# Path planning and optimization in the traveling salesman problem: Nearest neighbor vs. region-based strategies

J.M. Wiener<sup>1 2</sup>, Nicole N. Ehbauer<sup>1</sup> & H.A. Mallot<sup>1</sup>

November 23, 2006

<sup>1</sup> Cognitive Neuroscience	<sup>2</sup> CNRS/Collège de France
University of Tübingen	LPPA
Auf der Morgenstelle 28	11, place Marcelin Berthelot
72076 Tübingen	75231 Paris
Germany	France

**Keywords:** spatial cognition, navigation, route planning, path complexity, traveling salesman problem, regions, hierarchical planning, nearest neighbor strategy

#### Abstract

For large numbers of targets, route planning can be a very complex and computationally expensive task. Human navigators, however, usually solve route planning tasks fastly and efficiently. Here two experiments are presented that studied human route planning performance as well as the cognitive strategies and processes involved. 25 places were arranged on a regular grid in a large room. Each place was marked by a unique symbol. Subjects were repeatedly asked to solve traveling salesman problems (TSP), i.e. to find the shortest closed loop connecting a given start place with a number of target places. For each trial, subjects were given a so-called 'shopping list' depicting the symbols of the start place and the target places. While the exact TSP is computationally hard, approximate solutions can be found by simple strategies such as the nearest neighbor strategy. Experiment 1 tested whether humans employed the nearest neighbor (NN) strategy when solving the TSP. Results showed that subjects outperformed the NN strategy in cases in which the NN strategy did not predict the optimal solution, demonstrating that the NN strategy is not sufficient to explain human route planning behavior. As a second possible path planning strategy a region-based approach was tested in Experiment 2. When optimal routes required more region transitions than other, sub-optimal routes, subjects preferred these sub-optimal routes. This result suggests that subjects first planned a coarse route on the region level and then refined this route plan during navigation. Such a hierarchical planning strategy allows to reduce both, computational effort and working memory load during path planning.

## 1 Introduction

**Path planning is a non-trivial problem.** Consider planning a route in order to visit multiple locations in your home town. According to the number of target locations, planning a reasonably short path can be a very complex and computationally expensive task. This is best demonstrated by the well-known traveling salesman problem (TSP). The TSP can be stated as follows: Given a number of target places and the cost of traveling from one to the other (usually distance), what is the cheapest round trip route that visits each target place and returns to the start place. The number of possible round trips is computed as (N-1)!, with N being the number of places to visit. For visiting 5 places and returning to the starting place 120 different round trips are possible, for visiting 9 cities, already over 360 000 different round tips are possible. For humans, route planning tasks similar to the TSP are actually quite common, for example, on a typical shopping route on which multiple stores have to be visited. Usually, humans solve such route planning tasks fastly and efficiently. Obviously, rather than actually calculating and comparing all possible path alternatives, human navigators rely on strategies and heuristics allowing for the reduction of cognitive effort while resulting in reasonably short routes.

**Strategies and heuristics involved in path planning.** Route planning and path selection behavior in navigation, both for animals and humans, has been investigated in only few studies. Consequently, knowledge about the underlying mechanisms, strategies, and heuristics as well as the cognitive components and processes involved is far from comprehensive as emphasized by Golledge (1995): *"Traditionally, the path selection problem has been ignored or assumed to be the result of minimizing procedures such as selecting the shortest path, the quickest path or the least costly path."*.

Gärling & Gärling (1988), for example, investigated pedestrian shopping behavior with respect to distance minimization in multi-stop shopping routes. Most shoppers, that minimized the distance of their shopping routes, first chose the location farthest away, most probably to minimize effort to carry bought goods, and then minimized distances locally between shopping locations (see also Gärling, Säisä, Böök, & Lindberg, 1986). This so called locally-minimizing-distance (LMD) heuristics is similar to the nearest neighbor algorithm (NN) in artificial intelligence approaches (e.g. Golden, Bodin, Doyle, & Stewart, 1980). The NN-algorithm is a simple algorithm to solve TSPs fastly. From its current location, which corresponds to the starting location in step one, the NN algorithm visits the closest target location that has not been visited before. By simply repeating this procedure until all target locations have been visited and by returning to the starting place, the NN algorithm usually finds good or near-optimal solutions for TSPs of small sizes. On TSPs of large sizes, however, the NN-algorithm is known to often fail in finding reasonably cheap (short) solutions. Christenfeld (1995) studied human subjects' preference to choose a certain route from a series of almost identical routes. In three conditions (route choice from artificial maps, from street maps or in real world environments) subjects had the choice between a number of routes, identical with starting place and target place, metric length, and the number of turns. The only difference between the alternative routes was when along the route subjects had to make the turns. In all three conditions subjects delayed the turning decision as long as possible. Christenfeld suggested that this resulted from subjects' tendency to minimize mental effort, i.e. subjects did not worry about when and where to turn until they had to turn. This strategy offers a possible explanation for the fact that people's route choices are often asymmetric; i.e. people choose different routes from A to B than from B to A (e.g. Stern & Leiser, 1988). Bailenson, Shum, & Uttal (1998) investigated route planning from maps and formulated the *road climbing* principle, which states that instead of calculating the globally shortest route, subjects relied on routes that allowed to leave the region containing the start place sooner rather than later. In addition, subjects take the straightness and length of the initial route segment into account (Bailenson, Shum, & Uttal, 2000). This so-called Initial Segment Strategy (ISS) assumes that subjects prefer routes with the longest initial straight segment above alternative routes of equal length.

Wiener & Mallot (2003) studied the influence of environmental regions on human route planning behavior in active navigation. For this, subjects first learned virtual environments that were divided into different regions by active navigation and were then asked to solve different route planning and navigation tasks, i.e. to find the shortest route connecting a given starting place with a single or multiple target places. During navigation, subjects minimized the number of region boundaries they crossed during navigation and preferred paths that allowed for fastest access to the region containing the target. These results not only demonstrate that regional information is explicitly represented in spatial memory, but also demonstrates that route planning takes into account this regional information and is not based on place information and place-connectivity alone. Wiener & Mallot proposed a cognitive model, the *fine-to-coarse* planning strategy, to account for the empirical findings. This hierarchical route planning scheme reduces mental effort and working memory load during route planning by using fine spatial information for the close surrounding exclusively and coarse spatial information for distant places. In everyday route planning, multiple information sources are available, allowing for various wayfinding strategies. In a series of navigation experiments in virtual environments consisting of multiple regions, Wiener et al. (2004) studied the use and interaction of different route planning strategies. In addition to the *fine-to-coarse* planning strategy, two other wayfinding strategies could be identified, the *cluster*strategy and the *least-decision-load* strategy. Essentially, the cluster-strategy states that route planning takes into account the distribution of target locations within an environment, predicting that subjects try to increase the number of visited targets as fast as possible. The least-decision-load strategy states that subjects prefer paths that minimize the number of possible movement decisions. The latter strategy could be employed because the risk of getting lost is smaller on less complex routes.

