08221 Abstracts Collection
Geometric Modeling
— Dagstuhl Seminar —

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Abstract. From May 26 to May 30 2008 the Dagstuhl Seminar 08221 "Geometric Modeling" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Geometry, engineering, volumetric modeling, computer graphics

Executive Summary

Geometric Modeling is an area drawing from computer science, mathematics, engineering, and the life sciences. It is concerned with the computer representation of objects as diverse as

– brain scans
– mathematical functions
– terrains
– airplane wings

and many more. The seminar succeeded in bringing together leading researchers to present and discuss radically different approaches to the challenge of modeling complex geometric phenomena on the computer. Acquisition, representation and analysis of 3-dimensional geometry call for the combination of technically complex and often interdisciplinary approaches that are grounded both in classical mathematics and computer science data structures and theory.

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1537

Planar rational quadratics and cubics: parametrization and shape control

Gudrun Albrecht (University of Valenciennes, FR)

This talk is concerned with planar rational curves of degree two and three, and addresses the following two issues:

In the case of rational quadratics a simple analytical solution to the problem of determining the optimal parametrization is given. Optimality is measured with respect to arc length by means of the L_2–norm. The presented result is based on a method of Farouki [1] and Jüttler [2], who solve the optimal parametrization problem analytically in the case of *polynomial* curves, but suggest a numerical procedure for *rational* curves. This is joint work with I. Cattiaux-Huillard and V. Hernandez-Mederos.

In the case of planar rational cubics the issue of determining inflection points and singularities is dealt with. Given a planar cubic in standard Bézier form based on the work of Sakai [3], the distribution of its characteristic points (inflection points, cusps, loops) is determined, depending not only on the position of the control points, but also on the variable two inner weights. This is joint work with J.P. Bécar and X. Xiang.

[1] R. T. Farouki, Optimal parameterizations, Computer Aided Geometric Design, 14, 153-168, 1997.

[2] B. Jüttler, A vegetarian approach to optimal parameterizations, Computer Aided Geometric Design, 14, 887-890, 1997.

[3] M. Sakai, Inflection points and singularities on planar rational cubic curve segments, Computer Aided Geometric Design, 16, 149-156, 1999.

Hermite Point Set Surfaces

Marc Alexa (TU Berlin, DE)

Point Set Surfaces define a (typically) manifold surface from a set of scattered points. The definition involves weighted centroids and a gradient field. The data points are interpolated if singular weight functions are used to define the centroids. While this way of deriving an interpolatory scheme appears natural we show that it has two deficiencies: convexity of the input is not preserved and the extension to Hermite data is numerically unstable.

We present a generalization of the standard scheme that we call Hermite Point Set Surface. It allows interpolating given normal constraints in a stable way. It also yields an intuitive parameter for shape control and preserves convexity in most situations. The analysis of derivatives also leads to a more natural way to define normals, in case they are not supplied with the point data. We conclude by comparing to similar surface definitions.

Keywords: Point-based modeling, interpolation, Hermite data

Joint work of: Alexa, Marc; Adamson, Anders

Angle-based harmonic interpolation

Alexander Belyaev (Heriot-Watt University - Edinburgh, GB)

Given a convex polygon and a function defined at the vertices of the polygon, we consider the problem of harmonic interpolation inside the polygon. First a support polygon for the initial polygon is created. Then, since the visibility angle of a segment is a harmonic function (a double layer potential), the visibility angles of the sides of the support polygon are used as building blocks of the proposed interpolation scheme. The 3D extension of the developed interpolation scheme is straightforward.

We also consider a sequence of polygons such that each next polygon in the sequence is constructed as the support polygon for the previous polygon. This leads to a new simple flow acting on convex polygons. We show that the flow quickly evolves an arbitrary convex N-gon into a regular N-gon.

Finally we study properties of Möbius-invariant natural neighbor interpolation of Marshall Bern and David Eppstein.

Keywords: Angle-based interpolation

Massive mesh hole repair minimizing user intervention

Pere Brunet (TU of Catalonia - Barcelona, ES)

The talk will present the algorithms that have been used for the automatic detection and repair of an important number of holes (about twenty thousand holes) in the digital model of a large monument from the XII Century. The total size of the triangular mesh is in the vicinity of 173 M-triangles.

