraise the status of elements of Δ as candidates for admission into the expanded state of full belief. Once the degree of boldness is fixed and a decision is taken, the inquirer is committed to becoming absolutely certain of the truth of the propositions initially assigned degrees of belief short of the maximum.[19]

If the inquirer is not allowed to engage in inductive expansion as a matter of principle, the appraisals in terms of d-functions, b-functions and q-functions would serve no useful purpose. Indeed, this is true of any candidate measure of degree of belief in the satisficing sense.

Many authors are interested in characterizing a "qualitative" notion of belief that is not equivalent to full belief. Wolfgang Spohn, for example, introduces a notion of plain belief as belief to a positive degree. (Spohn, 1990, 149, 150-1.) Such a notion makes eminently good sense according to the account I favor. Plain beliefs are elements of Δ that would be added to K were the inquirer maximally bold – i.e., were the inquirer to adopt q = 1. Elements of Δ that cannot be added to K even when q = 1 cannot be added to K under any circumstances as long as K, U_K, *Q* and *M* are held fixed.

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[19] For this reason, the inquirer can iterate the operation of inductive expansion on the truncation of U_K resulting from rejecting elements of this partition with boldness q. The credal probability and *M*-functions can be updated and rejections using the same level of boldness q applied once more. The process can be iterated until no further rejections are achieved. As long as the ultimate partition initially used is finite, a fixed point will be achieved. We may look on the entire process as a single inductive expansion according to a stable inductive expansion rule with boldness q. New functions q(h), d(h) and b(h) can be defined for such a stable inductive expansion rule. (Levi, 1967a , 1980a, 1996, 2004.) Stable inductive expansions form the basis for the account of inductive expansion I offer for cases where the ultimate partition is infinite.

The set of plain beliefs PB_1 relative to K constitutes an expansion of K. This expansion is closed under consequence as is every coherent expansion of K. The set of plain beliefs is either error free (true) or false. However, from the point of view of the inquirer prior to expansion, the expansion is not judged error free or erroneous. So the inquirer cannot claim to have judged truth rightly or wrongly in terms of his or her beliefs. No risk of error is incurred. No erstwhile serious possibilities have been eliminated. Nor have the credal probability judgments over elements of Δ been altered. Nonetheless, there is an important relation between plain belief and full belief. If X plainly believes that *h* when X's state of full belief is K without fully believing that *h* and exercises maximum boldness to *change* X's state of full belief K by inductive expansion, X should add expand K so that *h* becomes one of the consequences.

We can consider sets of sentences PB_q consisting of all doxastic propositions that carry degree of belief greater than 1 – q for any value of q from 0 to 1. These too will characterize a conception of qualitative belief where the set of beliefs is closed under consequence. These are the sentences that would be added if the degree of boldness q were exercised. When the state of full belief is K, they are not fully believed but only believed to a degree greater than 1 – q.

There are thus many conceptions of qualitative belief that differ from full belief. The full beliefs in K are the elements of PB_0 – i.e., the elements of Δ consistent with K that fail to be rejected when q = 0. These are, of course, the elements of Δ consistent with K.

Thus, the elements of Δ consistent with K are ranked with respect to how bold the inquirer needs to be (given K, U_K, *Q* and *M*) in order to believe them in some qualitative sense. But the ranking has no significant application except in the context of evaluating inductive expansions aimed at changing states of full belief. If inductive expansion is never legitimate, as many authors seem to think, the ranking itself is useless.

L.J. Cohen (1977) has contended that degree of belief measures similar to the one I have just proposed is a measure of Baconian probability in contrast to the Pascalian probability structured according to finitely and possibly countably additive measures of mathematical probability. I think Cohen is right to maintain that d-functions have as much legitimate claim (but no more) to represent a presystematic conception of belief probability as *Q*-functions do. But it is not worthwhile to engage in fruitless disputes over this matter. Inductivists like Cohen and myself can recognize that both types of indices have important uses. We need to take note of these measures and their uses and to respect their differences. In particular, we should stop confusing degree of belief in the satisficing sense with expectation determining credal probability. Once we do that, we may avoid the temptation to indulge in tedious and pointless discussions of lottery and preface paradoxes.