Further insights into route planning and navigation strategies comes from the animal literature. Gallistel & Cramer (1996), for example, studied vervet monkeys ability to navigate the shortest route connecting multiple locations, by arranging baited locations in a group of four to one side and a group of two to the other side. As the nearest baited location of both food patches were equidistant from the starting

point, an algorithm like the nearest neighbor algorithm (NN) predicts that the monkeys choose to first visit both of the food patches equally often. However, the vervet monkeys first visited the richer food patch in all trials (c.f. cluster-strategy). In a second experiment Gallistel & Cramer (1996) arranged baited locations in a diamond shape. If the monkeys intended to return to the starting position that was part of the diamond, because it was baited only after the monkey left it, the monkeys generally chose the shortest route in this traveling salesman task. Here a NN strategy would predict that the monkeys followed a different non-optimal route. Gallistel & Cramer (1996) concluded that the vervet monkeys' route planning algorithm not only took into account the next step (as predicted by the NN), but is indeed planning three steps ahead. Rats, in contrast, have been suggested to use the NN strategy, i.e. to repeatedly visit the nearest not yet visited target, when trained to visit an array of cylindrical feeders in an open field (Bures, Buresova, & Nerad, 1992) (see also Menzel (1973) for chimpanzees' performance in a modification of the TSP).

Visual versions of the TSP. Human path planning and optimization behavior has been investigated in several studies by means of visual versions of the Traveling Salesman Problem (e.g., MacGregor & Ormerod, 1996; MacGregor, Ormerod, & Chronicle, 1999, 2000; MacGregor, Chronicle, & Ormerod, 2004; Van Rooij, Stege, & Schactman, 2003; Graham, Joshi, & Pizlo, 2000; Vickers, Lee, Dry, & Hughes, 2003b; Vickers, Bovet, Lee, & Hughes, 2003a; Vickers, Lee, Dry, Hughes, & McMahon, 2006). In these experiments subjects are usually confronted with a number of dots on a computer monitor. Subjects' task is to connect these dots by straight line segments such that the resulting path (tour) is optimal with respect to overall length. Generally, results from these studies show that humans are very good in solving visual TSPs. There is an ongoing debate on the strategies subjects applied in these experiments. MacGregor & Ormerod (1996), for example, have proposed that humans apply the convex hull method, assuming that subjects used the convex hull as part of their strategy (see also MacGregor et al., 2000, 2004). The convex hull is easily visualized by imagining an elastic band stretched open to encompass all dots; when released, it will assume the shape of the convex hull, touching all boundary dots of the TSP (the remaining cities are referred to as *interior* cities). Afterwards, the segment of the elastic band which is closest to an unconnected dot, will be stretched to include that dot into the tour. This latter step is repeated until all dots are incorporated in the overall tour. MacGregor & Ormerod (1996) argue that the fact that a tour that follows the convex hull method is by definition free of crossings and that humans tend to avoid crossings is one important piece of supporting evidence for the convex hull method. Van Rooij et al. (2003), however, argue that subjects know that crossings will result in sub-optimal solutions. They have proposed the crossing avoidance hypothesis, stating that humans avoid crossings when solving TSPs, rather than following the convex hull method. Vickers et al. (2003a) proposed a hierarchical nearest neighbor (NN) method, assuming that subjects first establish clusters of several cities based on NN distances, which they then sequentially link into a tour, using some variant of the nearest neighbor algorithm. Graham et al. (2000) proposed another hierarchical model, assuming that from the original stimulus (dot pattern) a series of images are generated which are increasingly blurred and compressed. By these means a hierarchy of images is generated in which neighboring dots collapse to clusters. The algorithm then starts with generating a tour upon an image which is so high in the hierarchy that only 3 clusters exist. By progressively moving to the next lower layer in the hierarchy further clusters, and eventually dots, are inserted into the tour.

The objective of this study is to develop an increased understanding of the cognitive components and processes involved in human navigation by studying planning and navigation behavior when solving TSPs. It is therefore important to consider differences between visual TSPs, as introduced above, and navigational TSPs, as studied here. For example, as subjects actively move through the environment in navigational TSPs, their spatial relation to the different target places constantly changes. Accordingly, subjects permanently have to deal with perspective changes and they usually do not have an overview of the environment as a whole. Also, in the visual TSPs, the path from the start to the current location is displayed as a polygonal line segment during the trial and subjects are allowed to correct their choices by undoing links between cities. Both of the latter properties are usually not available during real world navigation. Moreover, in everyday path planning, the navigator is faced with a number of memory tasks that are absent in visual TSPs. For example, if no external representation of space (e.g., a street map) is available, the target locations have to be retrieved from spatial memory. Additionally, once the targets have been localized, they have to be held active in a working memory during the actual process of planning a path. Here, it can be assumed that navigators have to deal with different memory related constraints, such as imprecise spatial knowledge or capacity limits of working memory. While this list is

far from being comprehensive, it demonstrates that solving visual TSPs and navigational TSPs can not, a priori, be assumed to be based on the same cognitive principles.

**Local and global path planning.** In the field of artificial intelligence, route planning algorithms are usually classified as either local or global. Global route planning algorithms take into account all the information available. The entire route is planned from the start to all goal locations before the first move is executed. Local planning algorithms, on the other hand, plan only few steps or even only a single step ahead. A prototypical example for a local planning algorithm is the nearest neighbor algorithm (NN) for solving the TSP. As introduced before, at each step the NN simply selects and visits the closest non-visited target location. In the field of human spatial cognition, another class of algorithms, the hierarchical planning algorithms, have been used to explain subjects navigation behavior. Hierarchical planning algorithms reduce computational effort during route planning by using spatial information at different levels of abstraction (e.g., fine-to-coarse planning strategy introduced above Wiener & Mallot, 2003). Kuipers, Tecuci, & Stankiewicz (2003), for example, have proposed a route planning scheme that is mainly based on a 'skeleton', i.e. a well-known subset of all paths in the environment. Rather than taking all paths into account during route planning, primarily the skeleton of well-known routes is used. Using such an abstraction of the environment reduces search space during route planning and therefore reduces computational effort.

**Synopsis.** In this work, two navigation experiments studying cognitive strategies underlying path planning behavior are presented. In the experiments subjects were repeatedly asked to solve different traveling salesman problems (TSP). By varying the number of target places to visit, subjects general performance in solving TSPs in active navigation was studied. The cognitive strategies that subjects applied during path planning and navigation were studied by varying the characteristics of the experimental environment and the specific route planning tasks. For each strategy in question, we designed two types of TSP tasks, one in which application of the strategy yields optimal solutions (strategy-adequate tasks) and one in which application of the strategy yields suboptimal solutions (strategy-inadequate tasks). If a particular strategy is used by the subjects, we expect better performance in the according strategy-adequate tasks. Special interest concerned the role of the Nearest Neighbor (NN) Strategy, the role of environmental regions, and the role of spatially clustered targets for path planning.

## 2 Experiment 1

## 2.1 Motivation

This experiment pursued two main purposes. First, it was designed to test for subjects' general performance in solving TSPs by active navigation. For this, subjects' performance of finding the shortest path in TSPs with varying number of targets was evaluated. Second, the experiment tested for the contribution of two simple route planning strategies, the Nearest Neighbor (NN) strategy and the clusterstrategy (see Section 1), when solving TSPs.

## 2.2 Material & Methods

### 2.2.1 The experimental setup

The experiment was conducted in a  $6.0 \times 8.4$  m experimental room. 25 small cardboard pillars were arranged on a  $5 \times 5$  squared grid with a mesh size of 1.10m. 25 symbols were randomly distributed about the 25 pillars (see Figure 1). In order to control for effects of the specific symbol-configuration, two versions of the setup were created that only differed in the specific arrangement of the symbols. Half of the participants conducted the experiment in one configuration, the other half conducted the experiment in the alternative configuration.