Keywords: Mesh repair, gigantic models, smoothing

Joint work of: Brunet, Pere; Chica, Antoni; Navazo, Isabel; Vinacua, Alvar

Efficient parallel thinning on the bcc lattice

Guido Brunnett (TU Chemnitz, DE)

We consider the computation of 1d skeletons for 3d objects. These can be used for applications as 3d object classification, retrieval, decomposition and deformation. The skeletons are computed by a thinning approach that performs on the body-centered cubic lattice. The Voronoi cells associated with this lattice are truncated octahedrons. Complexes of such bcc-voxels always form a bounded 3d manifold. We will show that bcc-voxels satisfying the BM condition can be removed without changing the topology of a given manifold. We will also introduce conditions that allow the simultanous removal of bcc-voxels without change in the topology. Based on these conditions we will formulate an algorithm for parallel directional thinning on the bcc grid.

Keywords: Skeletons, thinning, bcc lattice

Joint work of: Brunnett, Guido; Brunner, David

Towards NURBS-Compatible Subdivision

Tom Cashman (University of Cambridge, GB)

Non-Uniform Rational B-Spline (NURBS) surfaces can be non-uniform and defined for any degree, but existing subdivision surfaces are either uniform or of fixed degree. The resulting incompatibility forms a barrier to the adoption of subdivision for Computer-Aided Design. In this talk I will discuss our work towards a superset of NURBS which can handle irregularities in the control mesh. We call this target NURBS-compatible subdivision, as the resulting scheme would be able to represent any existing NURBS patch exactly. I will outline our strategy, discuss work which is already complete, and look at the challenges which lie ahead.

Joint work of: Cashman, Tom; Dodgson, Neil; Sabin, Malcolm

Computing Sigularities of Rational Ruled Surfaces

Falai Chen (Univ. of Science & Technology of China - Anhui, CA)

An algorithm is presented to compute the self-intersection curves of a rational ruled surface based on the theory of μ-bases. The algorithm starts by constructing the principal subresultants of a μ-basis of the rational ruled surface. The principal subresultant coefficients provides the information about not only the parametric loci of the self-intersection curves, but also the orders of the self-intersection curves. Based on this observation, an efficient algorithm is provided to compute the parametric locus of the self-intersection curves as well as their corresponding orders. The isolated singular points of the rational ruled surface are also computed.

Keywords: Ruled surface, self-intersection curve, singular point, μ-basis, moving plane, subresultant

Surface Self-Intersection Computation via Algebraic Decomposition

Gershon Elber (Technion - Haifa, IL)

This work draws upon a recent result on self-intersection detection and elimination for planar curves, which attempted at eliminating redundant algebraic components. We extend this result to surfaces and bivariate functions. An algebraic decomposition is presented that reformulates the self-intersection problem using an alternative set of constraints, while removing the redundant components. Extensions to higher dimensions are also briefly discussed.

Keywords: Surface self-intersection;multivariate polynomial constraints; algebraic decomposition;

Joint work of: Elber, Gershon; Grandine, Thomas; Kim, Myung Soo

An Analogue of the Lane-Riesenfeld Subdivision Algorithm for B-splines with Non-Uniform Knots

Ron Goldman (Rice University - Houston, US)

The Lane-Riesenfeld knot insertion algorithm for uniform B-splines is one of the prime examples of a subdivision procedure. The Lane-Riesenfeld algorithm simultaneously inserts one new knot between each consecutive pair of the original knots, but unlike Boehm's knot insertion algorithm or the Oslo algorithm which are valid for arbitrary knot sequences, the Lane-Riesenfeld algorithm works only for uniformly spaced knots. Here we shall present a new knot insertion algorithm that just like the Lane-Riesenfeld algorithm simultaneously inserts one new knot between every consecutive pair of the original knots, but is valid as well for arbitrarily knot sequences. Using blossoming, we provide a simple proof of the validity of our algorithm.

Joint work of: Goldman, Ron; Schaefer, Scott

A Global/Local Approach to 3D Mesh Parameterization

Craig Gotsman (Technion - Haifa, IL)

Inspired by recent methods developed for 3D mesh deformation, we present a simple method for mesh parameterization.

The method includes conformal and as-rigid-as-possible parameterization and is achieved through an iterative algorithm incorporating both local and global processing of the mesh elements.

Solid Modeling Applications of the Assignment Problem

Thomas Grandine (The Boeing Company - Seattle, US)

In the ongoing attempt to increase the level of automation of geometry and solid modeling processing, the assignment problem has proven to be a remarkably effective way to handle a large variety of matching and sequencing problems. This talk will review two important Boeing applications that make effective and extensive use of the assignment problem.

