Uncertainties in stochastic programming models – The minimax approach

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Extended abstract

50 years ago, stochastic programming was introduced to deal with uncertain values of coefficients which were observed in applications of mathematical programming. These uncertainties were modeled as random and the assumption of complete knowledge of the probability distribution of random parameters became a standard. In practice, complete knowledge of the probability distribution is rare and using this assumption, we introduce a new type of uncertainty which concerns the probability distribution.

Moreover, to apply stochastic programming one tends to use various simplifications, e.g., to relax integrality of variables, nonlinearities in constraints, to simplify the dynamic and stochastic specification, etc., and various approximations are used to get a solution in an efficient way.

The obtained results, recommendations for the decision maker, should be then carefully analyzed. This is done partly under the heading "stress testing", backtesting, out-of-sample analysis, or more generally within output analysis techniques. Applicability of the method depends on the structure of the model, on the assumptions concerning the probability distribution, on the available data and on hardware and software facilities.

Using a hypothetical, ad hoc distribution may lead to bad, costly decisions. Besides of a subsequent output analysis it pays to include the existing, possibly limited information into the model and to use the minimax approach.

The rapid development of hardware allows to solve large optimization problems, including some of minimax problems in real time. However, the commercial optimization systems fail to support extracting data from other applications. This causes difficulties when loading scenarios and puts limitations on output analysis. Concerning the minimax problems, there are new interesting suggestions how to extend the existing software for stochastic programs to solution of minimax problems. They apply to special problem formulations, see e.g. [8] for scenario-based mixed-integer minimax stochastic programs.

The minimax approach has been developed for special types of stochastic programs and special choices of $\mathcal{P}$. To illustrate the basic ideas we shall consider here stochastic programs of the form

$$\text{minimize } F(x, P) := E_P f(x, \omega) \text{ on the set } x \in \mathcal{X} \tag{1}$$

where the set $\mathcal{X}$ is fixed, independent of the probability distribution P of ω. The minimax approach is applied in cases when the probability distribution P is only known to belong to a specified class $\mathcal{P}$ of probability distributions. It means that

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one tries to hedge against the worst possible distribution by solving the minimax problem

$$\min_{x \in \mathcal{X}} \max_{P \in \mathcal{P}} F(x, P).$$

See [2] for an introduction and for a survey of various choices of the class $\mathcal{P}$.

To get the class $\mathcal{P}$ one often chooses compromises between the wish to exploit the existing, available information and the need to keep the minimax problem numerically tractable. Indeed, there are minimax problems which be solved using adapted algorithms for ordinary stochastic programs, cf. [1], [5], [6], [7].

Selected examples are given for the class $\mathcal{P}$ of probability distributions described by (generalized) moment conditions and a given support Ω. Assuming only knowledge of the support the minimax problem reduces to an instance of a robust optimization problem.

The minimax solution depends on specification of the class $\mathcal{P}$, in our case on the moments values and on the support Ω. Hence, we face an additional level of uncertainty which influences the results. A suitable sensitivity analysis with respect to changes of the input parameters is important and it has to be tailored to the type of the solved minimax problem and to the considered input perturbations. Similarly as in output analysis for ordinary stochastic programs (1) with respect to the probability distribution P, one may exploit results of parametric optimization and statistics. See e.g. [3], [4], [5], [7] for some attempts in this direction.

The surveyed results are based on the following papers:

1. M. Breton, S. El Hachem (1995), Algorithms for the solution of stochastic dynamic minimax problems, *Comput. Optim. Appl.* **4**, 317–345.

2. J. Dupačová (2001), Stochastic programming: minimax approach. In: Ch. A. Floudas, P. M. Pardalos (eds.) *Encyclopedia of Optimization*, Vol. V., pp.327–330 and *references therein.*

3. J. Dupačová (1984), Stability in stochastic programming with recourse – Estimated parameters, *Math. Progr.* **28**, 72–83.

4. J. Dupačová (1997), Moment bounds for stochastic programs in particular for recourse problems. In: V. Beneš, J. Štěpán (eds.) *Distributions with given Marginals and Moment Problems*, Kluwer, pp. 199–204.

5. M. Riis, K. A. Andersen (2004), Applying the minimax criterion in stochastic recourse programs, to appear in *European J. of Oper. Res.*

6. A. Shapiro, S. Ahmed (2004), On a class of minimax stochastic programs, *SIAM J. Optim.* **14**, 1237–1249.

7. A. Shapiro, A. Kleywegt (2002), Minimax analysis of stochastic programs, *GOMS* **17**, 532–542.

8. S. Takriti, S. Ahmed (2002), Managing short-term electricity contracts under uncertainty: A minimax approach. TR, Georgia Inst. of Technology.