Representing preferences in the possibilistic setting

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Abstract. The accurate and easy representation of users' preferences in information engineering systems becomes an important issue. Possibility theory provides a generic framework for the qualitative representation of preferences, where several equivalent information formats co- exist (distribution, logical bases, conditionals, graphical networks). Moreover, a bipolar representation distinguishing between positive and negative preferences has been developed in this setting. The paper offers a comprehensive survey of these representation issues.

Keywords. possibility, preference, possibilistic logic.

1 Introduction

Possibility theory has been initially introduced as a framework for representing partial ignorance and uncertainty [17,19,11]. However, it has rapidly appeared that it also allows us to represent preferences [18,12]. This paper considers possibility theory from this latter point of view.

Generally speaking, it amounts to expressing flexible constraints, that restrict the possible values of parameters that we can adjust. Thus, in a scheduling problem for example, one may have preferences on the starting date of a task, which should be fixed (whereas on the other hand its exact duration may be pervaded with uncertainty that cannot be controlled). Flexible queries [5] to a database provide another example of preference representation, such as, for instance, the user looking for "a house to let near the sea, at an affordable price". Preferences are not only expressed, directly on values of attributes as in the previous example, but also relatively between possible classes of choices, such as "I prefer a house near the sea to a house in the country". Such partial specifications of preferences are in general to be interpreted as ceteris paribus, i.e. every thing else being equal [6]. Then, the problem is to build a global preference relation from such specifications.

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Another important aspect of preference expression is often their "bipolar" character, in the sense where an agent which expresses its preferences, often states on the one hand what it does not want, what it dislikes, and on the other hand what it would like, what is particularly satisfactory for it, without there necessarily exists a complementarity between what it rejects and what it wishes.

This paper proposes a survey of the existing tools in possibility theory and possibilistic logic for preference representation. It is partly based on results of previous papers [2,1,3]. Section 2 shows how a possibility distribution, which is the basic notion in this framework, can be interpreted in two distinct ways, according to whether one stresses on what is impossible, or on what is satisfactory. The handling of the levels of importance of requirements when expressing preferences is also addressed. Section 3 discusses the modeling of conditional preferences, while section 4 is interested in the bipolarity of preferences. Section 5 briefly points outlines for further research in the possibilistic framework in order to allow the representation of more sophisticated preferences.

2 Two logical readings of a possibility distribution

A possibility distribution π [19] is a mapping from a referential U in a totally ordered scale, such as the real interval $[0, 1]$, or a finite scale $E = \{\alpha^1 = 0, \alpha^2, \cdots, \alpha^{n-1}, \alpha^n = 1\}$, where we also denote the smallest and the largest elements by 0 and 1 respectively, with $0 < \alpha^2 < \cdots < \alpha^{n-1} < 1$ and $n \geq 3$. A possibility distribution is interpreted as follows: $\pi(u) = 0$ means that the value or the state of the world u is completely impossible, while $\pi(u) = 1$ expresses that u is completely satisfactory. The more $\pi(u)$ is close to 0, the less u is possible, or in other words, the more u is impossible, the more it is rejected. Also, the more $\pi(u)$ is close to 1, the less u is impossible, or the more u is possible, the more it is satisfactory.
We denote by C^i the strict cuts of level α^i of π, i.e. $C^i = \{u : \pi(u) > \alpha^i\}$ for $i = 1, \cdots, n-1$.
A possibility distribution can be represented either by a conjunction of prioritized goals, or as a disjunction of classes of situations with a uniform level of satisfaction.