The geometry of the Moineau pump

Jens Gravesen (Technical University of Denmark, DK)

The Moineau pump was invented in 1931 by the French engineer René Moineau and exhibits an intriguing geometry. The original design is based on hypo- and epi-cycloids and all except one design has either cusps or less severe inflexion points with infinite curvature. By using the support function to represent planar curves it is possible to make an explicit analysis of a general design and we can show that points of infinite curvature are unavoidable.

Keywords: Moineau pump, envelope, support function

Smoothness of the support function representation

Jens Gravesen (Technical University of Denmark, DK)

By considering a real function h on the unit sphere, S^n as the support function for an n dimensional hyper-surface, it is possible to write down the inverse Gauss map, $\sigma : S^n \to \Sigma \subset \mathbb{R}^{n+1}$. Explicitly, $\sigma(x) = h(x)x + \nabla_{S^n} h(x)$, for $x \in \S^n$, where ∇_{S^n} is the intrinsic gradient on S^n.

If h is a C^k function then σ is C^{k-1}, but if $k \geq 2$ then the surface Σ is C^k. The case $k = 1$ is more involved, in order to make the same proof work it is among other things necessary to assume the existence of tangent spaces.

Here we give an example of a C^1 function on S^1 such that the corresponding planar curve is without a tangent in a dense set, it is in particular not a C^1-curve.

Keywords: Support function, smoothness

Interpolation of irregular meshes with bicubic G1 Bezier surfaces

Stefanie Hahmann (University of Grenoble, FR)

We consider the problem of creating a smooth parametric polynomial surface interpolating the vertices of an irregular quadrilateral mesh.

Surfaces of any topological type can be designed. They are overall tangent plane continuous without any singular points and without any singular parameterization.

In particular we will show that using a four-split of the parameter domain solves the problem of joining an arbitrary number of patches together at a common vertex with G1-continuity, known as the vertex consistency problem.

The resulting surface consists of a small number of very low degree surface patches, bi-cubic tensor product Bezier patches.

Explicit formulas of the Bezier control points are derived. Several degrees of freedom are available.

They can either be used as additional design parameters or they can be used for shape optimizations.

Keywords: G1 continuity, Bezier surfaces, interpolation, arbitrary topology

Joint work of: Hahmann, Stefanie; Bonneau, Georges-Pierre

Geometric Data Representations for Simulation and Visualization

Martin Hering-Bertram (Fraunhofer ITWM - Kaiserslautern, DE)

We present two simulation applications where geometric data representations play an important role. The first application is concerned with the simulation of medication applications to the inner ear. Here, a tetrahedral mesh obtained from segmenting the inner ear of a guinea pig is used to simulate diffusion and clearance of medication. In order to identify important parameters like transfer coefficients, the complexity of this model is reduced by unwrapping the ear and translating the simulation problem from three into one dimension.

The second application is concerned with wave based acoustics where the solution is represented by a combination of so called eigenmodes which are complex-valued pressure fields corresponding to resonance frequencies of a room geometry with absorption coefficients. The problem is to geometrically represent these eigenmodes in order to accelerate the sound field synthesis.

Both applications reflect work in progress and show the benefits of sophisticated data representations in scientific applications.

Keywords: Geometric modeling, simulation algorithms, scientific visualization

Curve subdivision with control of the arc-length

Victoria Hernandez-Mederos (ICIMAF - La Habana, SAM)

In this talk we present a new interpolatory subdivision method with control of the arc length of the limit curve. We prove that under certain conditions the scheme converges and the subdivision curve is continuous. Additionally we show that the length of segment of the polygonal subdivision curve between two consecutive points of the starting polygon is proportional, with the same proportionality factor, to the length of the corresponding edge. A bound for the Hausdorff between the limit curve and the starting polygon is also obtained.

Keywords: Subdivision curve arc-length

Joint work of: Hernandez-Mederos, Victoria; Ivrissimtzis, I.; Estrada, J.; Morales, S.

Designing Subdivision Schemes with High Approximation Orders

Kai Hormann (TU Clausthal, DE)

This talk is about some new insights regarding the reproduction of polynomial functions by a linear, binary, uniform, and stationary subdivision scheme.

