An essay on *msic*-systems

Jan Odelstad

1) Department of Mathematics, Natural and Computer Sciences, University of Gävle, Sweden, 2) DSV, KTH, Sweden, jod@hig.se

Abstract. A theory of many-sorted implicative conceptual systems (abbreviated *msic*-systems) is outlined. Examples of *msic*-systems include legal systems, normative systems, systems of rules and instructions, and systems expressing policies and various kinds of scientific theories. In computer science, *msic*-systems can be used in, for instance, legal information systems, decision support systems, and multi-agent systems. In this essay, *msic*-systems are approached from a logical and algebraic perspective aiming at clarifying their structure and developing effective methods for representing them. Of special interest are the most narrow links or joinings between different strata in a system, that is between subsystems of different sorts of concepts, and the intermediate concepts intervening between such strata. Special emphasis is put on normative systems, and the role that intermediate concepts play in such systems, with an eye on knowledge representation issues. In this essay, normative concepts are constructed out of descriptive concepts using operators based on the Kanger-Lindahl theory of normative positions. An abstract architecture for a norm-regulated multi-agent system is suggested, containing a scheme for how normative positions will restrict the set of actions that the agents are permitted to choose from.
Key-words: Concept formation, Intermediary, Intermediate concept, Legal concept, Normative system, Normative position, Norm-regulated system, Agent architecture.

1 Introduction

1.1 Conceptual systems in computer- and systems sciences

In the famous Schilpp-volume where established scholars discuss Einstein's work in physics and philosophy, Einstein, in his reply to criticisms, states the following about the relationship between epistemology and science:

> The reciprocal relationship of epistemology and science is of noteworthy kind. They are dependent upon each other. Epistemology without contact with science becomes an empty scheme. Science without epistemology is—insofar it is thinkable at all—primitive and muddled. However, no sooner has the epistemologist, who is seeking a clear system, fought his way through to such a system, than he is inclined to interpret the thought-content of science in the sense of his system and to reject

Dagstuhl Seminar Proceedings 09121
Normative Multi-Agent Systems
http://drops.dagstuhl.de/opus/volltexte/2009/1914

> whatever does not fit into his system. The scientist, however, cannot afford to carry his striving for epistemological systematic that far. He accepts gratefully the epistemological conceptual analysis; but the external conditions, which are set for him by the facts of experience, do not permit him to let himself be too much restricted in the construction of his conceptual world by the adherence to an epistemological system. He therefore must appear to the systematic epistemologist as a type of unscrupulous opportunist ... (Einstein, 1949.)

The science Einstein has in mind is primarily physics, but even for sciences that are rather unlike physics its reciprocal relationship to epistemology is of a noteworthy kind. The external conditions that, according to Einstein, restrict the adherence to an epistemological system is “the facts of experience”, with what Einstein probably meant the results of observations and experiments. But for the sciences that are rather unlike physics “the facts of experience” may better be characterized in some other way. For computer- and systems sciences, “the facts of experience” may perhaps be described as “useful applications”.

Every science ought to critically question its foundational assumptions. How urgent the researchers in a field experience these foundational questions may vary greatly from time to time. But probably all sciences go through stages when the need for revisions and elaborations of the basic principles and fundamental conceptions seem inevitable. In a young science, the foundational problems are important and at the same time not seldom overlooked, since researchers working in the field are so enthusiastic over the flow of new results. In such situations, philosophy (which includes epistemology as one of its sub-disciplines) may have a role to play to make clear—and sometimes even to remedy—weak points in the base of the new discipline. In this essay, some problems in the foundations of computer- and systems sciences are addressed and theories and tools which could be useful in the further development of some aspects of this discipline are outlined.[1]

Concepts are a fundamental tool for all kinds of human communication and concept formation is an important process in all branches of science. Information science is of course not an exception. An information system is, when all technical “embeddings” have been stripped off, a set of concepts and relations between these concepts. The skeleton of an information system is a conceptual structure, and this structure must have a solid formal representation, otherwise it cannot function in a computer context.

