Partial Matching by Structural Descriptors

Simone Marini, Silvia Biasotti, Bianca Falcidieno

stituto di Matematica Applicata e Tecnologie Informatiche
Consiglio Nazionale delle Ricerche
16149, Genova, Via De Marini 6, Italy
{simone,silvia, bianca}@ge.imati.cnr.it

Abstract. The extended abstract describes a method for recognizing similar sub-parts of objects described by 3D polygonal meshes. The innovation of this method is the coupling of structure and geometry in the matching process. First of all, the structure of the shape is coded in a graph where each node is associated to a sub-part of the shape. Then, the matching between two shapes is approached using a graph-matching technique relying upon a geometric description of each sub-part.

Keywords. Partial Matching, 3D Structural Shape Descriptor, Graph Matching

1 Introduction

Assessing the similarity among 3D shapes is a very complex and challenging research topic. Whilst there are already techniques for rapidly extracting knowledge from massive volumes of texts, there is an increasing demand for tools supporting the automatic search for 3D objects and their subparts in digital archives.

There is a growing consensus that shapes are recognized and coded mentally in terms of relevant parts, or features, and their spatial configuration, or structure. Methods approaching the problem from a geometric point of view do not take into account the structure of the shape and generally the similarity distance between two objects depends on their spatial embedding.

The work herein presented is based on results of differential topology, which deals with the description of shape by means of shape properties of one, or more, real-valued functions defined over the shape. Studying these properties, topological descriptions of the shape can be defined, namely the Reeb graphs, which induce a decomposition of the shape into significant regions. Such a decomposition defines a structural description of the shape, which is coupled with an error-correcting subgraph isomorphism to provide a system for shape similarity analysis. Moreover, the proposed framework makes it possible to plug in heuristics for tuning the matching algorithm to the specific application, in particular the 3D shape sub-parts correspondence and the partial matching.

Aim of this extended abstract is to describe a method for recognizing the sub-parts of two objects the most similar both in geometry and structure. Main

Dagstuhl Seminar Proceedings 06171
Content-Based Retrieval
http://drops.dagstuhl.de/opus/volltexte/2006/651

innovation of the method is the coupling of a structural descriptor, like the Reeb graph proposed in [1], with a geometric descriptor and the inexact graph-matching techniques [2,3,4,5].

Since the structural descriptor can be always coded as a directed graph, the partial matching can be solved by finding the common sub-graphs of the two objects. This is achieved through the construction of a common sub-graph (maybe not connected) between the two input graphs, that highlights where two shapes are similar or dissimilar. In figure 1, an example of partial matching between two mechanical parts, explains the overall approach of the matching process. The light-blue parts represent the sub-parts with similar geometry and structure, while the grey ones denote the sub-parts whose shape significantly differs. The reminder of the paper is organized as follows. First, a brief overview

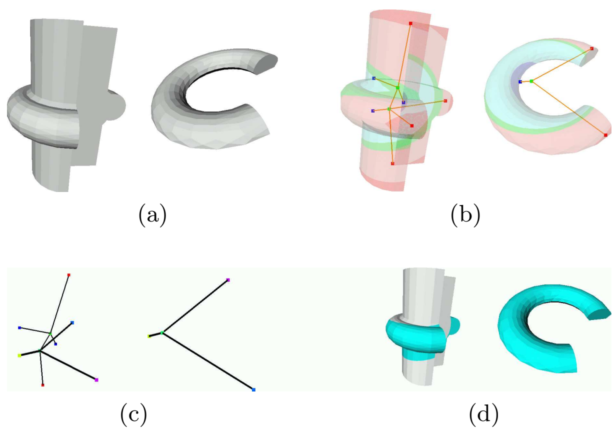


Fig. 1. Partial matching between two mechanical parts (a). The structural descriptor is extracted from the two objects (b); The two structural descriptors are compared through a graph matching technique (c); The partial matching obtained by comparing the two descriptors is represented on the two model objects.

on existing techniques for shape retrieval and partial matching is given. Then, the description of our approach is proposed; since solving the complete graph matching is computationally expensive, a new heuristic method which speeds up the process is proposed. Finally, results are presented and discussed. Conclusive remarks and suggestions on future work end the paper.