9. Indeterminacy in Credal Probability and Changing Confirmational Commitments

When an inquirer is in a state of full belief, X is committed to being certain of the truth of some (doxastic) propositions and remaining in suspense (and, in that sense, in ignorance) of others. No inquirer should claim to be opinionated in the sense of being certain as to which doxastic propositions are true and which are false in all cases. Such a claim presupposes that there are maximally consistent potential states of full belief in some space of potential states accessible "in principle" to all rational agents. The inquirer's opinionated claim is a claim of certainty as to which is the true one. This would be a manifest bit of arrogance if it were comprehensible. But no matter how finely we discriminate between possibilities, we cannot rule out ever more subtle distinctions. There are no possible worlds. Maximal opinionation in the sense just considered is incoherent. No matter how much we strive, occasions for inquiry prompted by doubt should remain.

In the face of ignorance, many authors turn to probability for solace. I submit, however, that opinionation concerning credal probability judgment can avoid the charge of absurd arrogance only if it, like full belief, is partial and modest.

Those authors who acknowledge the intelligibility of objective chance have, of course, recognized that inquirers may be in doubt as to the truth of rival statistical hypotheses. Statistical hypotheses are doxastic propositions and like other potential states of full belief carry truth-values. States of credal probability judgment, by way of contrast, lack truth-values. One cannot coherently suspend judgment with respect to the truth of a credal probability judgment or assign a credal probability as to the truth of such a judgment.

Yet, it does make sense to withhold numerically determinate credal probability judgment. A distribution of credal probability over some algebra of doxastic propositions is not a truth-value bearing proposition, so one cannot suspend judgment as to its truth. But a credal probability distribution has a use in evaluating expected values of options in decision problems. In deliberating in the context of some decision problem, the deliberating agent may rule out some probability distributions as *impermissible* for use in evaluating expected values of options and fail to rule out others so that they remain permissible for use. In this respect, X's state of credal probability judgment at a given time may serve not as a standard for serious possibility but instead as a standard for serious permissibility.

This is not the occasion to enter into a detailed account of the way such states of indeterminate credal probability judgment should be used to evaluate options; but something needs to be said so as not to leave the impression that the reference to a standard for serious permissibility is nothing more than a rhetorical flourish.[20]

Given a set of available options and a specification of the payoffs for outcomes of each option in each state of nature in terms of a determinate utility function, a permissible expected utility function for the available options may be determined for each permissible credal probability distribution over the states of nature. The set of optimal options with respect to expected utility can then be determined for that permissible expected utility function and, hence, for that permissible probability function. This can be done for each permissible credal probability function. The set of options that are *admissible with respect to expected utility* (*E-admissible*) is the set of options that come out optimal according to some seriously permissible probability distribution in the credal state.

[20] For further elaboration, see Levi, 1980a and 1999.

Further elaboration leads to an account of E-admissibility when not only the agent's credal probability state but state of utility judgment is numerically indeterminate. Additional criteria can then be added to reduce the set of E-admissible options to smaller subsets.[21]

Our topic in this paper is degrees of belief or evidential support. Strictly speaking, credal probability is a matter of degree only in very special cases where credal probability judgment is numerically determinate. I do not mean therefore to question the thesis that credal probability judgment is a type of degree of belief. Credal probability is what it is. It functions as a means for evaluating options with respect to E-admissibility and, as a consequence, serves as a "guide in life". It provides finer grained discriminations between serious possibilities than the state of full belief K does but like full belief contributes to practical deliberation in making such discriminations. There is no reason to refuse to recognize credal probability as degree of belief. We should avoid, however, granting it exclusive title to that epithet. And we should recognize that degrees of belief so conceived may and often should go indeterminate.

The significance of the indeterminacy has been often been obscured because of a tendency on the part of those who insist that rationality requires determinacy in probability judgment to acknowledge that inquiring agents are not capable of measuring either the credal probability judgments of others or their own with absolute precision.

There is no disputing that this claim is true. It is obviously true when X seeks to determine the credal probability judgments of Y but it is also true when X assesses the credal probability judgments of X. Acknowledgment of the limitations on our capacities to be perfectly rational include not merely recognition of our lack of logical omnisicience and parallel limitations on our capacity to be probabilistically coherent but also limitations on our capacity for fulfilling our commitments to self knowledge. We are incapable of fulfilling our commitments to fully believe the logical consequences of our full beliefs and to make credal probability judgments in conformity with the calculus of probability. We are likewise incapable of fully believing that we believe that h when we do so and that we do not believe that h when we do not do so even

[21] See Levi, 1980a, 1986, 1999 for further details.

though we are committed to both sorts of self-knowledge by ideal standards of rationality. And parallel remarks apply to X's about Y's credal state and X's credal state.