The formal representations of conceptual systems has a long history in philosophy and in several scientific disciplines. This essay is focused on the relation between layers or strata of concepts of different sorts in a conceptual system and on intermediate concepts that function as links between different strata. This study is brought about using algebraic tools, which implies that the representation is algebraic in character. The result is a theory of many-sorted implicative conceptual systems, *msic*-systems.

[1] This essay is a revised version of Odelstad (2008b).

I argue for an *anti-nivelistic* approach to theoretical systems, which implies the recognition of the multitude of layers or strata that usually are parts of such systems.[2] As a consequence, I also argue for an anti-nivelistic approach to knowledge representation. The following sketch is very vague and metaphorical, however, my message is more adequately found in the formalism below. Suppose that an *msic*-system M represents knowledge or information of a domain D. The implicative relation between concepts represents knowledge of some kind and the kind of knowledge it represents may differ in different parts of the system. In some parts of the system, it may represent conceptual knowledge, the knowledge of definitions of concepts and the logical relations between concepts. In other parts of the system, it may represent for example empirical knowledge about some kind of phenomena and in yet another part of the system it may represent empirical knowledge of another kind. Different strata of concepts of different sorts may thus express knowledge of different kinds. The knowledge represented by links between different strata often represent knowledge of a kind still different from the knowledge represented by the strata, for example knowledge of rational actions or appropriate rules. The revision of an *msic*-system can be done very partially. In many cases, the necessary revision is effected by the modification of the narrowest links between some strata of different kinds.

It is often argued that, for example, rule-based expert systems cannot be modified by the expert system itself. The following quotation from a text book may illustrate this idea:

> Knowledge in a rule-based expert system is represented by IF-THEN production rules collected by observing or interviewing human experts. This task, called knowledge acquisition, is difficult and expensive. In addition, once the rules are stored in the knowledge base, they cannot be modified by the expert system itself. Expert systems cannot learn from experience or adapt to new environments. Only a human can manually modify the knowledge base by adding, changing or deleting some rules. (Negnevitsky, 2005, p. 261.)

One of the advantages with the anti-nivelistic approach to knowledge representation expressed by *msic*-systems is, as I see it, that this may not be true. This is discussed in connection with forest cleaning below.

1.2 Stratification of concepts in theoretical systems

In an article from 1936, Albert Einstein discusses, among other things, the stratification of the scientific system. According to Einstein, there is a multitude of different layers or strata of concepts in science, where higher layers are more abstract than lower layers. As regards to the final aim of science, Einstein suggests, intermediary layers are only of temporal nature and must eventually disappear

[2] *Nivelistic* is constructed out of the French verb *niveler*, meaning “Mettre au même niveau, rendre égal".

as irrelevant. But in the science of today, these strata represent partial success, though problematic. (See Einstein, 1973, p. 295.)

Many theoretical systems show the same kind of phenomena as theoretical physics in the following respects: In the system there is a hierarchical ordering of the concepts in different strata and the status of the concepts in intermediate strata is not obvious. In theoretical physics, the ordering of the layers is based on degrees of abstraction. In other contexts, the stratification of the system can be grounded on quite different principles, for example: descriptive versus normative, state versus action or physical versus mental. One of the main issues to be examined in this essay is the stratification of concepts in theoretical systems, especially the connections between different strata and the function and status of intermediate layers.

The kind of theoretical systems that will come into focus in this study can, in a fairly general way, be characterized as *conceptual systems* and two essential characteristics of these systems are the following: They have an implicative form and they are many-sorted, i.e. a system consists of different sorts of concepts (at least two). They are thus *many-sorted implicative conceptual systems*, in the sequel abbreviated *msic*-systems. Different kinds of systems belong to the class under study, for example legal systems, normative systems, systems of rules and instructions, systems expressing policies and some varieties of scientific theories. Such systems have an important role to play in the discipline artificial intelligence, which has as one of its aims to bring forth "smart" behaviour of computers.

In the investigation reported here, *msic*-systems are studied from a logical and algebraic perspective aiming at clarifying their structure and developing effective methods for representing them. Special emphasis is put on the most "narrow" links between subsystems of different sorts in a system and intermediaries (intermediate concepts) mediating or intervening between subsystems of different sorts. Such links and intermediaries are of great interest when there are reasons for changing the system.