Figure 1: Left: schematic drawing of the experimental setup; right: subject solving a navigation task.



Figure 2: Example of a 'shopping list' for a navigation task with 9 target places.

### 2.2.2 Navigation tasks

During the experiment, subjects were asked to solve different navigation tasks, i.e. to solve different traveling salesman tasks (TSP). For each TSP they received a specific '*shopping list*' depicting the symbol that defined the start place and the symbols that defined the target places that had to be visited during navigation. Subjects were given the lists one at a time and upside-down, such that they could not see what was on the list. They were, however, verbally informed about the start place and asked to move to that start place. After subjects reached the start place, they were allowed to turn around the *shopping list* and the trial started. Subjects' task was to navigate the shortest route connecting the start place with all target-places and return to the start place. During navigation, subjects kept the shopping list and marked the target places they had already visited with little black markers.

In order to control for the influence of the specific sequence of the symbols depicted on the *shopping list*, two versions of each shopping list were generated. Half of the participants received one version of the shopping lists, while the other half received the other version of the shopping lists.

**Types of navigation tasks.** Each subject solved 36 different TSPs consisting of 4, 5, 6, 7, 8 or 9 target places (see Table 1). The 36 navigation tasks could be further subdivided into three types, the *NN-adequate tasks*, the *NN-inadequate tasks*, and the *cluster-tasks* (*NN-ambiguous tasks*) (see Figure 3).

- 1. **NN-adequate tasks:** For these set of navigation tasks, the predictions of the NN algorithm were identical with the optimal, i.e. the shortest possible, path (see Figure 4).
- 2. NN-inadequate tasks: For these set of navigation tasks, the NN strategy did not generate the optimal path (see Figure 4). If subjects applied the NN strategy on these TSPs, they were systematically lead on paths longer than the optimal paths.
- 3. **Cluster tasks (NN-ambiguous):** On cluster tasks the target places were distributed in two distinct target clusters of unequal size. In contrast to NN-adequate and NN-inadequate tasks, these TSPs were NN-ambiguous, i.e., the nearest neighbor algorithm did not make clear predictions for a single path. First, because the closest target places were always equidistant from the starting place, and second, because similar situations also re-occur during navigation, i.e. close target places were equidistant from the current position.



Figure 3: Example TSPs with six target places for the *NN-adequate tasks* (left), the *NN-inadequate tasks* (middle), and the *cluster tasks* (right). Start places are represented by grey circles, target places are represented by solid black circles, and black lines depict the optimal paths.

Navigation Tasks	Number of target places	Start place (target places)
cluster	4	3 (17,16,22,19), 15 (8,17,23,18)
	5	19 (20,15,14,7 ), 21 (19,20,15,14),
	6	4 (1,6,18,23,24,19), 25 (10,5,4,9,8,22)
	7	11 (18,24,10,9,4,3,8), 5 (2,1,6,7,19,24,20)
	8	23 (14,15,10,5,4,9,6,12), 24 (22,21,16,11,17,14,9,15)
	9	20 (9,10,5,4,3,8,17,18,23), 21 (6,1,2,7,8,3,4,20,24)
	control	16 (6,1,2,7,18,19,23), <b>1</b> (3,4,12,17,22,21,16,11), <b>3</b> (12,11,17,19,24,25,20,15,14)
NN-adequate	4	3 (11,22,19,10), 6 (17,24,15,4)
	5	11 (16,23,14,3), 21 (24,15,4,1,12)
	6	22 (23,19,15,8,2,16), 25 (14,9,8,2,6,16)
	7	5 (15,25,18,16,11,1,2), 24 (15,10,3,7,11,17,18)
	8	20 (19,24,22,12,6,2,4,9), 3 (8,12,11,22,23,19,20,15)
	9	25 (20,14,9,10,5,2,1,12,21), 23 (24,25,20,9,7,6,11,16,17)
	control	15 (19,24,22,16,12,1,4), 1 (4,10,14,19,22,16,12,7), 10 (4,8,2,1,11,16,18,24,20)
NN-inadequate	4	3 (16,23,25,13), 1 (13,10,19,21)
	5	21 (24, 14,13,3,2), 11 (21,13,10,4,7)
	6	23 (19,10,5,13,7,16), 4 (8,1,11,12,24,20)
	7	16 (12,13,19,25,9,8,1), 5 (9,20,13,18,22,16,2)
	8	1 (11,22,25,18,13,8,5,2), 3 (2,12,21,18,20,14,9,5)
	9	24 (18,21,12,7,6,1,3,5,19,24), 23 (19,14,10,5,8,2,6,16,22,23)
	control	15 (25,23,22,13,14,4,5), 6 (3,9,10,15,19,18,21,12), 25 (24,18,22,21,11,12,13,9,15,25)

Table 1: The table lists all navigation tasks of Experiment 1. The starting place is followed by the target places (in brackets). The numbers correspond to the place numbers in the schematized drawing of the experimental environment (see Figure 1).

#### 2.2.3 Control condition.

In the control condition, subjects solved 12 TSPs. Here, the target places themselves were marked with black markers, i.e. all the target locations were visually identifiable from the start position without the help of the shopping lists. In contrast to the experimental condition, here subjects did not have to identify, localize, and remember the target positions. By these means the contribution of these cognitive processes for subjects' performance when solving the TSP could be studied.

Before each trial, subjects were asked to close their eyes. They were led to the start place and were oriented towards the wall rather than towards the experimental setup. The experimenter now distributed the markers according to the specific TSP. Only afterwards subjects were allowed to turn around, now facing the experimental setup, and to start the trial. While half of the subjects performed the control condition before the experimental condition, the other half performed the control condition after the experimental condition.

### 2.2.4 Participants

24 subjects (12 females, 12 males, mean age: 22.88 years) participated in the experiment. They were mostly university students and were paid 8 Euro an hour.

### 2.2.5 Analysis

Subjects trajectories, i.e. the sequence of places visited, were recorded for each TSP and the length of this trajectory was calculated, assuming linear route segments between target points. For each navigation task also the shortest possible path was computed. By dividing the length of the traveled path by the length of the shortest possible path an *overshoot* value was obtained. By subtracting 1 and multiplying the result with 100 the overshoot in percent was obtained. An overshoot value of 100% therefore corresponded to a path with twice the length of the shortest possible path. Furthermore, the percentage of trials in which subjects actually found the shortest possible route was calculated (*found optimal route*). For each trial also the *start time*, i.e. the time from turning around the shopping route until starting to navigate was recorded.

The error bars of all barplots in this study display standard errors of the mean (s.e.m.).

## 2.3 Hypotheses and predictions

**General predictions.** It was expected that subjects' navigation performance, i.e. subjects' performance of finding the shortest possible route, decreased with increasing number of targets of the TSP. This expectation was supported by two considerations. First, the number of route alternatives that has to be considered in route planning increases with higher numbers of targets. Second, working memory load is higher if more targets have to be memorized and dealt with. At some point, it will not be possible to simultaneously hold the positions of all target places in working memory such that paths can not be planned taking all targets into account. Accordingly, as in the control condition, subjects did not have to identify and remember the positions of the target places based on the shopping list, it is expected that performance in the control condition clearly exceeded performance in the experimental condition.

