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POLYHEDRAL RISK MEASURES AND LAGRANGIAN RELAXATION IN ELECTRICITY PORTFOLIO OPTIMIZATION

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ABSTRACT. We present a multistage stochastic programming model for mean-risk optimization of electricity portfolios containing physical components and energy derivative products. We consider a medium-term time horizon of up to one year. Stochasticity enters the model via the uncertain (time-dependent) prices, electricity demand, and heat demand. The objective is to maximize the expected overall revenue and, simultaneously, to minimize a multiperiod risk measure, i.e., a risk measure that takes into account the intermediate time cash values. We compare the effect of different multiperiod risk measures taken from the class of polyhedral risk measures which was suggested in our earlier work. Furthermore, we discuss how such a mean-risk optimization problem can be solved by dual decomposition techniques (Lagrangian relaxation). Hence, the scope of this presentation, beside the model itself, is the impact of polyhedral risk measures on stochastic programming models with respect to both, results and decomposition structures.

1. INTRODUCTION

The deregulation of energy markets has lead to an increased awareness of the need for profit maximization with simultaneous consideration of risk management, adapted to individual risk aversion of market participants. Mathematical modeling of such problems with uncertain input data results in mixed-integer large-scale stochastic programming models. We refer to a wide range of literature dealing with power management in a hydro-thermal system and simultaneous optimization of power production and electricity trading, e.g. [6, 9, 15, 14, 3, 5].

We present a model tailored to the needs of the sales and trading department of a municipal power utility, i.e., our considerations are from the perspective of a small player at the electricity market. It is assumed that there is a combined heat and power (CHP) facility that serves for the heat demand and partly for the electricity demand. Further, we consider the possibility of trading electricity at the electricity spot market of the *European Energy Exchange* (EEX) in Leipzig, Germany. This spot market is a day-ahead auction. We allow only for price independent bids, hence, there is 100% volume safety on the one hand, and spot price risk on the other hand. Finally, within our model it is possible to participate in the electricity future market at EEX which provides a means for hedging spot price risk. In addition, it would be possible to evaluate the benefit of electricity delivery contracts offered by larger power concerns by incorporating them into the model, but we won't focus on this possibility here.

We suppose that each historical observation at time t of electricity demand D_t^e, spot price C_t^s, and heat demand D_t^h is a realization of certain trivariate random variables. The joint distribution of this stochastic process will be characterized by a time series model consisting of separate models for intra daily behavior and average daily behavior. The latter is modeled by an multivariate ARMA model with additional trend components and a jump-diffusion model. In addition, future prices are assumed as so-called *fair prices* with respect to the spot prices of the corresponding time period.

We generate a large number of Monte-Carlo scenarios from this time series model. By means of scenario reduction techniques according to [10] we generate from this initial approximation of the underlying probability distribution a specific form of an approximation - a scenario tree - representing the information structure of the optimization problem. Thus, spot prices and electrical load as well as most of the decision variables are defined on this tree and, hence, can be understood as random processes with discrete distribution.

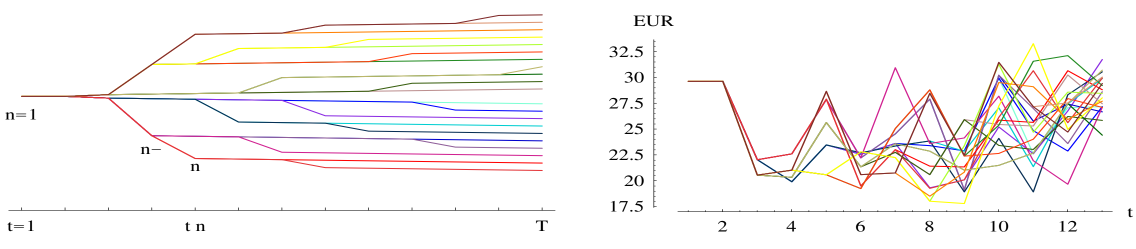

FIGURE 1. Scenario tree structure (left) and Scenario tree for spot prices (right).