Such ignorance gives rise to *imprecision* in judgments as to what X's credal state is even if X's credal state is numerically determinate. Strict Bayesians are quite comfortable about acknowledging such imprecision. They oppose indeterminacy in probability judgment. Recognizing indeterminacy is acknowledging a kind of ignorance that strict Bayesians refuse to countenance. This is a kind of ignorance that recognizes that there can be E-admissibility without optimality. In such cases, probabilistic ignorance generates ignorance as to what one ought to do.

The sort of ignorance in probability judgment strict Bayesians can allow is lack of self-knowledge and the attendant failure of commitment. The cure for such failure is some process of clarification whereby one comes to recognize commitments one already has. Those who acknowledge indeterminacy insist that there are forms of ignorance concerning probability judgment that are removed through inquiry rather than some process of therapeutic clarification. Whereas therapeutic clarification is a process of change of view where the current state is recognized to be rationally incoherent and the aim is to extricate oneself from that state, inquiry is undertaken to alter the rationally coherent point of view to which the inquirer is committed. This involves justifying one commitment in favor of another. Choosing an inductive expansion requires such justification of a change in commitment to full belief. I would like now to consider some aspects of changing confirmational commitments and how they are to be justified.

Suppose X faces a problem relative to K with ultimate partition U_K and informational value determining probability *M*. X's confirmational commitment *C* is such that X's credal state B over U_K is highly indeterminate. X may have come to this result because X and Y initially differed in their confirmational commitments and credal states even though they shared the same state of full belief. In order for X to explore differences with Y, X should shift to a confirmational commitment that recommends a credal state that recognizes the credal probability functions recognized as permissible according X's state and the credal probability functions recognized as permissible according to Y's all to be permissible. That is to say, B may be convex hull of their two credal states. Both X and Y are concerned to strengthen the credal state B prior to running some experiment obtaining data they mean to use in attempting to answer the question expressed by U_K and *M*.

One proposal for doing this is to recognize as permissible all credal distributions over U_K that when combined with U_K, *M* and the degree of boldness q adopted by the inquirers X and Y fail to reject any of the elements of U_K. The strengthened credal state would avoid bias in one way or another with respect to any element of U_K. Only the information obtained from additional investigation could then be used to eliminate elements of U_K.

Given any element x of U_K (where U_K is finite), x is rejected by a probability distribution *Q* if and only if $Q(x) < qM(x)$. Consider all probability distributions *Q* such that $Q(x) = qM(x)$ for all elements x of U_K but one. The probability of the exception is the remaining probability required to yield total probability 1. Take the convex hull of all these distributions as the credal state. This credal state is the same as the so called "epsilon contaminated class" of probabilities" characterized by $(1-\varepsilon)M(x) + \varepsilon Q(x)$ where *Q* is allowed to be any distribution over U_K. (Berger and Berliner, 1986, 462.)[22]

Thus, the inquirers X and Y would be entitled to adopt as the set of "permissible prior credal probabilities" for use in subsequent investigation the class of those distributions none of which would recommend rejecting an element of U_K. Some may complain that the choice of priors to count as permissible is dependent on the informational value determining *M*-function. I respond that making such bias explicit permits us to keep bias under critical control. Thus, if the *M*-function is uniform over U_K, when q = 1, the prior distribution is the uniform distribution; but

[22] Berger and Berliner cite several uses of ε-contaminated classes of distributions going back to the 1960's most but not all of which embrace frequentist interpretations rather than credal interpretations of probability. In Levi, 1980, ch.13.4ff, I proposed adoption of a family of prior distributions derived along the lines suggested in the text. Only much later did Teddy Seidenfeld point out to me that the class of unbiased priors as I called them was an ε-contaminated class as understood in the robust Bayesian literature. The difference between my approach and that in the statistical literature is that the distribution *M* I picked out for distinction in defining the class was equated with an informational value determining probability characterizing the demands for information appropriate to some problem of inductive expansion. I avoided consideration of "most plausible" distributions that seem to me to be obscure.

rather than considering this distribution to be recommended by some principle of insufficient reason, it is recognized as the product of the inquirer's demands for information.