2 Previous work

Concerning 3D shapes, there is a great number of techniques for shape matching. Many methods for 3D object comparison return as output a positive real number which measures how much an object resembles to another one [6,7,8,9,10]. Since no information on the sub-part correspondence of the compared objects is stored, these approaches are not available for partial matching and mapping of objects. On the contrary, such an information is essential in applications like object modeling, registration and recognition.

The use of spin-images [11] for classifying sub-parts of 3D objects has been recently proposed in [12] and [13]. Both methods represent the 3D objects as a set of parts, which are compared to obtain an object sub-part mapping. Other approaches like those described in [14,15] try to match global shapes with a special emphasis on selected sub-parts. For example, in [14] the surface of an object is described by segmenting it into patches; the complete surface description separately represents each patch and their interrelationships. Complex surfaces are segmented into simpler meaningful components (the patches) through shape discontinuities, such as jump boundaries, limbs and creases. Therefore, such a description can be viewed as an attributed graph whose nodes correspond to the surface patches and the edges codify the relations between them.

Finally, the methods proposed in [16] represent the shape object as binary tree obtained by recursively subdividing the object into two parts. The similarity measure between two objects is obtained by matching the two trees, where the sub-part correspondence is induced by the node mapping provided by the matching algorithm.

All these methods provide only one and arbitrary description of the object shape, while the framework proposed in this paper is based on the consolidated Reeb graph theory that allows the use of different functions to analyze the shape, each one able to identify different relevant sub-parts of the object. In particular, methods based on the spin-images provide a point-to-point correspondence between the object surfaces and do not store any kind of structural and topological information. On the contrary, the structural information represented in [14,16,15] either does not guarantee the identification of the most meaningful sub-parts or is not able to modularly incorporate different heuristics capable of adapting the matching to specific application contexts (e.g. global matching, sub-part correspondence, partial matching) or is not fully automatic [15].

The use of structural descriptions for shape similarity has been firstly addressed in [17], where the Reeb graph is proposed in a multi-resolution fashion to build a graph and perform shape similarity by means of graph matching techniques. Similar criteria have been successively used in [18] and further enriched by [19], where for each slice the geometric attributes considered are the volume, a statistic measure of the extent and the orientation of the triangles, an estimation of the Koenderink shape index and a statistic of the texture. Other geometric descriptors have been proposed for associating to the nodes of a skeletal graph the description of the related model sub-parts. In particular, methods based on the medial axis [20] like [21,22,23] code in a vector the *relevance* of the skeletal edges incident in a node (e.g. edge length, diameters and average circumference of the skeleton loops) or use geometric descriptors, like the mean curvature histogram [22].

3 Sub-Part Correspondence

In our idea, the partial matching problem may be grouped in three main issues [4,5]:

1. recognizing similar sub-parts in objects that are both structurally and geometrically similar (that is, having similar overall shape);
2. recognizing analogous sub-parts in objects having different overall shape;
3. distinguishing if an object shape is itself a sub-part of another.

In particular, we observe that the third problem is a particular case of the second one. In the first case similar sub-parts of the two objects should be automatically recognized and mapped. The method proposed in this paper is able to approach all facets of the sub-part shape correspondence problem with major emphasis on the first one (due to the properties of the structural descriptor we have adopted).

The second kind of correspondence deals with objects having different overall shape but similar sub-parts. The partial correspondence should recognize similar sub-parts of the two objects and produce the correspondent mapping.

3.1 Shape descriptors

Our implementation of the matching process involves both structure and geometry of the two objects. At the moment, the structural information of the object is captured by the Reeb graph computed with respect to a position invariant function [24], see figure 2.a). The functions used to validate the matching framework proposed in this paper are the distance from the center of mass of the object and the integral geodesic distance in [17]. As shown in [1], the shape characterization and the Reeb graph construction naturally induce a decomposition of the shape in topologically significant regions, see figure 2.(a). The region decomposition obtained from our contouring approach does not admit slices with internal holes and each border component of the surface patches is shared by only two distinct patches. Since each border component is completely shared by two patches, cutting and pasting operations along such a contour may be performed independently from other contours. Moreover, this segmentation produces a directed graph [24], in which each node corresponds to an object patch and each edge connects two nodes, see figure 2.(b).

characterization, see figure 2.c). The value of f and a geometric descriptor are associated to each node in the simplified ERG. In our approach we move from a local description of the surface slice to a more general representation of the model sub-parts, based on the assumption that the larger the model portion associated to a node is, the more relevant the node should be.