2.1 Prioritized goals

A possibility measure $\Pi(C) = max\{\pi(u) : u \in C\}$ and the dual necessity $N(C) = 1 - \Pi(\neg C)$ are associated to a possibility distribution π defined on E. It can be checked that

$$N(C^i) \geq 1 - \alpha^i,$$

since $\pi(u) \leq \alpha^i$, $\forall u \in \neg C^i$, where by definition we have $1 - \alpha^i = \alpha^{n-i+1}$. In particular $N(C^1) = 1$. This means that all values outside the support C^1 of π are impossible ($\Pi(\neg C^1) = 0$), and choosing u outside C^i is only possible to a degree at most equal to α^i (indeed, $\Pi(\neg C^i) \leq \alpha^i$). In other words, it is imperative to a degree $1-\alpha^i$ to choose u in C^i. The distribution defined on E is then equivalent to a set of pairs $\{(C^i, 1-\alpha^i) : i = 1, \cdots, n-1\}$ which can be interpreted as a set of goals C^i with their priority $1-\alpha^i$ (i.e., $N(C^i) \geq 1-\alpha^i$). To each pair $(C^i, 1-\alpha^i)$, is associated the elementary possibility distribution defined by

$$\forall u \in U, \pi_{(C^i,1-\alpha^i)}(u) = \begin{cases} 1 & \text{if } u \in C^i \\ \alpha^i & \text{otherwise} \end{cases}$$

which indeed expresses that the values outside C^i are only possible to a degree α^i and the possibility distribution π associated with N can be obtained from the conjunctive combination of these elementary possibility distributions

$$\pi(u) = min\{\pi_{(C^i,1-\alpha^i)}(u) : (C^i, 1-\alpha^i), i = 1, \cdots, n-1\}. \tag{1}$$

In the above construction, the sets C^i form a nested sequence: $C^1 \supseteq C^2 \supseteq \cdots \supseteq C^{n-1}$. This construction can be generalized, in a similar way, for a collection $\mathcal{B} = \{(B_j, \beta_j) : j = 1, \cdots, m\}$ of general sets $B_1, B_j, \cdots, B_m \subset U$, associated to priorities β_j, obeying constraints of the form $N(B_j) \geq \beta_j$. This is the basis of possibilistic logic [10]. The necessity measure N is then associated to the following possibility distribution:

$$\forall u \in U, \pi_{\mathcal{B}}(u) = \begin{cases} 1 & \text{if } u \in B_1 \cap \cdots \cap B_m \\ min\{1-\beta_j : (B_j, \beta_j) \in \mathcal{B} \text{ and } u \in \neg B_j\} & \text{otherwise.} \end{cases}$$

Let us consider a flexible query involving two distinct attributes defined respectively on referentials U_1 and U_2 and represented by the possibility distributions π_1 and π_2. Suppose that the attributes of the query are non-interactive, i.e. if the query is the conjunction of two restrictions represented respectively by the distributions π_1 and π_2 on the referentials of these attributes, then this query will be represented by

$$\pi(u) = min(\pi_1(u_1), \pi_2(u_2)) \text{ with } u = (u_1, u_2).$$

Observe here that the non-interactivity expresses ceteris paribus preferences, since the preference relation between $u = (u_1, u_2)$ and $u' = (u'_1, u_2)$ is the same, in *the broad sense*, as the one between $u'' = (u_1, u'_2)$ and $u''' = (u'_1, u'_2)$.

Example 1. Suppose that 'near' (the sea) is represented for a given agent by $\pi_1(u_1) = 1$ if $u_1 \leq 5$; $\pi_1(u_1) = .7$ if $5 < u_1 \leq 10$; $\pi_1(u_1) = .2$ if $10 < u_1 \leq 15$; $\pi_1(u_1) = 0$ if $u_1 > 15$,
while 'affordable' (price) is represented by $\pi_2(u_2) = 1$ if $u_2 \leq 200$; $\pi_2(u_2) = .5$ if $200 < u_2 \leq 400$; $\pi_2(u_2) = 0$ if $u_2 > 400$.
The joint possibility distribution $\pi = min(\pi_1, \pi_2)$ is associated to the following possibilistic formulas base $\mathcal{B} = \{(d \leq 15, 1), (d \leq 10, .8), (d \leq 5, .3), (p \leq 400, 1), (p \leq 200, .5)\}$.
The possibility distribution corresponding to the disjunction 'near the sea' or 'affordable price' $\pi = max(\pi_1, \pi_2)$ is associated to the base $\mathcal{B}' = \{(d \leq 15 \vee p \leq 400, 1), (d \leq 10 \vee p \leq 400, .8), (p \leq 10 \vee d \leq 200, .5), (d \leq 5 \vee p \leq 200, .3)\}$.