While the necessary and sufficient conditions for such a scheme to *generate* polynomials of a certain degree as limit functions are well understood, little is known about polynomial *reproduction* in the sense that taking uniformly spaced samples from a polynomial as the initial data of the subdivision scheme yields the same polynomial in the limit. Nevertheless, this property is very important as it is directly connected to the approximation order of the subdivision scheme.

We present necessary and sufficient conditions for a subdivision scheme to have polynomial reproduction of degree d and thus approximation order $d+1$. Our conditions are partly algebraic and easy to check by considering the symbol of a subdivision scheme, but also relate to the parameterization of the scheme. Moreover, we show how they can be used to design subdivision schemes with a desired approximation order and minimal support.

Keywords: Subdivision, polynomial reproduction, approximation order

Joint work of: Hormann, Kai; Dyn, Nira

Fairing Wireframes in Industrial Surface Design

Shi-Min Hu (Tsinghua University Beijing, CA)

Wireframe is a modeling tool widely used in industrial geometric design.

The term wireframe refers to two sets of curves, with the property that each curve from one set intersects with each curve from the other set.

Akin to the u-, v-isocurves in a tensor-product surface, the two sets of curves in a wireframe span an underlying surface. In many industrial design activities, wireframes are usually set up and adjusted by the designers before the whole surfaces are reconstructed. For adjustment, the fairness of wireframe has a direct influence on the quality of the underlying surface. Wireframe fairing is significantly different from fairing individual curves in that intersections should be preserved and kept in the same order. In this paper, we first present a technique for wireframe fairing by fixing the parameters during fairing. The limitation of fixed parameters is further released by an iterative gradient descent optimization method with step-size control. Experimental results show that our solution is efficient, and produces reasonably fairing results of the wireframes.

Keywords: Wireframe fairing, industrial surface design

Surface reconstruction from cross-section curves

Tao Ju (Washington University, US)

Building surfaces from cross-section curves has wide applications including bio-medical modeling. Existig work in this area has mostly focused on connecting simple closed curves on parallel cross-sections.

Here we consider the more general problem where input data may lie on non-parallel cross-sections and consist of curve networks that represent the segmentation of the underlying object by different material or tissue types (e.g., skin, muscle, bone, etc.) on each cross-section. The desired output is a surface network that models both the exterior surface and the internal partitioning of the object. We will show how a well-known approach for surface reconstruction from parallel, closed curves can be adapted and extended to handle these general cases and still guarantees to produce a closed, interpolating surface network.

Keywords: Surface reconstruction, contour interpolation, cross-section curves

Joint work of: Liu, Lu; Bajaj, Chandrajit; Deasy, Joe; Low, Dan; Ju, Tao

Full Paper:
http://www.cse.wustl.edu/∼taoju/index.htm#eg08

See also: LIU, L., BAJAJ, C., DEASY, J., LOW, D., AND JU, T. 2008. Surface reconstruction from non-parallel curve networks. Eurographics(Computer Graphics Forum) 27, 2, 155–164.

Geometric computing with arcs and biarcs

Bert Jüttler (University of Linz, AT)

Arc splines are a very interesting representation of curves, since they offer several advantages. Various geometric queries can be answered exactly (e.g., closest point computation or the evaluation of arc-length). Also, arcs can be used as geometric primitives in algorithms of computational geometry. The talk will briefly address three topics: approximation of point clouds by spatial arc splines, medial axis computation of planar free-form shapes, and the approximation properties of biarc constructions.

Controlling torsion sign

Panagiotis Kaklis (National TU - Athens, GR)

We present a method for computing the domain, where a control point is free to move so that the corresponding spatial curve is regular and of constant sign of torsion along a subinterval of its parametric domain of definition. The approach encompasses all curve representations that adopt the *control-point paradigm* and is illustrated for spatial Bézier and B-spline curves.

Keywords: Spatial curve, torsion, shape

Joint work of: Karousos, Manolis; Ginnis, Alexandros; Kaklis, Panagiotis

Low degree curvature continuous multisided surfaces

Kestutis Karciauskas (Vilnius University, LT)

A new approach is presented for constructing low degree finite curvature continuous fillings of multisided holes in B-spline surfaces. This method allows to get fair curvature continuous patchworks even of bidegree 5. New schemes are discussed in a light of interplay: degree, number of patches, quality.

Another part of discussion is a natural by-product of this method – fair "almost" curvature continuous surfaces.

Though formally they are only G^1, their quality is similar to that of G^2 siblings.