2. POLYHEDRAL RISK MEASURES

The risk of high losses of uncertain outcomes is quantified with so-called risk measures, i.e., mappings from some space of random variables (or random processes) to the real numbers that have certain properties (cf. [1, 7, 12]). In particular, in the case that the risk of long or medium term activities is to be considered, multiperiod risk measures are needed that take also intermediate cash values into account (cf. [2, 11]).

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However, stochastic programs incorporating risk measures are usually harder to solve. Therefore, one has to restrict the choice of the risk measure to those with favorable properties for the structure of the respective stochastic program. To this end, the class of polyhedral risk measures was introduced in [4] for which theses favorable properties are guaranteed.

Consider a finite number T' of time periods, a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and a filtration $\mathcal{F}_1 \subseteq \mathcal{F}_2 \subseteq ... \subseteq \mathcal{F}_{T'}$ of σ-fields, e.g., $\mathcal{F}_{t'} = \sigma(\xi_1, ..., \xi_{t'})$ with some random process ξ. Suppose the (uncertain) value process is represented by random variables $z_1, z_2, ..., z_{T'}$ with $z_{t'} \in L_p(\Omega, \mathcal{F}_{t'}, \mathbb{P})$ $(p \geq 1)$ for which large outcomes are preferred to lower ones.

In this framework, multiperiod *polyhedral risk measures* were defined in [4] as optimal values of certain simple multistage stochastic programs:

$$(1) \qquad \rho(z_1, ..., z_{T'}) = \inf \left\{ \mathbb{E}\left[\sum_{t'=1}^{T'} \langle c_{t'}, y_{t'} \rangle\right] \;\middle|\; \begin{array}{l} y_{t'} \in L_p(\Omega, \mathcal{F}_{t'}, \mathbb{P}; \mathbb{R}^{k_{t'}}),\ y_{t'}(\omega) \in Y_{t'}\,, \\ \sum_{\tau=0}^{t'-1} \langle w_{t',\tau}, y_{t'-\tau}(\omega)\rangle = z_{t'}(\omega) \end{array} \ (t' = 1, \dots, T') \right\}$$

with some $k_{t'} \in \mathbb{N}$, $c_{t'} \in \mathbb{R}^{k_{t'}}$, $t' = 1, \dots, T'$, $w_{t'\tau} \in \mathbb{R}^{k_{t'-\tau}}$, $t' = 1, \dots, T'$, $\tau = 0, ..., t'-1$, and polyhedral cones $Y_{t'} \subseteq \mathbb{R}^{k_{t'}}$, $t' = 1, \dots, T'$.

Instances of this class were suggested in [4] for the multiperiod case. All these examples are multiperiod coherent, they can be understood as multiperiod extentions of the one-period Conditional-Value-at-Risk $CVaR_\alpha(z) = \inf_{r\in\mathbb{R}} \left\{r + \frac{1}{\alpha}\mathbb{E}[(z+r)^-]\right\}$ with $0 < \alpha \ll 1$ (cf. [12]), e.g. $\rho(z_1, ..., z_{T'}) = \inf_{r\in\mathbb{R}}\{r + \frac{1}{T'-1}\sum_{t'=2}^{T'} \frac{1}{\alpha(T'-1)}\mathbb{E}[(r+z_{t'})^-]\}$. Another instance was suggested in [11] based on the value of perfect information (VPI).

3. Optimization Model

3.1. **Trees.** The model is a multistage stochastic program with a finite number T of timesteps. The input scenario tree consists of a set on nodes $\mathcal{N} = \{1, ..., N\}$ with node probabilities $(\pi_n)_{n\in\mathcal{N}}$, the tree structure (every node n has a unique predecessor $n-$ and an associated time step $t(n)$), and the random data $\xi = (\xi_n)_{n\in\mathcal{N}}$, $\xi_n = (D_n^e, D_n^h, C_n^s)$. For the description and implementation of the dynamics of the decisions (e.g. "day-ahead") it is useful to define trees related to the input scenario tree. Decision variables will be defined on the node of these trees. This guarantees non-anticipativity of the decisions in the required manner.