The possibilistic base $\mathcal{B}'' = \{(d \leq 15, 1), (d \leq 10 \vee p \leq 200, 1), (d \leq 5, .5), (p \leq 400, 1)\}$ gives an example of interactive preferences, since it is associated to a non-decomposable possibility distribution $\pi(u_1, u_2) = 1$ if $d \leq 5$ and $p \leq 400$; $\pi(u_1, u_2) = .5$ if $5 < d \leq 10$ and $p \leq 400$; $\pi(u_1, u_2) = 0$ if $d > 15$ or if $p > 400$ or if $10 < d \leq 15$ and $200 < p \leq 400$.

2.2 Situations having a guaranteed satisfaction level

A guaranteed possibility measure [12] $\Delta(C) = min\{\pi(u) : u \in C\}$ is also associated to the possibility distribution π defined on E. It is easy to check that

$$\Delta(C^{i-1}) \geq \alpha^i$$

since $\pi(u) \geq \alpha^i > \alpha^{i-1}$, $\forall u \in C^{i-1}$ for $i = 2, \cdots, n$. In particular $\Delta(C^{n-1}) = 1$. This means that choosing u in C^{i-1} guarantees a satisfaction degree at least equal to α^i. The possibility distribution π defined on E is then also equivalent to a set of pairs denoted by $\{[C^{i-1}, \alpha^i] : i = 2, \cdots, n\}$ which can be interpreted as a set of situations C^{i-1} with their guaranteed satisfaction degrees α^i respectively. To each pair $[C^{i-1}, \alpha^i]$ is associated an elementary possibility distribution defined by

$$\forall u \in U, \pi_{[C^{i-1},\alpha^i]}(u) = \begin{cases} \alpha^i & \text{if } u \in C^{i-1} \\ 0 & \text{otherwise} \end{cases}$$

which indeed expresses that the values in C^{i-1} are possible at least to a degree α^i, and the possibility distribution π on which Δ is based, can be obtained by the disjunctive combination of these elementary possibility distributions

$$\pi(u) = max\{\pi_{[C^{i-1},\alpha^i]}(u) : i = 2, \cdots, n\}. \tag{2}$$

In the above construction, the sets C^i form a nested sequence: $C^1 \supseteq C^2 \supseteq \cdots \supseteq C^{n-1}$. As for (1), the construction is generalized in the same way for a collection $\mathcal{D}$ of general sets $D_1, D_k, \cdots, D_r \subset U$, associated to priorities δ_j, obeying constraints of the form $\Delta(D_k) \geq \delta_k$ [3]. The guaranteed possibility measure Δ is then associated to the following possibility distribution

$$\forall u \in U, \pi_{\mathcal{D}}(u) = \begin{cases} max\{\delta_k : [D_k, \delta_k] \in \mathcal{D} \text{ and } u \in D_k\} & \text{if } u \in D_1 \cup \cdots \cup D_r \\ 0 & \text{otherwise} \end{cases}$$

Example 1 (continued)
Suppose that 'near the sea' and 'affordable' are still represented as in Example 1 i.e., $\pi_1(u_1) = 1$ *if* $u_1 \leq 5$; $\pi_1(u_1) = .7$ *if* $5 < u_1 \leq 10$; $\pi_1(u_1) = .2$ *if* $10 < u_1 \leq 15$; $\pi_1(u_1) = 0$ *if* $u_1 > 15$,
and $\pi_2(u_2) = 1$ *if* $u_2 \leq 200$; $\pi_2(u_2) = .5$ *if* $200 < u_2 \leq 400$; $\pi_2(u_2) = 0$ *if* $u_2 > 400$ *respectively.*
The joint possibility distribution $\pi = min(\pi_1, \pi_2)$ *is then associated to the following base of* Δ*-possibilistic formulas* $\mathcal{D} = \{[d \leq 5 \wedge p \leq 200, 1], [5 < d \leq 10 \wedge p \leq 200, .7], [d \leq 10 \wedge 200 < p \leq 400, .5], [10 < d \leq 15 \wedge p \leq 400, .2]\}$.