In computer science, *msic*-systems can be useful in many problem areas, for example: legal information systems, computer security, knowledge representation, expert systems, architectures for multiagent-systems, decision-analytic support systems and agent-based simulations. This study of *msic*-systems is mainly a contribution to the tradition of constructing intelligible and explicit models and representations in contrast to case-based, connectivist and emergent approaches (cf. Luger, 2002, p. 228). But *msic*-systems also prepare the grounds for the use of machine learning, where the links and intermediaries between subsystems will play an important role.

1.3 The theory of *msic*-systems

When developing a theory of *msic*-systems, it is important to note that different parts of the theory are situated on different levels of abstraction, and as a consequence there are different levels of applications of the theory. The word 'theory' has several meanings and in this context it is important to distinguish between the following two meanings:

(1) Theory in the sense often used in logic; abstract theory, theory in contrast to model (in the model-theoretic sense)
(2) Theory in contrast to practice and application.

Here a theory of *msic*-systems is put forward in both senses of 'theory'. The theory of *msic*-systems, where 'theory' is taken in the second sense contains some theories of *msic*-systems in the first sense, of formal theories. The formal theories of *msic*-systems are characterized axiomatically as algebraic theories and among the models of these abstract (formal) theories are specific *msic*-systems. The abstract theories of *msic*-systems express the structure of such systems. The theory of *msic*-systems, in the second sense, contains other theoretical perspectives than the abstract, formal ones.

The theory of *msic*-systems will be abbreviated *msic*-theory, where 'theory' stands for sense (2). A formal, abstract theory of *msic*-systems where 'theory' is taken in sense (1) will be called a structural *msic*-theory, since such a theory characterizes the structure of *msic*-systems. Such abstract theories will usually be presented as axiomatized theories within set theory. The most abstract part of the *msic*-theory will be framed as a number of set-theoretical predicates.

2 Normative systems

2.1 What is a norm?

The theory of *msic*-systems has many applications and there are many different kinds of *msic*-systems. In this essay I will focus on the representations of normative systems as *msic*-systems. I take a first step in the analysis of norms in this section, and a great deal of simplification is needed. Modifications and elaborations of this oversimplified picture will be developed step by step in later sections.

Norms, normative sentences, are understood in contrast to descriptive sentences. Sentences of the latter kind express matters of fact but are not used for expressing evaluations or value judgments. A normative sentence, on the other hand, does not state what *is* the case but what *shall* be the case or what *may* be the case, or will have an *evaluating* function.

Let us preliminarily say that there are two kinds of normative sentences, viz. categorically normative sentences and conditional normative sentences. A categorically normative sentence consists of a descriptive sentence preceded by a 'norm creating operator', for example 'it shall be the case that' or 'it may be the case that'. If q is a descriptive sentence then 'it shall be the case that q', which is abbreviated $\mathrm{Shall}(q)$ and 'it may be the case that q', abbreviated as $\mathrm{May}(q)$, are examples of categorically normative sentences. A conditional norm is an *if-then* sentence (an implication) where the antecedent is descriptive and the consequent is purely normative. Hence, a conditional norm has the form

$$p \to B(q)$$

where p and q are descriptive sentences and B is a norm-creating operator, for example Shall or May. As suggested above, it is possible to extend ordinary propositional logic with propositional operators as Shall and May, etc. The branch of logic derived in this way is called deontic logic. 'Deontic' comes from the Greek word 'deont', which means "that which is binding". Expressed in a very general way, deontic logic is the logical study of obligation and permission. The modern study of this kind of logic is often said to have commenced with the article "Deontic Logic" by the Finnish philosopher Georg Henrik von Wright published in *Mind* in 1951.[3] This theory was anticipated by Ernst Mally in the 1920s and, much earlier, by Gottfried Wilhelm Leibniz (1646-1716) and Jeremy Bentham (1748-1832). The core of standard deontic logic is the formal study of the deontic operators 'it is permissible that' (May) and 'it is obligatory that' (Shall) and we can extend predicate logic as well as propositional logic with these operators.

2.2 Norms in predicate logic and as ordered pairs

A conditional norm is (usually) expressed as a universal sentence. For example:

(n_1) For any x, y and z : if x has promised to pay \$$y$ to z, then x has an obligation to pay \$$y$ to z.

