Open Problem Session Report

Dagstuhl Workshop on Graph Drawing
May 10, 2005

This is a report on an informal session intended to stimulate communication and sharing of problems. Hence the attributions and citations may contain inaccuracies, and are certainly not complete. [1]

1. **Stephen Kobourov** Simultaneous Tree Embedding [2]
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   It is known that pairs of paths, caterpillars, and other classes of graphs have simultaneous geometric embeddings while pairs of outerplanar graphs do not (see reference below). Does there exist a simultaneous geometric embedding for pairs of trees?

   P. Brass, E. Cenek, C.A. Duncan, A. Efrat, C. Erten, D. Ismailescu, S. G. Kobourov, A. Lubiw, and J. S. B. Mitchell, "On Simultaneous Graph Embedding", *in* 8th Workshop on Algorithms and Data Structures, 2003, pp. 243-255.

2. **Cesim Erten** Simultaneous Planar Graph Embedding
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   Given any pair of planar graphs $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$ such that $|V_1| = |V_2|$, can we always find a plane, straight-line drawing of G_1, G_2 on the same point set? Note that it's implicit in the problem definition that:

   - We do not care if there is crossing between an edge of G_1 and an edge of G_2; however, we do not want any crossings between edges of G_1 (or G_2);

   - There is no mapping between vertex sets V_1, V_2, i.e. the graphs are not labeled;

   - We can pick different point sets for different input pairs.

   Related Work:

   - Any number of outer-planar graphs $G_1, \ldots, G_r$ can be embedded with the required conditions. [Bose, CGTA'02]

   - Any number of outer-planar graphs $G_1, \ldots, G_r$ and one planar graph G, can be embedded with the required conditions on an $O(n^2) * O(n^2)$ grid. [Brass et al., WADS'03]

3. **Emden Gansner** Contact Graphs of Convex Shapes
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   The basic problem statement for contact graphs is:

   Draw a (directed) (tree) G using (closed) (rectangles) for nodes such that $A \to B$ is an edge in G (if and) only if the rectangles R_A and R_B (touch) in the rectangular layout, with R_B below or to the right of R_A, all within (width) W.

   All words and phrases in parentheses are variables, which can be deleted or replaced by other values. Thus, one could replace "directed tree" with "undirected graph". "Rectangle" can be replaced with any convex shape. The width condition comes from the motivation for the original problem: display the tree as rectangles in a GUI. Thus, one would like the layout to use as much of the given width as possible but no more. Width could be replaced by area or aspect ratio. To touch, the boundaries of the objects must share points, but the meaning of touch might or might not allow a single point.

   Once the variable phrases are fixed, there are 3 obvious types of questions:

   - Characterize graphs with such layouts

   - Construct such layouts efficiently (preferably "good" layouts)

   - Can anything be said about the growth rate of the width or area?

   For general graphs, characterizations are known for circles and rectangles. Complexity results are only known for rectangles. Basically nothing is known about the directed case.

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[2] Later at the workshop, an example demonstrating a negative answer was given by Michael Kaufmann. Thus, the problem becomes a complexity issue.

Related problems:

- contact graphs for curves, line segments, etc.

- intersection graphs

- rectangle of influence drawings (Biedl, Kant, Kaufmann, Algorithmica, 19 (1997))

- rectangular duals (graphs whose duals are partitions of rectangles into rectangles) (Gant and He, Theoret. Comp. Sci. 172 (1997))

There is an extensive literature on these problems. And the last two are closely related to rectangular layouts.

4. **Franz Brandenburg** Degree restricted one-sided crossing minimization
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   Given: a bipartite graph with one layer fixed and degree 3 for the vertices on the other layer.

   Question: How hard is crossing minimization? Recall that for degree 2 it is easy, and for degree 4 it is NP hard. See: Munos, Unger, Vrto at GD2001

5. **Franz Brandenburg** Drawings with Uniform Link Length
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   Which graphs can be drawn in the plane with all edges of the same length? In addition, with the restriction to planarity? As a generalization: with the edge length between two bounds, say $1 < l(e) < 2$ for each edge e.

6. **Michael Kaufmann** Comparing Trees via Crossing Minimization
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   It is well-known that the 2-layer crossing minimization problem as well as the maximum planar subgraph problem are NP-hard, even for the case that the vertices in 1-layer are fixed. For the case that the possible set of permutations of the vertices is restricted by underlying tree structures and corresponding leaves are connected by matching edges, we were able that for the arbitrary case, the crossing minimization problem is NP-hard, while it is polynomially solvable by dynamic programming for the 1-layer fixed case. For the maximum planar subgraph problem (1- and 2 layer version), the complexity status is open, which is somehow counterintuitive, since crossing minimization is usually considered to be the harder problem.

   Related work has been done by Tim Dwyer and Falk Schreiber, as well as Henning Fernau, Michael Kaufmann, Mathias Poths.

7. **Martin Kutz** Planar Reachability Substitutes
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   Setting: Let $G = (V, E)$ be a directed graph. A *reachability substitute* (RS) for G is another digraph $R = (V^{'}, E^{'})$ such that $V \subseteq V^{'}$ and for any two nodes $u, v \in V$, there is a path from u to v in R iff there is a path from u to v in G. Of course, G itself is an RS for G.

   Problems:

   - Classify all graphs that have *planar* reachability substitutes (i.e., R must be planar).

   - Show an upper bound (as a function of $|V|$) on the size of a minimum planar RS for all such graphs.

   - What is the complexity of deciding whether a given graph has a planar RS?

   - How can one find a (minimal) planar RS for a given graph?

   Background: We know that there are graphs that do not have a planar RS. So there actually is something to be classified here. Also we known that a graph that has a planar RS, does always have a very compact non-planar RS, namely of size $O(|V| \log^2 |V|)$. This is shown in [Katriel, Kutz, Skutella: "Reachability Substitutes for Planar Digraphs", Manuscript, 2005, http:www.mpi-sb.mpg.de mkutz/publications.html].

8. **James Abello** Internal Visibility Graph Recognition Problem for Simple Polygons
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   A fascinating open problem (posed in the mid 1980's, e.g., by David Avis) is the following: given a graph $G = (V, E)$, together with a hamiltonian cycle C through all its vertices, can the graph be represented as the internal diagonals of a simple polygon with boundary cycle C? The problem is known to be decidable (Hazel Everett).