The possibility distribution associated to the disjunction 'near the sea' or 'affordable' $\pi = max(\pi_1, \pi_2)$ *is associated to the base* $\mathcal{B}' = \{[d \leq 5, 1], [d \leq 10, .7], [d \leq 15, .2], [p \leq 400, .5], [p \leq 200, 1]\}$.
The non-decomposable possibility distribution considered above: $\pi(u_1, u_2) = 1$ *if* $d \leq 5$ *and* $p \leq 400$; $\pi(\pi_1, \pi_2) = .5$ *if* $5 < d \leq 15$ *and* $p \leq 200$; $\pi(u_1, u_2) = 0$ *if* $d > 15$ *or if* $p > 400$ *or if* $10 < d \leq 15$ *and* $200 < p \leq 400$, *is associated to the base* $\mathcal{D}'' = \{[d \leq 5 \wedge p \leq 400, 1], [5 < d \leq 10 \wedge p \leq 400, .5], [10 < d \leq 15 \wedge p \leq 200, .5]\}$.

2.3 Importance levels and thresholds

In a conjunction of two constraints, there is at least two qualitative ways to express that the restriction on the attribute 2 is "less important" than the one on the attribute 1. They are respectively defined by

$$\pi(u) = min(\pi_1(u_1), max(1 - \lambda, \pi_2(u_2))) \tag{3}$$

$$\pi(u) = min(\pi_1(u_1), \lambda \rightarrow_G \pi_2(u_2))) \tag{4}$$

with $\lambda \rightarrow_G \rho = 1$ if $\lambda \leq \rho$ and $\lambda \rightarrow_G \rho = \rho$ otherwise.

The first formula expresses that even if u_2 is totally unsatisfactory (i.e., $\pi_2(u_2) = 0$), the effect on the global evaluation will be limited and upper-bounded it by $1-\lambda$. On the contrary, the second formula says that we are indeed totally satisfied about the second attribute as soon as we reach a satisfaction level λ in the sense of π_2. We can check that the first way consists of upper-bounding by λ the priority of the goals associated to π_2 (i.e., if π_2 is associated with $\{(C_2^i, 1 - \alpha_2^i) : i = 1, \cdots, n-1\}$ then $max(1 - \lambda, \pi_2)$ is associated with $\{(C_2^i, min(\lambda, 1-\alpha_2^i) : i = 1, \cdots, n-1\})$. This is equivalent to ignore all the goals with priority greater than λ, since the C_2^i are nested. The second (different) way consists of ignoring all the goals with priority smaller than $1 - \lambda$.
In a disjunction of constraints, there are also two qualitative ways for expressing that the satisfaction on the attribute 2 is more difficult to guarantee than the one on the attribute 1. They are defined by

$$\pi(u) = max(\pi_1(u_1), min(\lambda, \pi_2(u_2))) \tag{5}$$

$$\pi(u) = max(\pi_1(u_1), (1 - \lambda)\&\pi_2(u_2)) \tag{6}$$

with $\lambda\&\rho = 0$ if $1 - \lambda \geq \rho$ and $\lambda\&\rho = \rho$ otherwise.

The first formula expresses that even if u_2 is totally satisfactory (i.e., $\pi_2(u_2) = 1$), the effect on the global evaluation on the satisfaction will be upper-bounded by λ. On the contrary, the second formula says that we are indeed totally unsatisfactory about the second attribute as soon as we are below a satisfaction level $1 - \lambda$ in the sense of π_2. We can check that the first way consists of upper-bounding by λ the satisfaction level associated with π_2 (i.e., if π_2 is associated with $\{[C_2^{i-1}, \alpha_2^i] : i = 2, \cdots, n\}$ then $min(\lambda, \pi_2)$ is associated with $\{[C_2^{i-1}, min(\lambda, \alpha_2^i)] : i = 2, \cdots, n\}$). This is equivalent to ignore all situations with satisfaction level greater than λ since the C_2^i are nested. The second (different) way consists of ignoring all situations with satisfaction level smaller than $1 - \lambda$.