Since our graph is directed, each node identifies a subgraph and the geometric attribute associated to the node is obtained from the surface related to its subgraph, see figure 2.d). For leaf nodes, whose sub-graph is empty, we consider only the slice of the shape that correspond to them. Once sub-parts have been associated to each node, we use the spherical harmonic analysis of the sub-part to describe its geometry. Spherical harmonic analysis has been defined in [10] and this descriptor is rotation invariant. In [10] the scale invariance is obtained by uniformly scaling a model in a cube whose edge length is two; therefore, in our case each sub-part is separately scaled.

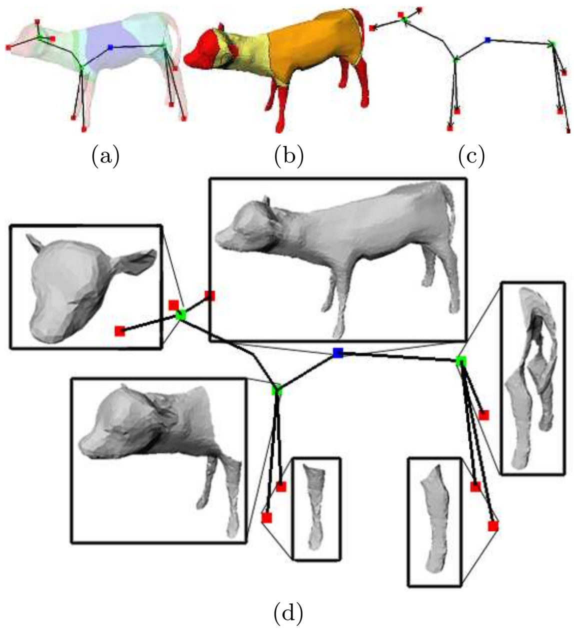


Fig. 2. A Reeb graph of a calf a), the surface segmentation associated b) and the oriented graph c). In d) the surface portions associated to some nodes are highlighted; these regions contain all patches associated to the subgraph nodes.

3.2 Graph matching

The sub-part shape correspondence between two objects is obtained by matching the directed attributed graphs. Inexact graph matching has been topic of research since many years and several techniques are available [25,26]: recently,the framework proposed in [2] formalizes the enumeration of all common subgraphs of two graphs in a way that makes it straightforward usable for plugging heuristics in it and, according to the specific case, achieves different approximations of the optimal solution. The matching algorithm proposed in this paper is a specialization of that described in [2] for partial graph matching applications.

According to the notation proposed in [2] we name a graph $\mathcal{G}$ a *common subgraph* of $\mathcal{G}_1$ and $\mathcal{G}_2$ if there exists a subgraph isomorphism from $\mathcal{G}_1$ to $\mathcal{G}$ and from $\mathcal{G}_2$ to $\mathcal{G}$. A *maximal common subgraph* is a common subgraph that can not be extended to another subgraph by the addition of nodes or edges. A *maximum common subgraph* of $\mathcal{G}_1$ and $\mathcal{G}_2$ is a common subgraph $\mathcal{G}$ such that there exists no other common subgraph having more nodes than $\mathcal{G}$. The maximum common subgraph in not necessarily unique.

The output of the matching process should be the largest maximal common subgraph that minimizes the geometrical and the structural differences. The proposed matching algorithm can be synthetically described by the two following steps:

1. Select a mappings M among the nodes of the two graphs $\mathcal{G}_1$ and $\mathcal{G}_2$. The mapping M is a set of node pairs (v_1, v_2), where v_1 is a node of $\mathcal{G}_1$ and v_2 is a node of $\mathcal{G}_2$;
2. compute the common subgraph between $\mathcal{G}_1$ and $\mathcal{G}_2$ by expanding the mapping M.

The step 2 expands the initial mapping M as much as possible while respecting the definition of common subgraph.

As discussed in [3], running the algorithm on M yields an approximation of the maximum common subgraph. Heuristic techniques can be used to select the best initial mappings and the subgraph expansion rules, in order to better approximate the optimal solution.