The following list summarizes the predictions for the different types of navigation tasks designed to test the assumed path planning heuristics, i.e. the NN-strategy and the cluster strategy.

- 1. **NN-adequate tasks:** If subjects applied the NN-strategy, they should be led along the optimal route (see Figure 4).
- 2. **NN-inadequate tasks:** If subjects followed the NN-strategy, they should systematically fail to find the optimal paths on these routes (see Figure 4).
- 3. **Cluster tasks (NN-ambiguous):** The NN strategy did not make predictions for cluster-routes. The cluster strategy, however, states that subjects plan their routes such that they visit as many targets places as fast as possible (see Wiener et al., 2004), thus predicting that subjects first visit the large target cluster rather than the small target cluster.



Figure 4: Predictions of the NN strategy for the NN-adequate task (left) and NN-inadequate task (right) depicted in Figure 3.

### 2.4 Results

#### 2.4.1 Experimental Condition

**Overshoot** Subjects' overshoot performance when solving the TSPs was remarkably good. On average, overshoot in the experimental condition was 5.86%. Even for the most complex navigation tasks (9 targets), subjects produced less than 10% overshoot (see Figure 5). For three of the TSPs with 9 targets, the overshoot values for all 362880 path alternatives was exemplarily calculated (see histogram in Figure 6). Less than .04% of all path alternatives produced comparable or smaller overshoot values than subjects.

An ANOVA revealed a highly significant main effect of the number of targets (F=17.25, df=5, p<.001), while no main effect for the type of navigation task (F=1.57, df=2, p=.22) and no interaction (F=1.53, df=10, p=.13) were found. While no significant significant effect between the different types of navigation tasks was found, overshoot values in NN-adequate tasks showed the highest variance (RS-adequate:120.1, RS-inadequate: 100.38, cluster: 71.67). Subjects' overshoot increased with increasing number of targets (Pearson's product-moment correlation: r=.94, p<.01). In experimental block 1 subjects produced slightly higher overshoot values than in experimental block 2 (6.43% vs. 5.04%, paired t-test: t=2.29, df=23, p=.03). Overshoot performance did not differ between female and male subjects (6.71% v 4.76%, t-test: t=1.63, df=22, p=.12).

**Found Correct Route.** On average subjects found the shortest possible route in 47.3% of the trials. An ANOVA revealed a highly significant main effects for the number of targets (F=25.37, df=5, p<.001) and the type of navigation task (F=79.09, df=2, p<.001) as well as a significant interaction (F=6.88, df=10, p<.001). While a Pearson's product-moment correlation revealed only a marginally significant correlation between performance of finding the optimal route and the number of target places (r=-.80, p=.055), a highly significant difference was found between 'easy' tasks, i.e. tasks with 4-6 targets, and 'difficult' tasks, i.e. tasks with 7-9 targets (32.7% vs 61.6%, paired t-test: t=8.89, df=23, p<.001). Performance of finding the optimal route and male subjects (44.13% vs 50.87%, t-test: t=-1.43, df=22, p=.17)

Subjects' performance in finding the optimal route did not differ between cluster tasks and NN-inadequate tasks (34.28% vs 35.86%, paired t-test: t=.47, df=23, p=.64), but differed both, between cluster tasks and NN-adequate tasks (34.28% vs 72.32%, paired t-test: t=10.88, df=23, p<.001), and between NN-adequate tasks and NN-inadequate tasks (72.32% vs 35.86%, paired t-test: t=10.19, df=23, p<.001).

**Start time**. On average subjects' start time was 22.10 seconds. An ANOVA revealed a highly significant main effect for the number of targets (F=24.02, df=5, p<.001) while no main effect for type of navigation task (F=1.75, df=2, p=.19) and no interaction (F=1.21, df=10, p=.29) was found. Subjects' start time increased with increasing number of targets (Pearson's product-moment correlation: r=.95, p<.01). Start time did not differ between female and male subjects (23.2 sec v 20.9 sec, t-test: t=.63, df=22, p=.53).



Figure 5: Results of the experimental condition of Experiment 1.



Figure 6: Overshoot distribution for all 362880 path alternatives for 3 exemplary TSPs with 9 target places. Note that on navigation tasks with 9 target places, subjects on average produced just below 10% overshoot (marked by arrow). In all of these particular examples less than .04% of the 362880 paths generated overshoot values below 10% and less than .005% of the 362880 paths generated overshoot values below 4%.

	mean		Т	df	p-value
Shopping list	list A	list B			
overshoot in %	6.29	5.18	.89	22	.38
start time in seconds	21.56	22.64	.30	22	.77
found correct routes in %	44.71	50.28	1.16	22	.26
Configuration	config A	config B			
overshoot in %	5.18	6.29	.88	22	.38
start time in seconds	24.44	19.75	1.37	22	.19
found correct routes in %	49.10	45.89	.65	22	.52
Control condition	before	after			
overshoot in %	7.26	4.21	2.79	22	.01 *
start time in seconds	18.14	26.05	2.52	22	.02 *
found correct routes in %	42.46	52.53	2.26	22	.04 *

Table 2: Results of t-tests on the influence of the various control factors.

**Control parameters.** Neither the control parameters *shopping list*, i.e. the specific arrangement of the symbols on the shopping lists (see Section 2.2.2) nor *configuration*, i.e. the specific arrangement of the symbols in the environments (see Section 2.2.1) had a significant influence on overshoot performance, start time, or performance in finding the optimal route (see Table 2). However, systematic differences were found, depending on whether the control condition was carried out before or after the experimental condition (see Section 2.2.3). Subjects showed better navigation performance and longer start times when they conducted the control condition after the experimental condition (see Table 2).

**Predictions of the NN-algorithm.** The overshoot predictions when using a NN strategy were calculated for the different types of navigation tasks: for NN-adequate tasks it was obviously 0%, for NN-inadequate tasks it was 16.92% and for cluster-tasks it was 8.13% (Note that cluster tasks were NN-ambiguous: the NN strategy did not predict a single but multiple solutions as it was faced with one or multiple situations along the path in which the closest target places were equidistant from the current position). Subjects' overshoot for both, the cluster-tasks and the NN-inadequate tasks was significantly smaller than predicted by the NN-algorithm (cluster-tasks: 5.38% vs 8.13%, t-test: T=4.39, df=23, p<.001; NN-inadequate tasks: 6.60% vs 16.92%, t-test: T=10.56, df=23, p<.001).

**Correlations between subjects' start time and overshoot performance.** Subjects' mean start time was negatively correlated with subjects' overshoot performance (r=-.42, p=.04), demonstrating that subjects who took longer before initiating the trial showed better navigation performance.

**Cluster tasks.** On cluster tasks the target places were distributed in two distinct target clusters of unequal size. Overall, subjects showed a significant preference to first visit the large cluster (59.02% vs chance level [50%], t-test: t=3.09, df=23, p<.01).