3 Conditional preferences

Preferences are also often expressed in a relative way. Let us consider the following simple example: "I prefer to take a tea (t). If there is no tea then I will take a coffee (c)".
This can be interpreted by the two following possibilistic constraints:

$$\Pi(t) > \Pi(\neg t),$$

$$\Pi(\neg t \wedge c) > \Pi(\neg t \wedge \neg c),$$

which express on the one hand that there are situations where there is tea which are more satisfactory than any situations where there is no tea, and on the other hand, if there is no tea then it is more satisfactory to have coffee rather than nothing. In the general case, there exists a unique possibility distribution with a minimal specificity which satisfies a set of consistent constraints such as the above ones.

Thus these two constraints induce the following possibility distribution:
$\pi(ct) = 1$, $\pi(\neg ct) = 1$, $\pi(c\neg t) = \alpha$, $\pi(\neg c\neg t) = \beta$ with $\alpha > \beta$.
This distribution describes the fact that totally satisfactory worlds are those where there is tea. Those where there is only coffee are less satisfactory, although being preferred to those where there is neither tea nor coffee. This distribution is also associated to the following $N-$ and $\Delta-$type possibilistic bases:
$\mathcal{B} = \{(c \vee t, 1 - \beta), (t, 1 - \alpha)\}$ which expresses that "having tea or coffee" is the goal with the highest priority, while the goal "having tea" has a smaller priority, and $\mathcal{D} = \{[t, 1], [c \wedge \neg t, \alpha]\}$ which expresses that tea is fully satisfactory, and that having coffee instead is less satisfactory.

General procedures exist allowing us to go from a representation format to another (possibility distribution, conditional possibilistic constraints, possibilistic logic bases of the form $N(p) \geq \alpha$ or of the form $\Delta(p) \geq \delta$). There is also another representation format which is graphical and given by a possibilistic Bayesian network, where the graph expresses conditional non-interactivities between variables, and where the arrows are associated to conditional possibilities. This graphical representation format is also related to the previous representation formats by translation procedures without any loss of information. See [2,1,8] for more details.

Note that a constraint of the form

$$\Pi(p \wedge q) > \Pi(p \wedge \neg q) \tag{7}$$

which expresses that in the context where p is true, it is preferred to satisfy q than to falsify it, is obviously weaker than a ceteris paribus preference of q over $\neg q$ which would require also that

$$\Pi(\neg p \wedge q) > \Pi(\neg p \wedge \neg q). \tag{8}$$

However

$$\Pi(q) > \Pi(\neg q) \tag{9}$$

which is equivalent to $max(\Pi(p \wedge q), \Pi(\neg p \wedge q)) > max(\Pi(p \wedge \neg q), \Pi(\neg p \wedge \neg q))$ which is then equivalent to $max(\Pi(p \wedge q), \Pi(\neg p \wedge q)) > \Pi(p \wedge \neg q)$ and $max(\Pi(p \wedge q), \Pi(\neg p \wedge q)) > \Pi(\neg p \wedge \neg q)$ which are different from (7) and (8) respectively. Indeed (9) only requires the existence of one world where q is true which is preferred to all worlds where q is false.

The constraints of the form

$$\Delta(p \wedge q) > \Delta(p \wedge \neg q)$$

are different from those of type (7) since they focus, in the context p, on the less satisfactory models of q and $\neg q$. They correspond to a very cautious attitude. These constraints have been studied recently in [8]. The mixed constraints $\Pi(p \wedge q) > \Delta(p \wedge \neg q)$ and $\Delta(p \wedge q) > \Pi(p \wedge \neg q)$ have been discussed in [4] and [16].

Example 2. Assume we want to express preferences among candidates on the basis of their levels in science (S) and literature (L). For S and L, we use three levels: good (G), average (A) and Insufficient (I). Having level X in science and literature is respectively denoted by X_S and X_L. The requirements are the following ones:

- $\Pi(I_S) = 0$ (the level in literature does not matter if the candidate is insufficient in science).
- $\Pi(\neg I_S \wedge G_L) > \Pi(\neg I_S \wedge \neg G_L)$ (*)
  (if the candidate is average or good in science, a candidate good in literature is preferred).
- $\Pi(G_S \wedge A_L) > \Pi(G_S \wedge I_L)$ and $\Pi(A_S \wedge A_L) > \Pi(A_S \wedge I_L)$ (Partial enforcement of Pareto ordering).