Depending on the attributes and on the topology of the graph, some nodes are more relevant than others. Since the considered input graph $\mathcal{G}$ is directed, each node v identifies a subgraph $\mathcal{G}'$ induced by the set of nodes having v as ancestor plus v itself [24]. The notion of node relevance is used to select the initial mapping M and it is captured by the subgraph associated to the node: a large subgraph corresponds to a large amount of structural and geometric information and then to a more relevant articulated object sub-part. Another useful heuristic has been constructed associating to the pair of nodes (v_1, v_2) the information about how much the common subgraph would expand with the addition of that pair. To this end, the distance function between two nodes v_1 and v_2 that involves node and edge attributes and the approximation of the structure of the subgraphs related to v_1 and v_2 has been shown in the equation (1), for details see [3].

$$d(v_1, v_2) = \frac{w_1 G\_S + w_2 St\_S + w_3 Sz\_S}{w_1 + w_2 + w_3}, \tag{1}$$

In equation (1) G_S, St_S and Sz_S are real numbers, G_S, St_S, $Sz\_S \in [0, 1]$. G_S and St_S respectively represent the geometric similarity between the node attributes and the structural similarity between the node subgraphs. Sz_S evaluates the similarity between the size of the sub-parts associated to nodes, where the size corresponds to the sum of the lengths of the subgraph edges. Finally, the three weights w_1, w_2 and w_3 belong to the range $[0, 1]$ and combine the three components of d. G_S compares the geometric signatures associated to v_1 and v_2. The signature of each node is obtained decomposing the sub-part surface into a collection of functions defined on concentric spheres and using spherical harmonics to discard orientation information for each one, [10].

Then, the similarity between the subgraph structures is defined as:

$$St\_S = \frac{\overline{in} + \overline{out} + \overline{sub\_n} + \overline{sub\_in} + \overline{sub\_out}}{5},$$

where

$$\overline{X} = \begin{cases} 0 & \text{if } \max(X(v_1), X(v_2)) = 0 \\ \frac{|X(v_1)-X(v_2)|}{\max(X(v_1),X(v_2))} & otherwise \end{cases}$$

in and out represent the in-degree and the out-degree of the two nodes, sub_n the number of the subgraph nodes, sub_in and sub_out the in-degree and out-degree sum of the subgraph nodes. Finally $Sz\_S = \overline{sub\_s}$, where sub_s is the sum of the edge attributes of the subgraph.

Finally $Sz\_S = \overline{sub\_s}$, where sub_s is the sum of the attributes of the subgraph that represent the size of the sub-parts associated to nodes.

The graph matching algorithm follows the framework proposed in [2,3]. In the listings 1.1 and 1.2 a brief description of the algorithm pseudo-code shows how the expansion process produce the common sub-graph starting from the initial set of relevant nodes.

The relevance of a node is computed with respect to the average size of all subgraphs induced, and the set of the initial nodes mapping (`CANDIDATES`) is obtained by combining all relevant graph nodes, see listing 1.1. The node pairs

Listing 1.1. The initial mapping procedure

```
Initial_Mapping(G1, G2, CANDIDATES)
{
  Relevant_Nodes(G1, RG1);
  Relevant_Nodes(G2, RG2);
  for each node v1 of RG1
       for each node v2 of RG2
            Add((v1, v2), CANDIDATES);
}
```

belonging to the set `CANDIDATES` are ordered with respect to the distance d shown in equation (1); then, the candidate node pair with the smallest value of d is extracted. New node pairs are added to `CANDIDATES` by combining all the nodes out-coming from the initial pair.

The process that produce the common subgraph by expanding the set of node pair `CANDIDATES` is outlined in the listing 1.2. The function `Pop` extracts the candidate node pair with the smallest value of d, while the statement `Check` between the edges `e1` and `e2`, guarantees the construction of the common subgraph `CS` is correct with respect to the definition of common sub-graph. `Updates` adds new node pairs to `M` combining all the nodes out-coming from `v1` and `v2`.