Within predicate logic, we can formalize (n_1) as follows:

(n_2) $\forall x, y, z : \mathit{PromisedPay}(x, y, z) \to \mathit{Obligation\_to\_Pay}(x, y, z)$

Thus, a typical conditional norm is a universal implication. Syntactically it consists of three parts: the sequence of universal quantifiers, the antecedent formula and the consequent formula. Note that the norm (n_2) correlates open sentences: $\mathit{PromisedPay}(x, y, z)$ is correlated to $\mathit{Obligation\_to\_Pay}(x, y, z)$. A norm like ($n_2$) can therefore be represented as a relational statement correlating a *ground,* $\mathit{PromisedPay}$, to a *consequence,* $\mathit{Obligation\_to\_Pay}$:

$\mathit{PromisedPay}\ \mathcal{R}\ \mathit{Obligation\_to\_Pay}.$

Generally, $p\mathcal{R}q$ represents the norm

(n_3) $\forall x_1, ..., x_\nu : p(x_1, ..., x_\nu) \to q(x_1, ..., x_\nu)$

given that p and q are ν-ary predicates. It is important here that the free variables in $p(x_1, ..., x_\nu)$ are the same and in the same order as the free variables in $q(x_1, ..., x_\nu)$. $\mathcal{R}$ is a binary relation, and $p\mathcal{R}q$ is a relational statement equivalent to $\langle p, q\rangle \in \mathcal{R}$. Thus, a norm can be represented as $p\mathcal{R}q$ or $\langle p, q\rangle \in \mathcal{R}$. If, in the

[3] The development of deontic logic is closely related to another, better known part of logic, namely modal logic. The core of modal logic is the formal study of the operators 'it is possible that' and 'it is necessary that' (the so-called alethic modalities) and modal propositional logic is propositional logic extended with the possibility- and necessity-operator.

actual context, $\mathcal{R}$ can be tacitly understood and therefore omitted, it is only a small step to the representation of (n_3) as the ordered pair $\langle p, q\rangle$.

Note that $p\mathcal{R}q$ as a representation of (n_3) does not generally presuppose that q is a normative (or deontic) predicate, so $p\mathcal{R}q$ can be used as a representation of any sentence which has the same form as (n_3). Therefore, in many contexts of application the implicative relation $\mathcal{R}$ can be such that only some of the sentences $p\mathcal{R}q$ are norms. For reasons that will be explained when the formal framework is discussed, $p\mathcal{R}q$ will be abbreviated as the ordered pair $\langle p, q\rangle$ only when p and q are conditions of different sorts.

In the above discussion of the representation of norms, p and q, as well as $PromisedPay$ and $Obligation\_to\_Pay$, appear as predicates. But the term predicate is often used for syntactical entities, and, therefore, interpreting $p\mathcal{R}q$, p and q will here instead be conceived of as *conditions*. If p is a ν-ary condition and i_1,...,i_ν are individuals, then $p(i_1, ..., i_\nu)$ is a statement. Antecedents and consequences of norms are represented as conditions and are called *grounds* and *consequences* respectively. A norm is represented as a statement relating (or correlating) a ground to a consequence, or represented as an ordered pair consisting of a ground and a consequence. In the preliminary analysis put forward in this section, grounds are descriptive and consequences are normative conditions.[4]

Note that $Obligation\_to\_Pay$ is a normative condition but that the sentence $Obligation\_to\_Pay(x, y, z)$ can be analysed as

Obligatory $Pay(x, y, z)$.

where Obligatory is a deontic operator resulting in a new predicate when it is applied to a given predicate. Pay is a descriptive condition and by applying the deontic operator Obligatory we can in a sense construct a normative condition Obligatory Pay out of the descriptive condition Pay. It is presupposed here that 'Obligatory Pay' is equivalent to $Obligation\_to\_Pay$ and I will return to this way of constructing normative conditions out of descriptive conditions using deontic operators.

Within the framework of the above preliminary analysis of norms, we can view a normative system $\mathcal{N}$ as consisting of a system $\mathcal{B}_1$ of potential *grounds* (descriptive conditions) and a system $\mathcal{B}_2$ of potential *consequences* (normative conditions). The set of norms in $\mathcal{N}$ are the set J of *links* or *joinings* from $\mathcal{B}_1$ to $\mathcal{B}_2$. The Figure 1.1 is an attempt to illustrate the situation, where a norm is represented by an arrow from the system of grounds to the system of consequences.