Keywords: Curvature continuity, multisided surface

Joint work of: Karciauskas, Kestutis; Peters, Jörg

Approximation of Implicit Surfaces by Local and Singularity Free G1 Triangular Spline Surfaces

Tae-Wan Kim (Seoul National University, KR)

We propose a simple and efficient method for the approximation of implicit surface by G^1 triangular spline surface. Compared with the polygonizaion technique, the proposed method employs piecewise polynomials of high degree, archives G^1 continuity and is capable of interpolating the position, normal, and normal curvature at each vertex of an underlying base mesh. To satisfy vertex enclosure constraints, we develop a new scheme to construct a C^2 consistent boundary curves network which is based on the geometric Hermite interpolation of normal curvatures. By carefully choosing the degrees of the scalar weight functions, boundary Bézier curves and triangular Bézier patches, we achieve a local and singularity free algorithm for constructing a G^1 triangular spline surface of arbitrary topological type. This method achieves high precision at low computational cost, and involves only local and linear solvers which leads to a straightforward implementation. Analyses of freedom and solvability are provided, and numerical experiments demonstrate the high performance of algorithms and the visual quality of results.

Keywords: G^1 continuity, Geometric Hermite interpolation, Boundary curves network, Equality constrained optimization, Vertex enclosure constraint

Joint work of: Kim, Tae-Wan; Tong, Wei-hua

Green Coordinates

Yaron Lipman (Tel Aviv University, IL)

We introduce Green Coordinates for closed polyhedral cages. The coordinates are motivated by Green's third integral identity and respect both the vertices position and faces orientation of the cage.

We show that Green Coordinates lead to space deformations with a shape-preserving property. In particular, in 2D they induce conformal mappings, and extend naturally to quasi-conformal mappings in 3D. In both cases we derive closed-form expressions for the coordinates, yielding a simple and fast algorithm for cage-based space deformation. We compare the performance of Green Coordinates with those of Mean Value Coordinates and Harmonic Coordinates and show that the advantage of the shape-preserving property is not achieved at the expense of speed or simplicity. We also show that the new coordinates extend the mapping in a natural analytic manner to the exterior of the cage, allowing the employment of partial cages.

Fitting Curves and Surfaces by Geometric Algorithms

Takashi Maekawa (Yokohama National University, JP)

We introduce methods to fit a set of data points by curves/surfaces, which are defined by control points, using geometric algorithms.

Our algorithm iteratively updates the control mesh in a global manner based on a simple point-curve/surface distance computation followed by translations of the control vertices along the displacement vectors. The main advantages of our approach compared to existing surface fitting methods are simplicity, generality, and speed for some cases. We also present method to interpolate a set of data points as well as unit tangent vectors or unit normal vectors at the data points by means of B-spline curve interpolation technique using geometric algorithms. We demonstrate our technique with a variety of complex examples.

Keywords: Fitting, intyerpolation, approximation, geometric algorithm

See also: T. Maekawa, Y. Matsumoto and K. Namiki, "Interpolation by Geometric Algorithm", Computer-

Hermite Polynomial Least Interpolation

Stephen Mann (University of Waterloo, CA)

In this talk, I will discuss multivariate Hermite interpolation, interpolating values and derivative values at scattered points. I will leverage the Least interpolant introduced by Amos Ron and Carl de Boor in the 1990s, discussing both computational and theoretical aspects of this problem.

Joint work of: Mann, Stephen; Haller, Kirk

Subdivision methods for geometric processing of semi-algebraic sets

Bernard Mourrain (INRIA - Sophia Antipolis, FR)

Subdivision methods are powerful techniques that can be used to approximate isolated solutions of non-linear equations.

In this talk, we will discuss an extension of this approach to the treatment of curves, surfaces, volumes defined by polynomial equalities or inequalities. Regularity criteria which allow to determine the topology structure inside a cell from information on its boundary are used. We will present some mathematical ingredients of this approach (topological degree, Whitney stratification) and show some applications to arrangement computation in 2D and 3D, Voronoi diagram of parametrised curves.

Joint work of: L. Alberti and J. Wintz

PNG1 Triangles

Kerstin Müller (TU Kaiserslautern, DE)

The new generation of graphics hardware released with DirectX 10 offers the functionality to generate new triangles from an input triangle and its restricted neighborhood in the geometry shader.

In our talk we present our new PNG1 surface patch that is designed for the efficient use of this new functionality.