In particular has to be observed that the initial mapping among relevant nodes makes the algorithm robust with respect to structural noise allowing the

Listing 1.2. The comparison procedure

```
Match(G1, G2, CS)
{
 Initial_Mapping(G1, G2, M);
 while(M Not Empty){
      c = (v1, v2) = Pop(M);
      e1 = (Parent(v1), v1);
      e2 = (Parent(v2), v2);

      if(Mapped(v1) and Mapped(v2))
           if(Check(e1, e2)){
                Add((e1, e2), CS);
      }else if(Not Mapped(v1) and
                Not Mapped(v2)){
                    Add(c, CS);
                    Add((e1, e2), CS);
              }

      Update(CANDIDATES, c);
 }
}
```

construction of a not necessarily connected common subgraph enabling the recognition of similar sub-parts even if the overall objects shape/structure is dissimilar.

3.3 Computational complexity

The computational cost of the algorithm is given by the sum of the costs of two main steps: the extraction of the structural algorithm and the graph matching phase. The first step of the algorithm (that includes the extraction of the ERG structure and the coding of the models sub-parts using the spherical harmonic descriptor in [10]) may be stored in an out-of-core pre-processing phase. As shown in [1] the complexity of the ERG extraction process is $O(\max(m+n, n\log(n)))$ with m the number of vertices inserted in the triangulation during the slicing phase and n the number of vertices in the original mesh. The storage of the graph nodes using the spherical harmonic descriptor requires $O(b^4))$ operations using a volumetric regular grid having $O(b^3)$ cells, see [27] for details.

The computation of the common subgraph between two graphs, with n and m nodes respectively, is polynomial with respect to $k = \max(n, m)$. In fact, identifying the relevant nodes is linear with respect to the number of nodes of the graph $O(k)$, while the mapping M is obtained by combining all relevant graph nodes $O(s * k^2)$, where s is the computational cost of the comparison of two geometric attributes associated to the graph nodes. The construction of the ordered set `CANDIDATES` takes $O(k * log * k)$ while the extraction of the smallest value of d is constant; new elements of `CANDIDATES` are obtained by combining all the nodes out-coming from the pair of nodes $O(s * k^2)$ and this operation is repeated at most once for each node belonging to the common subgraph. By all these observations the computational complexity of the graph matching algorithm is $O(s * k^3)$.

4 Examples and discussion

In this section some experimental results are provided and discussed. Sub-part correspondence represented in the figures of this section is obtained by giving the same color to similar sub-parts.

In figure 3 is shown the partial correspondence obtained by comparing similar models, in our case three horses. Although the overall shape of the animals is the same, the models differ over some details: for example, the structure of the head, the tail and the posture. Our partial matching correctly recognizes the correspondence among the bodies and the front/rear sub-parts of the models. If some shape features have no correspondence in the other models (like the tail) they are not mapped at all. Nevertheless it could happen that features like the legs may be switched. This is caused by a lack of structural information into the leg description, in these cases the sub-parts mapping is completely demanded to the geometric descriptor that produces this output.

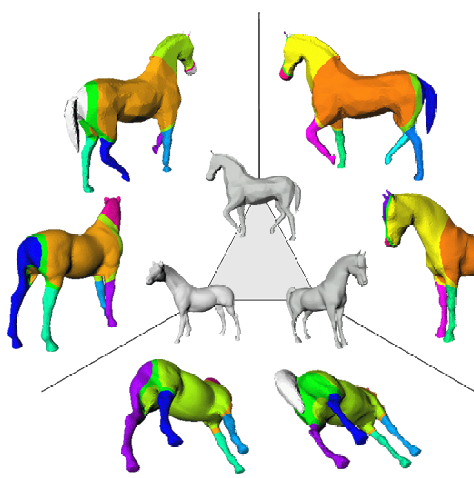


Fig. 3. Sub-part correspondence among three animals: these models have similar structure and geometry.

Other experimental results of the application of our matching method to models having same overall shape but different spatial embedding are shown in figure 4. The geodesic distance distribution on a human model does not change if the legs and the arms are stretched rather than curled up. Therefore, since our structural descriptor is independent of different poses of the same object, our method can recognize human features like hands, head, legs and body in arbitrary positions, see figure 4.