#### 2.4.2 Control condition

**Overshoot.** On average subjects' overshoot in the control condition was 2.71%. An ANOVA revealed a main effect of the number of targets (F=6.41, df=2, p<.01) and type of navigation task (F=11.80, df=2, p<.01), while no significant interaction was found (F=1,59, df=4, p=.18). Overshoot increased with increasing number of targets. In contrast to the experimental condition, here overshoot for NN-adequate tasks was smaller than for NN-inadequate tasks (.96% vs 2.60%, paired t-test: t=2.71, df=23, p=.01) and for cluster-tasks (.96% vs 4.55%, paired t-test: t=4.48, df=23, p<.001).

**Found correct routes.** In the control condition subjects' found the optimal route in 62.5% of the trials. An ANOVA revealed significant main effects of the number of targets (F=4.38, df=2, p=.02) and type of navigation tasks (F=45.35, df=2, p<.001) as well as an significant interaction (F=6.01, df=4, p<.001). Performance of finding the optimal route was stronger for NN-adequate tasks than for both, NN-inadequate tasks (88.9% vs 68.1%, t-test: t=4.73, df=23, p<.001) and cluster tasks (88.9% vs 30.6%,



Figure 7: Results of the control condition of Experiment 1.

t-test: t=9.56, df=23, p<.001). In contrast to the experimental condition, subjects' performance for the NN-inadequate tasks was higher than for the cluster-tasks (68.1% vs 30.6%, t-test: t=4.94, df=23, p<.001).

**Start time**. On average subjects' start time in the control condition was 5.31 seconds. An ANOVA revealed a main effect for the type of navigation task (F=4.19, df=2, p=.02) while no main effect of the number of targets (F=2.08, df=2, p=.14), and no interaction was found (F=1.39, df=4, p=.24).

**Comparison experimental-condition and control-condition**. In the control condition, subjects were tested only on TSPs with 7, 8, and 9 targets. They showed a significant reduction both for overshoot and start time as compared to the experimental condition on routes with 7,8, and 9 targets (overshoot control: 2.71%, overshoot experiment: 8.72%, paired t-test: t=5.98, df=23, p<.001, start time control: 5.31 sec, start time experiment: 26.66 sec, paired t-test: t=9.13, df=23, p<.001). In the control condition, subjects' performance of finding the optimal path almost doubled as compared to navigation tasks with 7,8, and 9 targets in the experimental condition (control: 62.5%, experimental: 32.7%, t-test: t=8.31, Df=23, p<.001).

#### 2.5 Discussion

Overall, subjects overshoot performance when solving the TSPs was remarkably good. On average, subjects produced less than 6% overshoot. Even for the most complex navigation tasks with 9 targets, subjects produced ~10% overshoot in the experimental condition and only ~4% overshoot in the control condition. The fact that less than .04% of all path alternatives of TSPs with 9 targets produce overshoot values below 10%, and less than .005% produced overshoot values below 4% emphasizes subjects' good performance.

**Number of target places.** With increasing size of the TSPs, subjects performance for finding the optimal solution decreased while subjects' start time increased. These results were expected and have

been predicted for two reasons. First, simply because with increasing number of target places, the computational complexity of a TSP increases as more alternative solutions have to be taken into account. Second, because the task of localizing and simultaneously remembering the positions of all target places depicted on the 'shopping list' becomes more challenging as the number of target places increases, i.e. working memory load increases. A comparison of subjects' performance between the experimental condition and the control condition allows to differentiate between these two explanations in more detail. In the control condition the target places were directly marked in the environment. By these means, all spatial information required for planning a path was directly visually accessible, subjects did not have to localize and remember the target positions with the help of the 'shopping list'. As a result, subjects' performance in the control condition dramatically increased as compared to the experimental condition. For the TSPs with 7 to 9 targets, overshoot in the experimental condition was raised with respect to the control condition by a factor of about 3.3. These differences in overshoot performance between the control condition and the experimental condition can be interpreted as reflecting the impact of limitations of working memory for target positions on route planning difficulty and performance. However, even in the control condition, subjects performance decreases with increasing TSP size. It is suggested that this increase reflects the impact of increasing complexity of large TSPs as compared to small TSPs on route planning.

Further support for the crucial role of working memory for route planning comes from the observation that in the experimental condition both, overshoot and start time reach a plateau for TSPs with 7 targets and no further increase can be observed for larger TSPs. This is surprising at first glance, as the number of possible solutions for a TSP increases from 5040 for 7 target places to over 360000 for 9 target places. A possible explanation for this saturation is that in traveling salesman problems with up to 4-6 target places, subjects are able to hold the positions of all target places simultaneously in working memory allowing them to plan the path globally, i.e. to take into account all target positions during route planning. For TSPs with 7 or more targets, however, the task of holding the positions of all target places in working memory at the same time and generating a global route plan becomes too challenging. Thus, planning heuristics and strategies but seem to be stable across the different numbers of target places tested here. This interpretation is well in line with numerous findings demonstrating that working memory is a limited capacity system (e.g., Millner, 1956; Baddeley, 2003).

**Types of navigation tasks.** The most important result with respect to the route planning strategies applied was that subjects outperformed the NN-algorithm on NN-inadequate tasks and on cluster-routes, i.e. on average subjects found shorter paths than the NN-algorithm. This results clearly demonstrates that the NN-algorithm is not sufficient to explain human route planning and navigation behavior in TSPs.

On cluster-routes, the target places were distributed in two distinct target clusters of unequal size. Subjects showed a preference to first visit the large target cluster as compared to the small target cluster. This result is in line with results from Wiener et al. (2004) and supports the cluster-strategy, stating that subjects plan their routes in order to visit as many targets as fast as possible (for similar results in route planning in vervet monkeys see Cramer & Gallistel, 1997).

While for both, overshoot performance and start time, no significant differences could be found between the three types of navigation tasks, subjects' performance of finding the optimal route was almost twice as good for NN-adequate tasks than for NN-inadequate tasks and cluster-routes. This dissociation between overshoot performance and performance of finding the optimal route is surprising as it suggests that the errors made when navigating NN-adequate tasks were more severe than errors on NNinadequate tasks and cluster-routes. This interpretation is supported by the finding that the variance of the overshoot was highest for NN-adequate tasks.

**The control condition** Significant differences in behavior were observed between subjects that conducted the control condition before the experimental condition and subjects that conducted the control condition after the experimental condition. The latter group showed better navigation performance in the experimental condition. A possible explanation for this effect is based on the large difference in start time between control condition (~5 secs) and experimental condition (~26 secs). If subjects first experienced the control condition, in which they were very fast in planning the path and initiating the navigation, they might have felt rushed in the experimental condition, as it took far longer to plan or

prepare the route. Accordingly, they spent less time in localizing and remembering the target places and planning the route, which could either result in clearly sub-optimal route plans or in errors such as forgetting target places along the route. Such errors usually have to be corrected by taking large detours. This interpretation is supported by the finding that the group of subjects who first conducted the control condition showed shorter start time in the experimental condition as compared to the group that first conducted the experimental condition.

**Interviews with subjects.** Further insights into possible strategies and mechanisms applied came from informal interviews with the subjects after the experiments. Most subjects reported to have applied one of two strategies when faced with large TSPs. The first strategy was based on a regionalization of the environment. Subjects reported that they individually subdivided the 25 target places into a number of different regions. During route planning they then assigned the target places to be visited to these regions, allowing them to first plan a coarse route visiting the relevant regions. Such a coarse route is simple and easily remembered and the fine route plan can then be created by inserting close target places during navigation. The second strategy reported was to first select a subset of target places depicted on the shopping list according to some criteria, for example color. Then a coarse route plan was generated taking into account only the selected subset of target places. Again this route plan is simple and easily remembered and can be refined either before or during navigation by inserting the missing target places into the route.