PNG1 patches are defined over a triangle with its vertex normals and the three edge neighbor triangles which are available in the geometry shader.

By blending two cubic Bt'ezier patches for each triangle edge we get a surface patch that is G1 continuous with its three neighboring patches.

Special Features such as creases are also available with PNG1 patches.

The resulting surface is especially suitable for efficient, high-quality tessellation on the GPU. We implemented the PNG1 as a plugin for Autodesk Maya and we will show some examples made with this tool.

Joint work of: Hagen, Hans; Müller, Kerstin; Fünfzig, Christoph; Hansford, Dianne; Farin, Gerald

Skinning T-spline Surfaces

Ahmad Nasri (American University of Beirut, LB)

T-spline surfaces are B-spline surfaces which could be defined by control meshes with T-junction. In this talk, we present an approach for generating T-spline surfaces through a set of cross-sectional curves defined over non compatible knot vectors. The method provides a solution to the open problem of lofting where the cross sections do not have to be defined over the same knot vectors.

Keywords: T-spline, Skinning, Lofting, NURBS.

Joint work of: Nasri, Ahmad; Sinno, Khaled; Zhang, J.

Adaptive, Implicit Modeling of Urban Terrain Noisy Point Cloud Data

Gregory M. Nielson (ASU - Tempe, US)

We describe some work in progress on a project focused on the mathematical and geometric modeling of scanner data (e.g. LIDAR) which has been obtained from urban terrains. The methods that are being developed are based upon the concepts of adaptively refined implicit models. There are special and particular advantages to using implicit models for this type of geometry including the ease of performing Boolean operations (union and intersection) and creating multiresolution models. The field functions for the implict models are selected on the basis of efficient compact models that can replicate complicated geometry and also faithfully reproduce sharp detailed features and artifacts. In addition some new, novel improved methods for estimating normal vectors is described which improve the quality and efficiency of the adaptive fitting process

Keywords: Adaptive, Point Cloud, Scattered Data, Modeling

Partial elimination in Constraint Solving

Jörg Peters (University of Florida, US)

Certain collections of rigid bodies allow a provably efficient partial elimination of distance and incidence constraints based on graph theory and rational parameterization.

The characterization of such a collection (and what goes wrong if the characterization is not met) is the key point to be explained.

Joint work of: Peters, Jörg; Sitharam, Meera

Full Paper:
http://www.cise.ufl.edu/research/SurfLab/papers.shtml

Topology During Approximation of Manifolds

Thomas J. Peters (Univ. of Connecticut - Storrs, US)

There is a long tradition in numerical analysis of approximating manifolds with upper bounds on the Hausdorff distance between the original manifold and its approximant. Increasing attention is being given to additional constraints that ensure that the approximant will also be topologically equivalent to the original manifold. An argument is given for using ambient isotopy as the topological equivalence relation, particularly for applications in animation and visualization, specifically for temporal anti-aliasing. Then, recent algorithms and open issues will be presented.

Joint work of: K. Abe, K. E. Jordan, L. E. Miller, E. L. F. Moore and A. C. Russell

Partial funding for T.J. Peters was from NSF grant CCF 0429477, from a 2005 IBM Faculty Award and from Kerner Graphics.

All statements here are the responsibility of the author, not of any of the listed funding sources.

Topology of Geometry in Motion

Thomas J. Peters (Univ. of Connecticut - Storrs, US)

There is a long tradition in numerical analysis of approximating manifolds with upper bounds on the Hausdorff distance between the original manifold and its approximant. Increasing attention is being given to additional constraints that ensure that the approximant will also be topologically equivalent to the original manifold. An argument is given for using ambient isotopy as the topological equivalence relation, particularly for applications in animation and visualization, specifically for temporal anti-aliasing. Then, recent algorithms and open issues will be presented.

The talk reports on joint work with K. Abe, K. E. Jordan, L. E. Miller, E. L. F. Moore and A. C. Russell.

Partial funding for T.J. Peters was from NSF grant CCF 0429477, from a 2005 IBM Faculty Award and from Kerner Graphics.

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Keywords: Computational topology, isotopy, computer graphics

Curved Folding

Helmut Pottmann (TU Wien, AT)

Fascinating and elegant shapes may be folded from a single planar sheet of material without stretching, tearing or cutting, if one incorporates curved folds into the design. Employing discrete differential geometric properties of developable surfaces we present an optimization-based computational framework for design and digital reconstruction of surfaces which can be produced by curved folding.