1. Trading day tree: based on original scenario tree, branching at any node is delayed in time until the beginning of the next trading day (mon-fri and not a holiday). Each node $n \in \mathcal{N}$ of the original scenario tree has a unique corresponding node $j(n) \in \mathcal{N}^{\text{trade}}$ such that for the timesteps of the nodes it holds that $t(n) = t(j(n)$.
2. Future tree: Based on the original scenario tree, the number of timesteps and, hence, the number of nodes is reduced such that there is one timestep at each trading day at 12 am. In addition, there are timesteps (and nodes) for the final billing of the futures at the end of each month (11 pm).
3. Risk tree: Based on the original scenario tree, the number of timesteps is reduced to the subset of timesteps $t' = 1, ..., T'$ in the same manner as for the future tree.

3.2. **Decision variables.** Decision variables will be denoted by the letter x. All of them are defined on one of the trees described in the previous paragraph. They will be indexed by the node number of the respective tree.
Future stock for month m (base): $x_d^{fb,m}$, $d \in \mathcal{N}^{\text{fut}}$, $m = 1, ..., 12$
Future stock for month m (peak): $x_d^{fp,m}$, $d \in \mathcal{N}^{\text{fut}}$, $m = 1, ..., 12$
Spot market volumes: x_j^s, $j \in \mathcal{N}^{\text{trade}}$
Contract volumes: x_j^c, $j \in \mathcal{N}^{\text{trade}}$.
Power production, electricity: x_n^{pe}, heat: x_n^{ph}, $x_n^p := (x_n^{pe}, x_n^{ph})$, $n \in \mathcal{N}$
with restrictions $|x_n^{pe} - x_{n-}^{pe}| \leq \delta_{\max}$, $A \cdot x_n^p \leq b$, with some $b \in \mathbb{R}^k$, $k \in \mathbb{N}$, $A \in \mathbb{R}^{k\times 2}$, and $x_n^{ph} \geq D_n^h$, $n \in \mathcal{N}$.

The collectivity of all these variables will be denoted by x and all the (non-coupling) constraints will be symbolized by the notation $x \in \mathcal{X}$.

3.3. **Cash value.** The wealth at time t depends on the scenario and the decisions up to time t. Therefor we define auxiliary scalar variables z_n at each node $n \in \mathcal{N}$ of the input scenario tree that will be called cash value. Of course, the cash value z_n at node $n \in \mathcal{N}$, i.e., at time $t(n)$ in scenario $s(n)$, is additively composed of the cost of each component of the portfolio at node n:

$$z_n = z_n^s + z_n^p + z_n^c + \sum_{m=1,\dots,12} \left(z_n^{f,b,m} + z_n^{f,p,m}\right) + \sum_{\tilde{n}\in\text{path}(n)} \left(P^{pe} \cdot D_{\tilde{n}}^e + P^{ph} \cdot D_{\tilde{n}}^h\right)$$

with z_n^f, z_n^s, z_n^p, and z_n^c denoting cash values originating only from future trading, spot market, power production, and supply contract, respectively. The last term represents the revenue for satisfying the heat and electricity demand, respectively, with $\text{path}(n)$ denoting the set of all nodes between node n and the root node of the tree. Note that $z = z(x, \xi)$, i.e., cash values depend (non-anticipatively) on the decisions and the stochastics. Note further that, of course, component separability ($z^s = z^s(x^s, \xi)$, $z^p = z^p(x^{pe}, x^{ph}, \xi)$, $z_n^{f,b,m} = z_n^{f,b,m}(x_n^{f,b,m}, \xi)$ ...) holds.

3.4. **Stochastic Program.** The optimization problem representing our model can be written in the following form:

$$(2) \qquad \min \left\{ \rho\left((z_n(x))_{n\in\mathcal{N}}\right) \;\middle|\; \begin{array}{l} x \in \mathcal{X} \\ x_{j(n)}^s + x_n^{pe} + x_{j(n)}^c \geq D_n^e,\ n \in \mathcal{N} \end{array} \right\}.$$