Both of the reported navigation strategies follow essentially the same logic: they simplify the route planning task by applying a hierarchical planning scheme. First, a coarse route that is simple and easy to remember is generated on basis of an abstraction of the environment. This route plan is then refined during navigation by inserting target places.

Also results from the cluster-tasks, demonstrating that subjects preferred to first visit the large target cluster, can be interpreted in terms of such hierarchical planning schemes. If subjects understood that the target places were arranged in two distinct target clusters in these tasks, this information essentially represented an abstraction of the environment that reduced the number of targets to two: the large cluster and the small cluster. If one assumes that the attractivity of a target cluster or target region, respectively, correlates with the number of actual targets residing within that cluster (or region), a coarse route plan takes the form of: (1.) visit the large cluster, (2.) visit the small cluster, (3.) return to start place. Only when entering a target cluster a fine route plan has to be generated that determines the sequence in which single target places within that cluster are visited.

## 3 Experiment 2

## 3.1 Motivation

In Experiment 1, informal interviews with subjects suggested that one possible route planning and navigation strategy was based on a subjective regionalization of the environment (see Section 2.5). If based on regions, route planing becomes a hierarchical process. First, a coarse route plan is generated on the level of the regions exclusively. This route plan is then refined during navigation. Employing such a region-based planning scheme proposes that subjects first visit all target places in one region before moving on to the next region. This experiment was designed to empirically test this region-based planning strategy. For this, the environment was subdivided into different regions and subjects had to solve similar TSPs as in Experiment 1.

#### 3.2 Material and methods

#### 3.2.1 The experimental setup

The experimental setup was identical to Experiment 1, but differed in the arrangement of the symbols on the 5x5 grid. Symbols of equal color were neighboring each other, thus creating 5 clearly distinct



Figure 8: Experimental setup of Experiment 2. By arranging the symbols according to their color, 5 distinct regions were generated.

regions in the environment (see Figure 8). As in Experiment 1, two versions of the setup were created that only differed in the specific arrangement of the symbols in order to control for effects of the specific symbol-configuration. Half of the participants conducted the experiment in one configuration, the other half conducted the experiment in the alternative configuration.

## 3.2.2 The navigation tasks

The general navigation task, i.e. solving the TSP, was identical to the experimental condition of Experiment 1 (see Section 2.2.2). Subjects were given a *shopping list* for each navigation task, depicting the symbol defining the start place and the symbols defining the target places that had to be visited during navigation. To control for the influence of the specific sequence of symbols depicted on the *shopping list*, two versions of each shopping list were generated. Half of the participants received one version of the shopping lists, while the other half received the other version of the shopping lists.

As in Experiment 1, each subject solved 36 TSPs consisting of 4, 5, 6, 7, 8, or 9 target places (for a detailed description of all routes see Table 1). Each navigation task could be assigned to one of two types, the *Region-Strategy adequate tasks (RS-adequate tasks)* and the *Region-Strategy-inadequate tasks (RS-inadequate tasks)* (see Figure 9).

- 1. **Region-Strategy-adequate tasks (RS-adequate)**: If subjects employed a region-based planning strategy, i.e. if subjects first visited all targets in one region before moving on to the next region, it was possible to find the optimal route (see Figure 9). By these means, a region-based strategy could support finding the optimal route.
- 2. **Region-Strategy-inadequate tasks (RS-inadequate**): Employing a region-based planning strategy on RS-inadequate tasks will systematically lead to sub-optimal paths (see Figure 9). Furthermore, if routes are planned on the region level, the resulting paths should systematically cross fewer region boundaries as compared to the optimal path.

It is important to note that the configuration of start place and target places was always identical for pairs of two TSPs, one of which always was a navigation task from the RS-adequate group while the other routes always was of the RS-inadequate group. Any differences in behavior between the RS-adequate and the RS-inadequate group can thus be clearly attributed the the region characteristics of the navigation tasks.

#### 3.2.3 Participants

24 subjects (15 females, 9 males, mean age: 25.13 years) participated in the experiment. They were mostly university students and paid 8 Euro an hour.



Figure 9: Examples for a *RS-adequate task* (left) and a *RS-inadequate task* (right). Grey circles represent the starting places, solid black circles represent target places, and the black line represents the optimal solution. If routes are planned on the region level, solution to the RS-adequate TSPs should be optimal while solution found for RS-inadequate TSPs should be more prone to error. Note that these two routes are identical with respect to number of targets, spatial configuration of start and target places, and therefore also with respect to the optimal solution. The two different types of navigation tasks used in Experiment 2 are generated by simply mirroring and/or shifting the configuration of start and target places.

Navigation task	Number of target places	Start place (target places)
RS-adequate	4	6(9,5,10,19), 22(12,3,13,20), 6(7,13,15,17)
	5	22(12,2,4,5,18), 17(11,13,4,15,19), 25(13,7,4,5,9)
	6	13(22,20,14,9,5,7), 25(13,7,3,4,5,9), 2(17,21,22,23,19,18)
	7	9(8,1,12,11,21,17,18), 11(6,8,14,10,15,25,19), 1(16,12,13,18,24,14,9)
	8	4(9,14,19,23,16,17,13,2), 10(9,8,13,12,11,16,21,19), 20(18,17,21,16,7,13,9,10)
	9	15(9,3,7,13,12,11,16,22,24), 9(4,5,10,14,19,25,24,18,8), 10(4,8,2,1,16,13,18,24,20)
RS-inadequate	4	23(18,14,4,20), 15(12,6,11,22), 5(15,24,14,7)
	5	20(24,14,7,4,10), 10(18,22,11,6,12), 1(3,5,15,20,7)
	6	12(21,19,13,8,4,6), 20(17,21,16,11,7,12), 10(18,22,16,11,6,12)
	7	6(1,3,9,5,10,20,14), 21(6,12,13,8,4,14,19), 20(19,22,13,12,2,8,9)
	8	11(13,14,20,15,4,8,2,1), 25(20,15,14,9,4,3,2,18), 1(2,3,4,10,19,14,8,11)
	9	20(19,24,23,18,14,9,3,4,10), 6(2,8,4,5,20,13,18,22,16), 11(22,24,20,15,14,13,9,3,7)

Table 3: The table lists all routes of Experiment 2. The starting place is followed by the target places (in brackets). The numbers correspond to the place numbers in the schematized drawing of the experimental environment (see Figure 8).



Figure 10: Analysis on the region level (each region is is represented by a capital-letter): On the region level the left route (RS-adequate) is described as C-C-D-D-A-E-B-B-C while the right route (RS-inadequate) is described as D-A-A-B-B-C-B-E-D.

#### 3.2.4 Analysis

In addition to the analysis described in Section 2.2.5, in this experiment subjects' trajectories were also analyzed on the region level. For this, every navigation task was described both at the place level as well as on the region level: For example, the left route displayed in Figure 10 can be described on the place level as follows: 20-18-17-21-16-7-13-9-10. On the region level the route is represented as C-C-D-D-A-E-B-B-C. From this region representation, the number of region crossings was calculated for every traveled path as well as for all the corresponding optimal solution. Furthermore, by comparing the region-representation of a traveled route with the region-representation of the optimal route, error at the region level were analyzed independent from errors at the place level.