Our work not only contributes to applications in architecture and industrial design, but it also provides a new way to study the complex and largely unexplored phenomena arising in curved folding.

Keywords: Curved folding, developable surfaces

Joint work of: Pottmann, Helmut; Kilian, Martin; Flöry, Simon; Chen, Zhonggui; Mitra, Niloy; Sheffer, Alla

Analysis and Construction of Subdivision Algorithms

Ulrich Reif (TU Darmstadt, DE)

In this talk, we outline some less well known facts about subdivision algorithms.

First, we revisit the seemingly well-known argument concerning the fact that a dominant eigenvalue 1 is necessary and sufficient for a convergent scheme.

Second, we provide an easy-to-verify criterion for the injectivity of the characteristic ring.

Third, we introduce the concept of the embedded Weingarten map, which is geared to the special needs of a shape analysis of subdivision surfaces.

Fourth, we suggest a general framework for the construction of C^2-algorithms, which generalizes the concept of guided subdivision.

Keywords: Subdivision surface, subdivision algorithm

Dynamic Geometric Computation of Interacting Models

Rich Riesenfeld (University of Utah, US)

In many diverse applications involving design and manipulation of free-form geometric modeling, it is important to compute continuously and robustly the precise interactions among objects undergoing dynamic deformations over a period of time. We transform the problem into a computation on a higher dimensional manifold where singularity theory is used to develop a rigorous framework for solving it in a robust manner, one that assures identifying all points of abrupt topological transitions.

Keywords: Deformations, intersection, B-splines

Joint work of: Riesenfeld, Richard; Chen, Xianming; Cohen, Elaine; Damon, James

A New Method for Sketching a Solid Model in a CAD System

Nikolaos S. Sapidis (Univ. of the Aegean, GR)

The subject of this paper is automatic construction of a Polyhedron (solid model) from a Single plane Sketch (PSS), where a plane sketch is a linear sketch without hidden lines. Existing PSS techniques view PSS as an Image-Processing or Computer-Vision problem, while the present research is based heavily on Graph Theory, Solid Modeling and Euclidean Geometry. A comprehensive PSS methodology is derived, whose preliminary results on standard examples are shown to be satisfactory.

Joint work of: Sofia Kyratzi and Nickolas Sapidis (: speaker)

Manifold Dual Contouring

Scott Schaefer (Texas A&M University, US)

Dual Contouring is a feature-preserving iso-surfacing method that extracts crack-free surfaces from both uniform and adaptive octree grids. We present an extension of Dual Contouring that further guarantees that the mesh generated is a manifold even under adaptive simplification. Our main contribution is an octree-based, topology-preserving vertex clustering algorithm for adaptive contouring. The contoured surface generated by our method contains only manifold vertices and edges, preserves sharp features, and possesses much better adaptivity than those generated by other iso-surfacing methods under topologically safe simplification.

Joint work of: Schaefer, Scott; Ju, Tao; Warren, Joe

Full Paper:
http://faculty.cs.tamu.edu/schaefer/research

Scattered Data Fitting on Manifolds

Larry Schumaker (Vanderbilt University - Nashville, US)

Suppose D is a C^2 manifold in $\mathbb{R}^3$, and that f is an unknown scalar-valued function defined on D.

We consider the problem of constructing an approximation s to f based on a given set of measurements $\{f(u_i)\}_{i=1}^n$ at a set of scattered data points $\{u_i\}_{i=1}^n$ lying on D. This problem arises in many applications, and has been studied in numerous papers. Here we discuss a method based on projection of the data into tangent planes, coupled with appropriate bivariate spline interpolation or approximation methods. Using Powell-Sabin splines, we show that the resulting method produces a C^1 surface, and give explicit error bounds for smooth functions f. Several computational examples are included.

Keywords: Scattered data fitting, manifolds

Isosurface Stuffing: Fast Tetrahedral Meshes with Good Dihedral Angles

Jonathan Shewchuk (Univ. of California - Berkeley, US)

The isosurface stuffing algorithm fills an isosurface with a uniformly sized tetrahedral mesh whose dihedral angles are bounded between $10.7°$ and $165°$. All vertices on the boundary of the mesh lie on the isosurface. The algorithm is whip fast, numerically robust, and easy to implement because, like Marching Cubes, it generates tetrahedra from a small set of precomputed stencils. A variant of the algorithm creates a mesh with internal grading: on the boundary, where high resolution is generally desired, the elements are fine and uniformly sized, and in the interior they may be coarser and vary in size. Isosurface stuffing is the first algorithm that simultaneously copes with boundaries of complex shape and rigorously guarantees the suitability of its tetrahedra for finite element methods. Our angle bounds are guaranteed by a computer-assisted proof. We illustrate the use of isosurface stuffing for dynamic remeshing in a fluid simulation with moving liquid surfaces.