A more serious difficulty is that the epsilon contaminated family recognizes for each element of U_K at least one permissible distribution according to which the prior credal probability of x is very high so that it becomes difficult to reject x at the given level of boldness for any data points unless likelihoods are very decisive. If this is a genuine objection, it can be remedied in several ways by imposing a maximum less than 1 on the value of *Q* in the formula for epsilon contamination.

Further issues need to be addressed if the *M* – function is allowed to go indeterminate. I shall not address them here.

What I mean to emphasize is that efforts to strengthen confirmational commitments may be justified in the context of inductive expansion by recommendations that reflect the inquirer's demands for information without being willful or arbitrary in any serious manner. Moreover, the fact that such changes are controlled by interest in inductive expansion points in a direction opposite to the one favored by those who wish to avoid the question of justifying changes in states of full belief.

Thus, I claim that just as degree of belief or evidential support in both the maximizing and satisficing senses presuppose the context of inductive expansion, in a broad class of cases, changes in confirmational commitments are justified in the context of inductive expansion.

10.The Lockean Thesis and Epiphenomenal Belief.

Richard Foley's view is representative of the ideas of many contemporary epistemologists who worry about the epistemology of belief and the epistemology of degree of belief. He advances what he calls "The Lockean Thesis".

> To say that we believe a proposition is just to say that we are confident of its truth for our attitude to be one of belief. Then it is epistemically rational for us to believe a proposition

just in case it is epistemically rational for us to have sufficiently high degree of confidence in it to make our attitude towards it one of belief. (Foley, 1992, 111.)

The notion of degree of belief used here need not be degree of belief in the satisficing sense. Indeed, Foley takes the notion of degree of belief to be degree of credal probability almost without argument. In effect, Foley is following an approach made explicit by H.E.Kyburg a long time ago (1961, 1974). As we have seen, credal probability could not be degree of belief in the satisficing sense where probabilities are determinate or indeterminate. Credal probability in the satisficing sense takes high probability to be sufficient for *changing* states of full belief (and, as a consequence states of credal probability judgment) by adding all highly probable propositions to the state of full belief. Since the consequences of such a state (the set of full beliefs) should be fully believed, all conjunctions of such full beliefs should also be fully believed. Otherwise the state of full belief could not serve as the standard for serious possibility.

The Lockean Thesis as understood by Foley is interpreted differently. Relative to the initial state of full belief K, proposition *h* carries a high degree of belief. Let us say for the sake of the argument that it is a high degree of credal probability. If the degree of belief is high enough, the inquirer believes in Foley's qualitative sense that *h relative to* K. The inquirer does not change belief state. The inquirer who believes that *h* in Foley's qualitative sense does not fully believe that *h* and does not come to full belief that *h*. Qualitative belief in this sense has no clear relation with warrant for changing belief. Rational qualitative belief that *h* is qualitative belief that *h* supported by the evidence in K just as rational degree of belief that *h* is degree of belief supported by K.

Qualitative belief so conceived need not be closed under conjunctions just as Kyburg has insisted for a long time and Foley echoing Kyburg insists. In response to the charge that qualitative belief as he following Kyburg understands it is merely an epiphenomenal expression of degrees of belief, Foley responds that rational qualitative belief is applicable to situations where one has three options: bet on *h* with a one dollar gain if *h* is true, one dollar loss if *h* is false, bet on ~h for similar terms and refuse to bet with no gain or loss. One may face the same issue for several probabilistically independent propositions.[23] Foley thinks such betting situations are of the kind where one might usefully deploy his notion of qualitative belief. Maybe this is so. But the probabilistic notion whether determinate or not can also be used. Qualitative belief remains an epiphenomenon.

In any case, rational qualitative belief relative to K cannot be degree of probability relative to K above a given threshold.