Inserting (1) into (2) leads to an expectation based stochastic program with additional variables y (i.e., an additional component) and additional (coupling) constraints:

$$(3) \qquad \min \left\{ \sum_{t'=1}^{T'} \mathbb{E}\left[(y_l c_{t'})_{l\in\mathcal{N}_{t'}^{\text{risk}}}\right] \;\middle|\; \begin{array}{l} x \in \mathcal{X},\ y \in \mathcal{Y} \\ x_{j(n)}^s + x_n^{pe} + x_{j(n)}^c \geq D_n^e,\ n \in \mathcal{N} \\ z_{n(l)}(x) = \sum_{k\in\text{path}^{\text{risk}}(l)} w_{t'(l),t'(l)-t'(k)} y_k,\ l \in \mathcal{N}^{\text{risk}} \end{array} \right\}$$

with $\mathcal{Y} = \{(y_l)_{l\in\mathcal{N}^{\text{risk}}} | y_l \in Y_{t'(l)}\ \forall\, l \in \mathcal{N}^{\text{risk}}\}$ denoting the feasible variables originating from the risk measure definition.

4. Effect of Risk Measures

4.1. **Cash value curves.** Optimizing without risk in the objective or with $CVaR$ applied to the value at the last time step only, leads to high spreading and to very low intermediate values for a considerably high number of scenarios. This may lead to serious liquidity problems. The usage of a multiperiod risk measure that takes intermediate time steps into account corrects both, spreading and negativity of values. The way how this is achieved, however, differs among the risk measures. Some risk measures tend to make the curves run closer together. Other risk measures in the objective try to find an maximal level such that cash values do not fall below that level at any time with high probability (cf. Fig. 2). Further, the VPI based risk measure according to [11] tends to reduce the uncertainty of two consecutive timesteps.

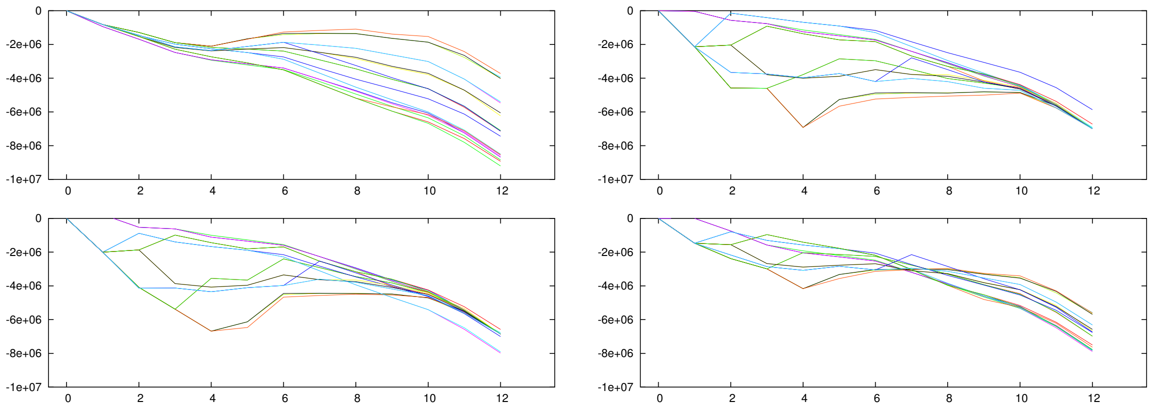

FIGURE 2. Cash value curves over time (one year) when optimization was carried out with expectation (top left) and Conditional-Value-at-Risk (top right) of the final value and with two different multiperiod risk measures (bottom) taking intermediate values into account.

4.2. **Decomposition structure.** Problem (2) is almost decomposable with respect to the portfolio components. The demand satisfaction restriction however couples the components additively. Moreover, since the risk measure ρ is not linear, there is further coupling induced by the cash values. Due to the special form (1) of ρ the latter coupling is transformed into another coupling constraint in (3). Hence, *component decomposition*, a dual decomposition technique based on Lagrangian relaxation of these coupling constraints (cf. [13, Chapter 3] and [9]), can be applied similarly to the case when ρ is linear, e.g. $\rho(z) = \mathbb{E}[-z_T]$. Namely, to relax these coupling constraints, one has to penalize their violation by introducing a suitable set of Lagrangian multipliers, thus, the minimization with respect to the remaining constraints can be carried out componentwise. Finally, one has to maximize over these multipliers in order to get (good) lower bounds for (3). However, it is shown in [4] that the latter maximization has to consider linear equality constraints (beside box constraints), which is not the case for the traditional situation $\rho(z) = \mathbb{E}[-z_T]$.

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