### 3.3 Predictions

Employing a region-based route planning strategy, i.e. first planning a coarse route at the region-level and refining the route only afterwards, will prevent subjects from finding the optimal solution when navigating RS-inadequate navigation tasks (see Section 3.2.2). It is therefore expected that subjects will show decreased performance on RS-inadequate tasks as compared to RS-adequate tasks.

More specifically, if subjects employed a region-based strategy, it was expected that they produce more errors on the region-level (see Section 3.2.4) when navigating RS-inadequate tasks as compared to RS-adequate tasks. On RS-inadequate tasks these errors on the region level should systematically lead to fewer region crossings as compared to the optimal solution.

## 3.4 Results

**Overshoot.** Average overshoot in Experiment 2 was 5.03%. An ANOVA revealed a highly significant main effect of the number of target places (F=6.89, df=5, p<.001), while no main effect for route type (F=2.34, df=1, p=.14), and no interaction was found (F=.14, df=5, p<.94). Generally, subjects' overshoot increased with increasing number of targets (Pearson's product-moment correlation: r=.98, p<.001). In experimental block 1 subjects produced higher overshoot values than in experimental block 2 (5.73% vs. 4.35%, paired t-test: t=3.21, df=23, p<.01). Overshoot performance did not differ between female and male subjects (4.76% vs. 5.47%, t-test: t=.95, df=22, p=.65).

**Found optimal routes.** On average, subjects found the optimal route in 29.85% of the trials. An ANOVA revealed a significant main effect for the number of target places (F=7.56, df=5, p<.001), a main effect for route type (F=22.98, df=1, p<.001), while no significant interaction was found (F=.70, df=5, p=.624). A Pearson's product-moment correlation did not reveal a significant correlation between performance of finding the optimal route and the number of target places (r=-.54, p=.27). Performance



Figure 11: Results of Experiment 2.



Figure 12: Results of the region based analysis of Experiment 2.

of finding the optimal route did not differ between female and male subjects (28.53% vs 32.26, t-test: t=.61, df=22, p=.55).

**Start time**. Average start time in Experiment 2 was 16.17 sec. An ANOVA revealed a highly significant main effect for the number of target places (F=8.86, df=5, p<.001), while no main effect for route type (F=.91, df=1, p=.35), and no interaction was found (F=.54, df=5, p<.75). Subjects' start time increased with increasing number of targets (Pearson's product-moment correlation: r=.98, p<.001). Start time did not differ between female and male subjects (14.78 sec vs 18.48 sec, t-test: t=.75, df=22, p=.47).

**Optimal region route.** An ANOVA revealed a significant main effect for the number of target places (F=3.87, df=5, p<.01), a highly significant main effect for route type (F=93.42, df=1, p<.001), as well as a significant interaction (F=9.24, df=5, p<.001). Subjects followed the optimal region route more often when navigating RS-adequate tasks than when navigating RS-inadequate tasks (55.02% vs 22.58%, paired t-test: t=10.04, df=23, p<.001, see Figure 12).

	mean		Т	df	p-value
Shopping list	list A	list B			
overshoot in %	4.37	5.69	1.09	22	.29
start time in seconds	15.15	17.19	.46	22	.65
found correct routes in %	28.13	31.70	.68	22	.50
Configuration	config A	config B			
overshoot in %	4.83	5.23	.32	22	.75
start time in seconds	16.31	16.03	.06	22	.95
found correct routes in %	28.28	31.58	.62	22	.54

Table 4: Results of t-tests on the influence of the control factors list and configuration.

**Region crossings.** When solving the RS-inadequate navigation tasks, subjects crossed less region boundaries than would have been expected for optimal solutions. On average, 4.11 region transitions were made on RS-inadequate tasks, which is a reduction by .61 from the expected 4.72 region transition for optimal solutions (4.11 vs 4.72, t-test: t=14.69, df=23, p<.001, see Figure12). On RS-adequate routes, on the other hand, on average, 4.37 region transitions were made, which is an increase of .17 from the expected 4.22 region transitions for optimal solutions (4.37 vs 4.22, t-test: t=6.36, df=23, p<.001, see Figure 12).

**Further control factors.** Neither of the control factors *shopping list* (see Section 3.2.2) and *configuration* (see Section 8) had a significant influence on overshoot performance, start time, or performance of finding the optimal route (see Table 4).

## 3.5 Discussion

With respect to overshoot performance, performance in finding the optimal route, and start time, results from Experiment 2 rendered a very similar picture as results from Experiment 1. Again, subjects showed remarkably good overshoot performance (~5%). Also, their performance in both, overshoot and finding the optimal solution decreased with increasing TSP size, while their start time increased.

This special purpose of this experiment was to test whether subjects employed a region-based planning strategy when faced with TSPs. Such a strategy states that during route planning first a coarse path is planned on the region-level that is then refined during navigation by including close target locations. To test this hypothesis two types of navigation tasks were generated, Region-Strategy-adequate tasks (RSadequate) and Region-Strategy-inadequate tasks (RS-inadequate). If routes are planned on the region level, solution to the RS-adequate tasks should be optimal while solution found for RS-inadequate tasks should be more prone to error, both on the region level as well as on the place level (see Section 3.2.2). It was therefore predicted that differences in overshoot performance between the route types were found (see Section 3.3). Contrary to this prediction, subjects' overshoot performance did not significantly differ between the two types of navigation tasks. However, performance in finding the optimal route was better when navigating RS-adequate tasks as compared to RS-inadequate tasks. This latter result was in line with the predictions. The dissociation between overshoot performance and performance of finding the optimal route between the two types of navigation tasks suggests that the errors made when navigating RS-adequate navigation tasks were more severe than errors on RS-inadequate tasks. It is interesting to note that a similar dissociation between overshoot performance and performance of finding the optimal route has already been observed in Experiment 1.

Although, contrary to the predictions, no overshoot difference was observed between RS-adequate tasks and RS-inadequate tasks, the use of a region-based planning strategy is not necessarily disproved. Subjects navigated sub-optimal paths in both types of navigation tasks. It is therefore possible that subjects employed a region-based planning strategy in both route types but the predicted performance differences were shadowed by non-specific errors. Furthermore, as also in Experiment 1 overshoot performance did not allow to differentiate between the different types of navigation tasks, it can be argued that in the current context the overshoot measure is not sensitive enough to distinguish between navigation strategies.

While the analysis of subjects' overshoot performance did not render a clear picture, the analysis of subjects' navigation behavior on the level of the regions revealed strong differences between the route

types. On the region level subjects followed the optimal route more often when navigating RS-adequate tasks than when navigating RS-inadequate tasks. Combined with the result, that on average subjects crossed fewer region boundaries on RS-inadequate tasks as compared to the optimal solution, demonstrated that subjects minimized the number of region crossings during route planning and navigation. Remember that on RS-inadequate tasks, employing a region-based strategy will not only result in sub-optimal paths, but also in paths that cross fewer region boundaries as compared to the optimal route. The analysis of navigation behavior on the region level therefore strongly suggests that subjects have employed a region-based planning strategy during route planning.