10. Plain Belief

There is a qualitative notion of belief relative to K that is equivalent to degree of belief in the satisficing sense above a given threshold relative to K. Indeed there is an entire family of such qualitative notions relative to K parameterized by the index of boldness q. Begin with the notion of *plain belief* (Spohn, 1990). Although Spohn's measures of degrees of belief bear some formal similarities to measures of degrees of belief in the satisficing sense, Spohn himself avoids concern with inductive expansion or cognate applications of his formalisms. But this need not deter us from pointing out that if his notion of degree of belief is interpreted as degree of belief in the satisficing sense, plain belief that *h* as he defines it becomes positive degree of belief that *h* in the satisficing sense where the latter means that ~*h* is rejected when the inquirer exercises maximum boldness q = 1. The set of plain beliefs is closed under the consequence relation not only when the ultimate partition is finite as I have been assuming here but when the proposals I have made elsewhere for extending the inductive expansion rules to countably infinite sets of alternatives are deployed.[24]

[23] Foley fails to mention the requirement of probabilistic independence that seems appropriate to his intention although exemplified (so one may suppose) by his examples.
[24] Levi, 2004, ch.6.2. The extended inductive expansion rules proposed are guaranteed to avoid inconsistency only if the *M*-function is finitely additive. These rules are guaranteed to avoid inconsistency when q = 1 only if the credal probability over U_K has a maximum value among elements of U_K. The distribution does not have to be countably additive although this is not forbidden. Spohn himself has used another interpretation according to which plain beliefs must receive probability 1 in the standard reals. This approach will insure that the set of plain beliefs will include meets of finite sets of plain beliefs but it will not include meets of a countable infinity of plain beliefs. The approach I favor does. Consider for example a countably infinite lottery where each ticket has the same credal probability of winning and the informational value of ticket j winning is the same for all tickets and, hence, *M*(j) = 0. On my account, inquirer X should lack plain belief in any hypothesis about the outcome of the lottery except the claim that exactly one ticket will be drawn. On Spohn's view, X should plainly believe that ticket i will not win for each i, should plainly believe that exactly one ticket will win but should not believe that no ticket will win.

Some plain beliefs relative to K are and some are not full beliefs that are consequences of K. However, if the inquirer is maximally bold in inductive expansion (relative to K, U_K,*Q*, *M*), X should expand from K to the set of propositions that are plainly believed relative to K. That is to say, X's new set of full beliefs should consist of the plain beliefs relative to K that are consequences of K and the set of plain beliefs relative to K that are not consequences of K.

However, inquirer X may be more cautious and refuse to expand by adding all plain beliefs relative to K. They remain plain beliefs relative to K and, indeed, relative to the set expanded at the lower value for q without being fully believed. So the notion of plain belief or belief to a positive degree remains a useful idea.

In the same manner we can define other qualitative notions under the rubric "belief to a degree greater than 1 – q" for each degree of boldness less than 1 except for q = 0 where we speak of degree of belief 1.

Each one of these notions is a qualitative notion of belief relative to K. With the partial exception of the case where q = 0, it represents a qualitative notion distinct from full belief. But with the exercise of degree of boldness q, it also marks the warranted inductive expansion of K. It is this circumstance that distinguishes qualitative notions of belief relative to K of this type from the epiphenomenal notions used by Kyburg, by Spohn and by Foley.

10, Conclusion.

If the argument of this essay has any merit, measures of evidential support in the satisficing and maximizing sense have useful application because of the importance of jusitifying changes in states of full belief via deliberate, inductive expansion. Evidential support in the maximizing sense is, so I have argued, expected epistemic utility. Evidential support in the satisficing sense is a measure of degree of belief exhibiting the formal structure of a Shackle measure. Expected epistemic utility is rarely considered to license a degree of belief. Shackle measures, on the other hand, are used to determine degrees of belief. In any case, neither expected epistemic utility nor degree of belief in the Shackle sense can be a probability in the sense of the calculus of probability. But we can recognize expectation determining credal probability as a conception of degree of belief. But this conception too is entangled with the idea of full belief. In the first place, X's state of credal probability should be a function of X's state of full belief in accordance with a confirmational commitment. As long as the confirmational commitment remains unchanged, changes in credal state are determined by changes in state of full belief. On the other hand, changes in confirmational commitment are often justified by considering how adopting revised confirmational commitments would modify the methods used to acquire new full beliefs. So degree of belief as credal probability, like the other notions of degree of belief, requires consideration of states of full belief and their modification.

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