This leads to the following ranking

$$G_S G_L \approx A_S G_L \succ G_S A_L \approx A_S A_L \succ G_S I_L \approx A_S I_L \succ I_S G_L \approx I_S A_L \approx I_S I_L.$$

Now assume we add the direct constraint on the ordering that we should have $G_S A_L \succ A_S G_L$.
Enforcing this new constraint will remain compatible with (*) (since the set of preferred interpretations of (*) remains not empty i.e., it is now $\{G_S G_L\}$) as it can be checked and then the revised ordering can be computed.
$G_S G_L \succ G_S A_L \succ A_S G_L \succ A_S A_L \succ G_S I_L \approx A_S I_L \succ I_S G_L \approx I_S A_L \approx I_S I_L$.

4 Bipolar preferences

The distinction between negative information and positive information applies in particular to preference representation, and this has been supported by psychologists since a long time. Positive information refers to what is desired while negative information refers to what is rejected. Indeed, we can describe desired, pursued alternatives on one hand and unsatisfactory, rejected alternatives on the other hand. In multiple-criteria decision framework, we can thus use a bipolar scale (or a Cartesian product of scales) to allow positive evaluations and negative evaluations. The joint handling of negative and positive evaluations, and in particular their aggregation (see for example [14]) may be problematic insofar it drops the difference of nature of the evaluations.

The possibilistic representation framework presented above can be easily "bipolarized" by means of a *pair* of possibility distributions (π_*, π^*), where π_* represents the fuzzy set of guaranteed possible elements or values, while $\pi^*(u)$ evaluates the non-impossible, i.e. acceptable character of u, and $1 - \pi^*(u)$ evaluates the extent to which u is impossible, rejected as inacceptable.

A (strong) coherence condition requires the pair (π_*, π^*) satisfy

$$\forall u, min(\pi_*(u), 1 - \pi^*(u)) = 0, \tag{10}$$

i.e. if u is somewhat guaranteed possible, i.e. $\pi_*(u) > 0$, u should be completely non-impossible, i.e. $\pi^*(u) = 1$. This implies the following weaker minimal coherence condition

$$\forall u, \pi_*(u) \leq \pi^*(u). \tag{11}$$

Clearly, this representation by pair of distributions (π_*, π^*) is naturally associated to the different compact representations described in the previous sections, in particular in the form of pairs of bases of types N and Δ, namely $\mathcal{B} = \{(p_i, \alpha_i) : i = 1, \cdots, n\}$ and $\mathcal{D} = \{[p_j, \gamma_j] : j = 1, \cdots, m\}$ corresponding semantically to the possibility distributions π^* and π_* respectively. Indeed it is intuitively clear that the guaranteed possibility measure is appropriate for expressing positive preferences, while the impossibility measure $1 - \Pi(.) = N(\neg\ .)$ allows us to express rejections of what is unsatisfactory, inacceptable.

An application of this idea to flexible queries on a database has been already proposed [13], where a distinction is given between constraints (possibly flexible), whose violation has a negative effect on the evaluation, and wishes to satisfy if possible, whose satisfaction has a positive effect (but the non satisfaction has no impact on the evaluation). These ideas translated in a logical framework allow to address a symbolic optimization problem, where negative preferences play the role of constraints and positive preferences play the role of criteria. Then we can look for a logical description of the set of preferred solutions [15].

5 Conclusion and perspectives

The possibility theory framework appears to be rich in representation formats, allowing us to express many aspects of an agent's preferences in a qualitative way. Each format may appear more or less simple or appropriate, according to the preferences we want to express and what we want to highlight.

Another powerful representation format of preferences is "CP-nets" and "T-CP-nets" [6]. Currently, we are working on a detailed comparison between the two frameworks [9]. Extensions of the possibilistic framework presented in this paper should allow to represent orderings of priorities which depend on the context, or to represent such preferences as those considered in [7], such as "if it is the same thing, I prefer the cheapest one".

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