A norm in a normative system $\mathcal{N}$, the norm here represented as an ordered pair $\langle p, q\rangle$, can be regarded as a mechanism of inference. We can distinguish two cases. Suppose that p and s are descriptive conditions and q and t normative. Then the following "derivation schemata" are valid given $\mathcal{N}$.

1.
$p(i_1, ..., i_\nu)$
$\langle p, q\rangle$

[4] Cf. Odelstad & Lindahl (2002), pp. 32 ff and Lindahl & Odelstad (2004), section 3.2.

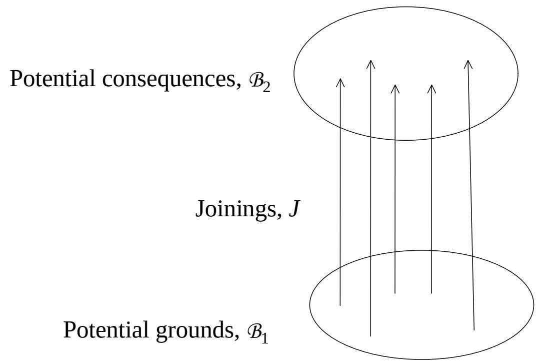

Fig. 1. A simple normative system.

$q(i_1, ..., i_\nu)$

2.
$s\mathcal{R}p$
$\langle p, q \rangle$
$q\mathcal{R}t$

$\langle s, t \rangle$[5]

In (1), $\langle p, q \rangle$ functions as a deductive mechanism correlating sentences by means of instantiation, while in (2), $\langle p, q \rangle$ plays an important role in correlating one condition, s, to another condition, t.[6]

A condition, as the term is used here, is very similar to a relation; in a sense a condition is used for "expressing" a relation.[7] Relations, and therefore also conditions, are a specific kind of concepts. A normative system is thus a system consisting of an implicative relation between concepts. Note that the kind of normative systems we have encountered so far consists of two sorts of concepts, descriptive and normative.

[5] Note that $s\mathcal{R}p$ relates conditions of the same sort and the same holds for $q\mathcal{R}t$; s and p are descriptive but q and t are normative. A norm consists of conditions of different sorts. As stated earlier, only implicative sentences that relate conditions of different sorts will be represented as ordered pairs.

[6] See Lindahl & Odelstad (2004) subsection 3.2 and Odelstad & Boman (2004) subsection 2.2. Cf. Alchourrón & Bulygin (1971) p. 28. Schema 1 corresponds to what Alchourrón and Bulygin call the correlation of individual cases to individual solutions, and schema 2 corresponds to what they call the correlation of generic cases to generic solutions.

[7] Properties are here regarded as unary relations and can be "expressed" by conditions.

Easily observable, conjunctions, disjunctions and negations of conditions can be formed by the operations $\wedge, \vee, '$, namely in the following way (where $x_1, ..., x_\nu$ are place-holders, not individual constants).

$(p \wedge q)(x_1, ..., x_\nu)$ if and only if $p(x_1, ..., x_\nu)$ and $q(x_1, ..., x_\nu)$.
$(p \vee q)(x_1, ..., x_\nu)$ if and only if $p(x_1, ..., x_\nu)$ or $q(x_1, ..., x_\nu)$.
$(p')(x_1, ..., x_\nu)$ if and only if not $p(x_1, ..., x_\nu)$.

$\perp$ (Falsum) is the empty condition, not fulfilled by any $\nu-$tuple, and $\top$ (Verum) is the universal condition, fulfilled by all $\nu-$tuples.

As is well-known, the truth-functional connectives can be used as operations in Boolean algebras. It is therefore possible to construct Boolean algebras of conditions. The role of the set of norms is to join two Boolean algebras:

- a Boolean algebra of grounds,
- a Boolean algebra of consequences.

The norms are links or joinings between the algebra of grounds and the algebra of consequences.

The outline of the algebraic approach to normative systems just presented is substantially simplified. The approach will be developed extensively below.