Keywords: Guaranteed quality tetrahedral mesh generation, isosurface stuffing

Joint work of: Shewchuk, Jonathan; Labelle, François

Full Paper:
http://www.cs.cmu.edu/∼jrs/jrspapers.html#stuffing

Fragment based Mesh Segmentation for Reverse Engineering

Hiromasa Suzuki (University of Tokyo, JP)

Recently reverse engineering applications have extended from style design to other design/manufacturing processes where they have faced new requirements. We are interested in reverse engineering of functional parts to generate their CAD models to be used for generating FEM meshes. Some typical examples are a cylinder head of an automotive engine and a transmission case. In these applications, high quality (Class-A) surfaces are not usually required, but manual operation must be minimized for shortening the product development time. In my talk, I will introduce our practical development with industry partners aiming at fully automatic reverse engineering system and in particular its core of a heuristic method for segmenting meshes of such functional parts. This method uses region growing based on VSA (Variational Shape Approximation) to first generate many small regions (fragments) which are then merged to form segments for three types of surfaces, primitive surfaces, base surfaces and fillets.

Keywords: Reverse engineering, mesh segmentation

Symmetry of shape charts with applications to energy-minimizing subdivision algorithms

Georg Umlauf (TU Kaiserslautern, DE)

For subdivision surfaces, the so-called shape chart can be used to characterize the curvature behavior at an extraordinary point a priori from the initial control net. Of late, it has been used in different approaches to tune subdivision algorithms to handle the so-called hybrid shapes. For this the shape charts are computed numerically. In this talk, symmetries of shape charts are discussed that can be used to simplify the computations and to reduce the computation costs significantly. Based on these properties I will present a method to fair the limit surface of a subdivision algorithm locally around an extraordinary point.

The goal of this method is to construct a sequence of spline rings around an extraordinary point by a modified subdivision matrix, such that each spline ring has minimal energy.

The dominant six eigenvalues of the subdivision matrix have to satisfy linear and quadratic equality- and inequality-constraints in order to guarantee normal-continuity and bounded curvature at the extraordinary point. All smaller eigenvalues can be chosen arbitrarily within certain intervals and therefore can be used to optimize the shape of the subdivision surface by minimizing quadratic energy functionals. A suitable functional should contain third order measures and reasonably approximate the variation of curvature, since second order measures may lead to flat surfaces. Additionally, if the sub- and subsub-dominant eigenvalues vary within predefined intervals, C^1-regularity of the surface and locality of the stencils can be guaranteed, although eigenvectors are changed.

Keywords: Subdivision algorithms, shape charts, energy minimziation

Discrete differential geometry of hexagonal meshes

Johannes Wallner (TU Graz, AT)

Recently a curvature theory for polyhedral surfaces has been established which associates with each face a mean curvature value computed from areas and mixed areas of that face and its corresponding Gauss image face.

Therefore a study of minimal surfaces requires studying pairs of polygons with vanishing mixed area. We show that the mixed area of two edgewise parallel polygons equals the mixed area of a derived polygon pair which has only the half number of vertices. Thus we are able to recursively characterize vanishing mixed area for hexagons and other n-gons in an incidence-geometric way. We use these geometric results for the construction of discrete minimal surfaces and a study of equilibrium forces in their edges, especially those with the combinatorics of a hexagonal mesh.

Joint work of: Wallner, Johannes; Müller, Christian

Full Paper:
http://www.geometrie.tugraz.at/wallner/mixedarea.pdf

C^1 homogeneous representation of G^1 composite rational Bezier curves

Jianmin Zheng (Nanyang TU - Singapore, SG)

A G^1 composite Bezier curve may not be C^1 continuous in its homogeneous representation. In this talk, we will examine possible approaches to converting the curve into a representation that is C^1 continuous in its homogenous form and their insights.

We also present a method which always works. This is useful for surface skinning applications.

Keywords: Rational Bezier curves, NURBS, continuity, knots, homogeneous representation