## 4 Conclusion

Planning a path covering multiple target locations is a complex and computationally expensive task. In this study, subjects were faced with path planning tasks in which they had to visit up to 9 target places. In such tasks more than 360 000 alternative paths for visiting all target places exist. Even though subjects had to first localize all target places from a so-called shopping list, on average they took less than 20 seconds before initiating their movement. Furthermore, on average they produced paths with less than 6% overshoot. While this performance was remarkably good, start time as well as navigation performance strongly suggest that, when faced with navigation tasks in which many target places had to be visited, subjects relied on planning heuristics, rather than computing and comparing all possible path-alternatives and selecting the optimal solution. The most simple heuristics to solve multi-target planning tasks is probably the well-known nearest neighbor algorithm (NN), in which a navigator is repeatedly visiting the closest non-visited target. The NN has therefore been suggested to be involved in animal and human navigation (e.g., Gärling & Gärling, 1988; Bures et al., 1992). Results of Experiment 1, however, showed that subjects were able to outperform the NN-algorithm, i.e. subjects found shorter paths than the NN-algorithm predicted. This result demonstrates that the NN-algorithm is not sufficient to explain human navigation behavior. Results of Experiment 2 suggest that subjects rather relied on a region-based planning heuristics. In such a region-based planning scheme, first a coarse path is planned on the level of the regions. This coarse route plan is then refined during navigation by the inclusion of close target places.

The outlined region-based planning strategy constitutes a hierarchical planning heuristic that reduces both, computational effort during route planning as well as memory load while resulting in reasonably short paths. During initial planning an abstraction of the environment is used. By these means not only the complexity of the environment is reduced but also target places residing in the same region are collapsed such that fewer target locations have to be taken into account during initial route planning. The resulting route plans are therefore simple and easy to remember. Following the classification introduced above (see Section 1), this first planning step resembles a global planning algorithm, in which the entire route is planned from the start to all goal locations. As the route plan was generated on the region level it is, however, coarse and not sufficient for actual navigation. The coarse route plan therefore has to be refined during navigation. Here several possibilities exist. For example, a simple local planning algorithm such as the NN-algorithm could be used for route planning and navigation within a given region.

Taken together, the presented experiments shed some light on how humans solve complex and computationally expensive navigation tasks. The general finding is that the complexity of the navigation tasks was broken down by applying fast but very efficient hierarchical planning schemes.

## 5 Acknowledgments

This work was supported by the DFG (Deutsche Forschungsgemeinschaft) MA 1038/9-1 and WI 2729/1-1. Special thanks to Dominik Seffer for his help during designing and carrying out the experiments.

## References

- Baddeley, A. (2003). Working memory: looking back and looking forward. *Nat Rev Neurosci*, **4**(10), 829–839.
- Bailenson, J. N., Shum, M. S., & Uttal, D. H. (1998). Road Climbing: Principles Governing Asymmetric Route Choices On Maps. *Journal of Environmental Psychology*, 18, 251–264.
- Bailenson, J. N., Shum, M. S., & Uttal, D. H. (2000). The initial segment strategy: A heuristic for route selection. *Memory & Cognition*, 28(2), 306–318.
- Bures, J., Buresova, O., & Nerad, L. (1992). Can rats solve a simple version of the traveling salesman problem?. *Behavioral Brain Research*, **52**(2), 133–142.
- Christenfeld, N. (1995). Choices from identical Options. *Psychological Science*, 6, 50–55.
- Cramer, A. E. & Gallistel, C. R. (1997). Vervet monkeys as travelling salesmen. *Nature*, **387**(6632), 464–464.
- Gallistel, C. R. & Cramer, A. E. (1996). Computations on metric maps in mammals: Getting oriented and choosing a multi-destination. *Journal of Experimental Biology*, **199**(1), 211–217.
- Gärling, T. & Gärling, E. (1988). Distance minimization in downtown pedestrian shopping. *Environment and Planning A*, **20**, 547–554.
- Gärling, T., Säisä, J., Böök, J., & Lindberg, E. (1986). The spatiotemporal sequencing of everyday activities in the large-scale environment. *Journal of Environmental Psychology*, **6**, 261–280.
- Golden, B., Bodin, L., Doyle, T., & Stewart, W. (1980). Approximate traveling salesman algorithms. *Operations Research*, **28**, 694–711.
- Golledge, R. (1995). Path selection and Route Preference in Human Navigation: A Progress Report. In A. Frank & W. Kuhn (Eds.), Spatial Information Theory: A Theoretical Basis for GIS (COSIT'95). Number 988 in Lecture Notes in Computer Science (pp. 207–222). Berlin, Springer.
- Graham, S. M., Joshi, A., & Pizlo, Z. (2000). The travelling sdalesman problem: A hierarchical model. *Memory & Cognition*, 28(7), 1191–1204.
- Kuipers, B., Tecuci, D., & Stankiewicz, B. (2003). The skeleton in the cognitive map: a computational and empirical exploration. *Environment and Behavior*, **35**(1), 80–106.
- MacGregor, J. N. & Ormerod, T. C. (1996). Human performance on the traveling salesman problem. *Perception and Psychophysics*, **58**, 527–539.
- MacGregor, J. N., Ormerod, T. C., & Chronicle, E. (2000). A model of human performance on the travelling salesperson problem. *Memory & Cognition*, 28(7), 1183–1190.
- MacGregor, J. N., Ormerod, T. C., & Chronicle, E. P. (1999). Spatial and contextual factors in human performance on the travelling salesperson problem. *Perception*, **28**(11), 1417–1427.
- MacGregor, J. N., Chronicle, E. P., & Ormerod, T. C. (2004). Convex hull or crossing avoidance? Solution heuristics in the traveling salesperson problem. *Memory & Cognition*, **32**(2), 260–270. Clinical Trial.
- Menzel, E. W. (1973). Chimpanzee spatial memory organization. Science, 182(4115), 943–945.
- Millner, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information.. *Psychological Review*, **63**, 81–97.
- Stern, E. & Leiser, D. (1988). Levels of spatial knowledge and urban travel modeling. *Geographical Analysis*, 20, 140–155.
- Van Rooij, I., Stege, U., & Schactman, A. (2003). Convex hull and tour crossings in the Euclidean traveling salesperson problem: implications for human performance studies. *Memory & Cognition*, **31**(2), 215– 220.

- Vickers, D., Bovet, P., Lee, M. D., & Hughes, P. (2003a). The perception of minimal structures: performance on open and closed versions of visually presented Euclidean travelling salesperson problems. *Perception*, **32**(7), 871–886.
- Vickers, D., Lee, M. D., Dry, M., & Hughes, P. (2003b). The roles of the convex hull and the number of potential intersections in performance on visually presented traveling salesperson problems. *Memory* & Cognition, 31(7), 1094–1104.
- Vickers, D., Lee, M. D., Dry, M., Hughes, P., & McMahon, J. A. (2006). The aesthetic appeal of minimal structures: judging the attractiveness of solutions to traveling salesperson problems. *Percept Psychophys*, 68(1), 32–42.
- Wiener, J. M. & Mallot, H. A. (2003). Fine-to-Coarse Route Planning and Navigation in Regionalized Environments. *Spatial Cognition and Computation*, **3**(4), 331 358.
- Wiener, J. M., Schnee, A., & Mallot, H. A. (2004). Use and Interaction of Navigation Strategies in Regionalized Environments. *Journal of Environmental Psychology*, **24**(4), 475 – 493.