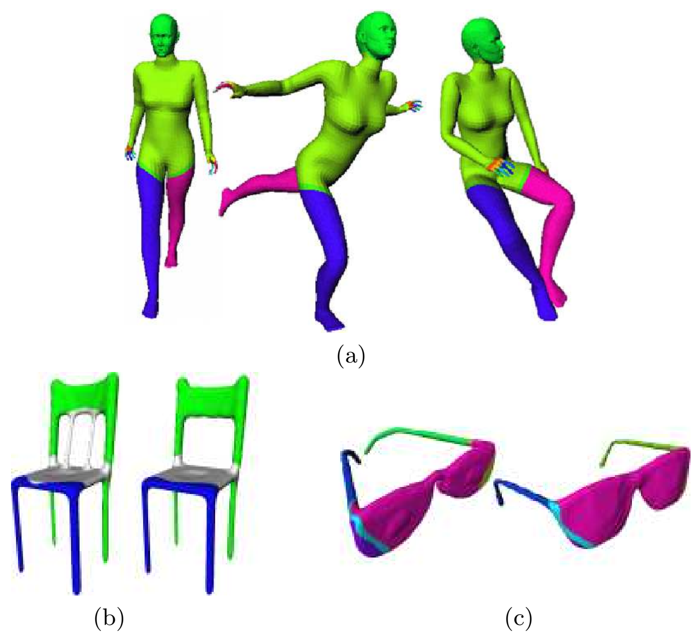
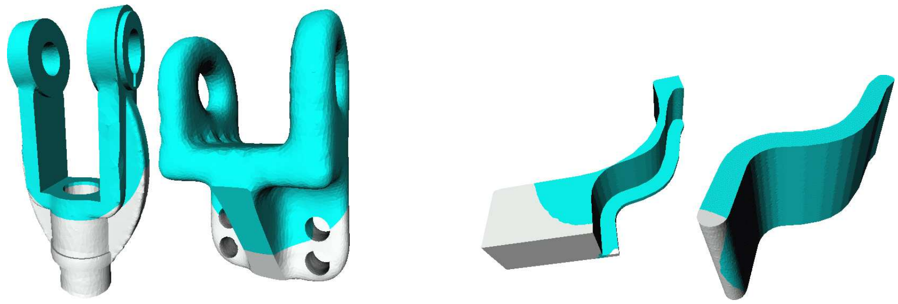


Fig. 4. Correspondence of shape features on a human model in different poses (a), between two similar chairs (b) and finally between two spectacles (c).

Two examples of partial correspondence among models having different overall appearance is given in figure 5(a) and 5(b). In this case the models may differ both in structure and geometry but they have similar sub-parts: even if the two objects shown in figure 5(a) are not similar from a geometric and structural point of view, they have two similar two big protrusions. The coupling of geometry and structure pursued by our algorithm correctly map the common sub-parts of the two objects. On the contrary, the central parts of the models are different, thus they are not recognized as similar sub-parts. Similar remarks hold for figure 5(b), where the two objects have a similar subpart.

In figures 6 and 7, two examples where a whole model is a sub-part of another one are shown. In figure 6 the graph of the cow head is a subgraph of the cow graph. The matching algorithm computes the common subgraph reasoning on the graph structure and on the geometric attributes, with the result that the mouth, the ears and the horns are correctly mapped. Even if the two shapes of figure 7(a) 7(b) are dissimilar, the elongated part shown in (b) is correctly recognized as sub-part of the model shown in (a). This is because the structural



(a) (b)

Fig. 5. The correspondence between the sub-parts of the models is highlighted by the color.

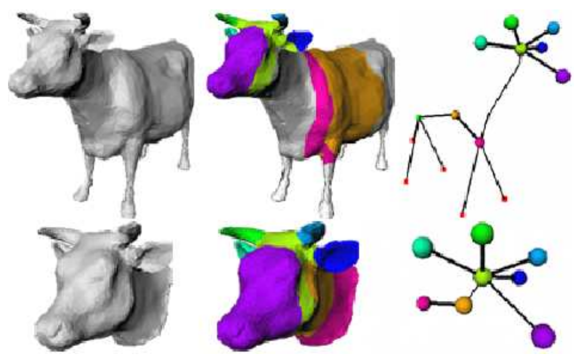
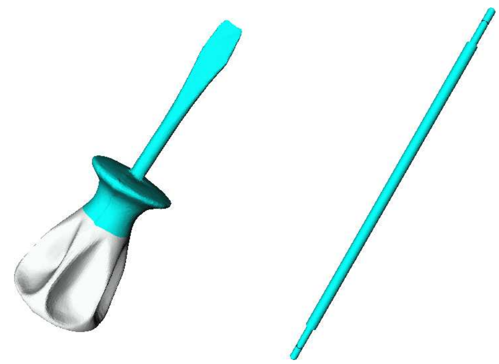


Fig. 6. Recognition of the head of a cow with respect to the whole animal model.

descriptor of (a) correctly characterizes both the geometry and the structure of the sub-part that identifies the long part of the screwdriver.