3 Conceptual systems

In the previous subsection, a simple normative system has been characterized as a two-sorted implicative conceptual system, where the concepts are, from a logical point of view, relations (expressed as conditions) and the two sorts of concepts involved are descriptive and normative conditions. However, relations (and therefore also conditions) are only one specific kind of concepts, where 'kind' is something else than 'sort'. Other kinds of concepts are, *inter alia*, aspects (in philosophy of science often called attributes) and measures (often termed scales). Examples of aspects are length, weight, temperature, intelligence, utility and probability. Examples of measures are meter, kilogram, degrees centigrade and the probability measure. Different *kinds* of concepts have different logical form (for example relations, structures and functions) while different *sorts* of concepts differ in their cognitive status (for example descriptive and normative respectively). From a logical point of view, aspects are structures and measures are functions.

When studying implicative conceptual systems where the concepts are conditions, the implicative relation is implication in a straightforward sense. However, when the concepts are aspects or scales, we are dealing with implicative relations that are implications only in a rather generalized sense. Implicative statements, i.e. statements expressing that an implicative relation holds, can in such cases, for example, be interpreted as determination or relevance. However, we will even in the generalized contexts talk about the antecedent and consequent of an implicative statement, and even of grounds and consequences.

As pointed out above, for concepts which are conditions we can in an obvious way define the operations conjunction, disjunction and negation and thereby arrive at a Boolean algebra. The situation is different for concepts that are aspects, since taking the negation of an aspect is not certainly a meaningful operation. However, aspects can form a lattice. We shall discuss this further below.

'Concept' is a complicated notion and is of great importance in many areas. It is tightly connected to the notion of 'meaning', and 'the meaning of concepts' is a philosophical minefield. But in this context, it is impossible to avoid the term 'concept'. The following short passage from the entry *Concept* in *The Encyclopedia of Philosophy* describes its usefulness:

> Concept is one of the oldest terms in the philosophical vocabulary, and one of the most equivocal. Though a frequent source of confusion and controversy, it remains useful, precisely because of its ambiguity, as a sort of passkey through the labyrinths represented by the theory of meaning, the theory of thinking, and the theory of being. (Heath, 1967.)

In the theory of *msic*-systems, the use of the notions 'concept' and 'meaning' is instrumental, and these notions function as passkeys to the main objectives of the work presented here. As the word 'concept' is used, complex combinations of concepts are still regarded as concepts. A concept can be defined in terms of other concepts in a more or less complicated way.

A notion connected to 'concept' that will play a role here is 'cognitive status'. The idea is that the different sorts of concepts constituting an *msic*-system are often different with respect to their cognitive status. As a source of inspiration for using the notion 'cognitive status' in the theory of *msic*-systems one can take Ernest Nagel's discussion of the cognitive status of scientific theories in his book *The Structure of Science*. But here the term 'cognitive status' is applied to concepts. Examples of different cognitive status include: logical, empirical, observational, operational, theoretical, physical, mental, descriptive, prescriptive, normative, evaluative, and—as we will see below—intermediate. (Note that several of the different sorts of cognitive status exemplified above can be applied to the same concepts.)

4 Intermediate concepts—form and function

4.1 Intermediaries

In the simplified presentation above, a normative system is represented as a two-sorted implicative conceptual system, consisting of a set of descriptive grounds and a set of normative consequences. However, many concepts for example in law are neither purely descriptive nor purely normative. Like Janus, the Roman god of beginnings and endings, they have two faces, one turned towards facts and description, the other towards legal consequences. These concepts are said

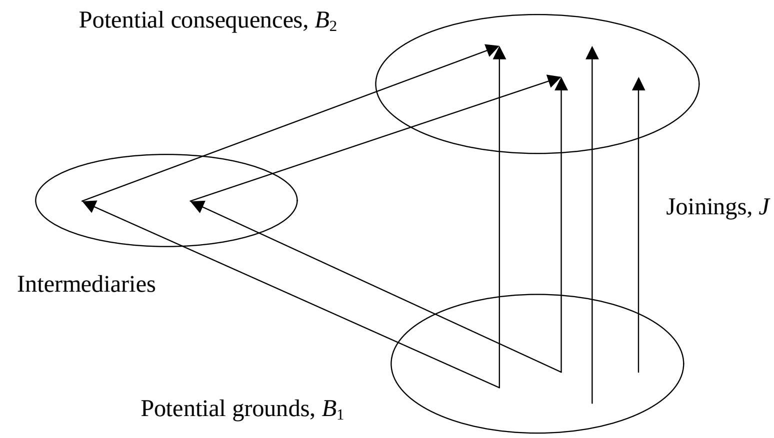

Fig. 2. A normative system with intermediaries.