5 Concluding remarks and future work

In summary, in this paper we have presented a new method for measuring similarity and recognizing sub-part correspondence between 3D shapes. Main research contribution is the new similarity matching mechanism to compare 3D shapes coupling geometry and topology. Since this method computes an approximation of the maximal common subgraph of two structural shape descriptors, it is particularly suitable for sub-part shape correspondence. In addition, it is flexible, because it can be applied to any skeletal structure with the same properties of our topological graph (attributed, directed and acyclic), and tunable, as it can be used in a multi-step query approach, to progressively refine the set of geometrically similar candidates. Even if the flexibility of the mapping function makes adaptable to various application contexts, other shape descriptors like the *shape graph* proposed in [28] may be considered.



(a) (b)

Fig. 7. The whole model (b) is recognized as sub-part of the screwdriver (a).

The method proposed in this paper produces an object segmentation without any user interaction while the approaches in [13] and [15] do. Furthermore, differently from [12] that always splits a shape into three parts, our segmentation depends on the shape complexity of the object and decomposes it into a set of significant sub-parts. The method proposed in [14] produces an automatic and structural subdivision of the object surface but it works only on simple surfaces where shape discontinuities are present and easily recognized. As shown in figures 3 and 4, coupling structure and geometry like in our approach is advantageous for comparing models having similar overall shape and structure but different posture.

Acknowledgments

The authors thank all the people of the Shape Modelling group at the IMATI-GE/CNR, in particular B. Falcidieno and M. Spagnuolo for their fruitful support. Models in figure 4(a) have been kindly provided by MIRALab, Univ. of Geneva.

This work has been partially supported by the EU Newtwork of Excellence "AIMSHAPE" (Contract n. 506766).

References

1. Biasotti, S.: Computational topology Topology Methods for Shape Modelling Applications. PhD thesis, University of Genova (2004)
2. Marini, S., Spagnuolo, M., Falcidieno, B.: From exact to approximate maximum common subgraph. In: Proceedings of the 5rd IAPR -TC-15 Workshop on Graph-based Representations in Pattern Recognition. Lecture Notes in Computer Science, Poitiers (France) (2005) 263–272
3. Marini, S.: 3D Shape Similarity Through Structural Descriptors. PhD thesis, University of Genova (2005)

4. Biasotti, S., Marini, S.: Sub-part correspondence using structure and geometry. In Gallo, G., Battiato, S., Stanco, F., eds.: Proc. of 4th Eurographics Italian Chapter Conference. (2006)
5. Biasotti, S., Marini, S., Spagnuolo, M., Falcidieno, B.: Sub-part correspondence by structural descriptors of 3d shapes. to appear on Computer-Aided Design: Special issue on Shape Similarity Detection and Search for CAD/CAE Applications (2006)
6. Osada, R., Funkhouser, T., Chazelle, B., Dobkin, D.: Matching 3D models with shape distributions. In: Proceedings of Shape Modelling and Applications, Genova, Italy, IEEE Press (2001) 154–166
7. Novotni, M., Klein, R.: A geometric approach to 3D object comparison. In: Proceedings of Shape Modelling and Applications, Genova, Italy, IEEE Press (2001) 167–175
8. Corney, J., Rea, H., Clark, D., Pritchard, J., Breaks, M., MacLeod, R.: Coarse filters for shape matching. IEEE Computer Graphics and Applications **22** (2002) 65–74
9. Veltkamp, R.C., Tangelder, J.W.: Polyhedral model retrieval using weighted point sets. In: Proceedings of Shape Modelling and Applications, Seoul, South Korea, IEEE Press (2003) 119–128
10. Kazhdan, M., Funkhouser, T., Rusinkiewicz, S.: Rotation invariant spherical harmonic representation of 3D shape descriptors. In Kobbelt, L., Schröder, P., Hoppe, H., eds.: Proceedings of Symposium in Geometry Processing, Aachen, Germany (2003) 156–165
11. Johnson, A.E., Hebert, M.: Recognizing objects by matching oriented points. In: Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. (1997) 684–689
12. Huber, D., Kapuria, A., Donamukkala, R., Hebert, M.: Parts-based 3D object classification. In: Proceedings of the International Conference on Computer Vision. Volume Vol. 2. (2004) 82–89
13. Ruiz-Correa, S., Shapiro, L., Meila, M.: A new paradigm for recognizing 3D object shapes from range data. In: Proceedings of the International Conference on Computer Vision. (2003) 1126–1133
14. Fan, T., G.Medioni, Nevatia, R.: Recognizing 3D objects using surface description. IEEE Transactions on Pattern Analysis and Machine Intelligence **11** (1989) 1140–1157
15. Funkhouser, T., Kazhdan, M., Shilane, P., Min, P., Kiefer, W., Tal, A., Rusinkiewicz, S., Dobkin, D.: Modeling by example. ACM Transactions on Graphics **23** (2004) 652–663
16. Bespalov, D., Shokoufandeh, A., Regli, W.C., Sun, W.: Local feature extraction using scale-space decomposition. In: ASME Design Engineering Technical Conferences, Computers and Information in Engineering Conference Conference (DETC 2004-57702), ASME Pres (2004)
17. Hilaga, M., Shinagawa, Y., Komura, T., Kunii, T.L.: Topology matching for fully automatic similarity estimation of 3D shapes. In: Proc. of SIGGRAPH 2001, Los Angeles, ACM Press (2001) 203–212
18. Bespalov, D., Regli, W.C., Shokoufandeh, A.: Reeb graph based shape retrieval for CAD. In: Proceedings of the 2003 ASME Design Engineering Technical Conferences, Chicago, Illinois, ASME (2003)
19. Tung, T., Schmitt, F.: The Augmented Multiresolution Reeb Graph approach for content-based retrieval of 3D shapes. Int. J. of Shape Modelling **11** (2005) 91–120

20. Blum, H.: A transformation for extracting new descriptors of shape. In Whaten-Dunn, W., ed.: Proceedings of the Symp. Models for Perception of Speech and Visual form, Cambridge MA: MIT Press (1967) 362–380
21. Sundar, H., Silver, D., Gagvani, N., Dickinson, S.: Skeleton based shape matching and retrieval. In: Proceedings of Shape Modelling and Applications, Seoul, South Korea, IEEE Press (2003) 130–139
22. Zhang, J., Siddiqi, K., Macrini, D., Shokoufandeh, A., Dickinson, S.: Retrieving articulated 3-D models using medial surfaces and their graph spectra. In: Proceedings of International Workshop on Energy Minimization Methods in Computer Vision and Pattern Recognition (EMMCVPR), St. Augustine, FL (2005)
23. Iyer, N., Jayanti, S., Lou, K., Kalyanaraman, Y., Ramani, K.: Shape-based searching for product lifecycle applications. Computer-Aided Design **37** (2005) 1435–1446
24. Biasotti, S., Marini, S., Mortara, M., Patanè, G., Spagnuolo, M., Falcidieno, B.: 3d shape matching through topological structures. In: Proc. of DGCI. Volume 2886 of Lecture Notes in Computer Science. (2003) 194–203
25. Messmer, B.T., Bunke, H.: A new algorithm for error tolerant subgraph isomorphism detection. IEEE Transactions on Pattern Analysis and Machine Intelligence **20** (1998) 493–504
26. Cordella, L.P., Foggia, P., Sansone, C., Vento, M.: A (sub)graph isomorphism algorithm for matching large graphs. IEEE Transactions on Pattern Analysis and Machine Intelligence **26** (2004) 1367–1372
27. Kazdhan, M.M.: SHAPE REPRESENTATIONS AND ALGORITHMS FOR 3D MODEL RETRIEVAL. PhD thesis, Princeton University (2006)
28. Mortara, M., Patané, G., Spagnuolo, M., Falcidieno, B., Rossignac., J.: Blowing bubbles for multi-scale analysis and decomposition of triangle meshes. Algorithmica **38** (2003) 227–248