to be intermediate between facts and legal consequences and will often be called intermediaries. Figure 1.2 will give a first illustration of this idea.

As an example, consider what it means to be a citizen according to the system of the U.S. Constitution. Article XIV, section 1 reads as follows:

> All persons born or naturalized in the United States, and subject to the jurisdiction thereof, are citizens of the United States and of the State wherein they reside. No State shall make or enforce any law which shall abridge the privileges or immunities of citizens of the United States; nor shall any State deprive any person of life, liberty, or property, without the due process of law; nor deny to any person within its jurisdiction the equal protection of the laws.

Two key concepts in the article are *citizen* and *person*. The article specifies the ground for the condition being a citizen in the United States:

persons born or naturalized in the United States, and subject to the jurisdiction thereof

and specifies a number of regal consequences of this condition expressed in terms of 'shall':

no State shall make or enforce any law which shall abridge the privileges or immunities of citizens of the United States.

The article does not state any ground for the condition to be a person but specifies a number of legal consequences connected to this condition:

nor shall any State deprive any person of life, liberty, or property, without due process of law; nor deny to any person within its jurisdiction the equal protection of the laws.

Within the constitutional system of United States, this article is supplemented with rules laid down by the Constitution and through court decisions. These rules determine together, by specifying grounds and consequences, the role the concept 'citizen' and 'person' have within the legal system.

Let us construct a simplified "condition-implicative" representation of the legal rules described above.[8] According to the rules, the disjunction of the two conditions

b: to be a person born in the U.S.

n: to be a person naturalized in the U.S.

in conjunction with the condition

s: to be a person subject to the jurisdiction of the U.S.

implies the condition

c: to be a citizen of the U.S.

That this implicative relationship holds according to the system is represented in the form $((b \vee n) \wedge s)\mathcal{R}c$. Since it is a settled matter that citizens who are minors do not have the right to vote in general elections, c does not imply the condition

e: to be entitled to vote in general elections.

Therefore: not $[c\mathcal{R}e]$, and hence not $[((b \vee n) \wedge s)\mathcal{R}e]$.

Let

a: to be adult.

Simplifying matters, suppose that,

(1) $(c \wedge a)\mathcal{R}e$.

It is easy to see that this is equivalent to

(2) $c\mathcal{R}(a' \vee e)$.

Going from (1) to (2) can be called *exportation*, and going from (2) to (1) *importation*.

We thus have within the system the following rules: $((b \vee n) \wedge s)\mathcal{R}c$ and $c\mathcal{R}(a' \vee e)$, stating that the condition $((b \vee n) \wedge s)$ is a ground for c and $(a' \vee e)$ is a consequence of c. These two rules determine partly the role of c (citizenship) in the constitutional system under study. But there can also be other grounds $g_1, g_2, \ldots$ for c and consequences $h_1, h_2, \ldots$ of c within the constitutional system. Suppose that $g_1, g_2, \ldots$ are the grounds of c and $h_1, h_2, \ldots$ the consequences of c. Hence, the role of c in the system is characterized by

$$g_1\mathcal{R}c, g_2\mathcal{R}c, \ldots, c\mathcal{R}h_1, c\mathcal{R}h_2, \ldots$$

The concept c thus couples a set of legal consequences to a set of legal grounds and c is situated "intermediate" between the set of grounds and the set of consequences. Concepts of this kind are called *intermediate concepts* or *intermediaries*. Over the past sixty years, there has been an on-going discussion in Scandinavia as regards the idea of intermediate concepts in the law. The debate was started

[8] The concept citizen regarded as an intermediary is discussed in Odelstad & Lindahl (1998), Odelstad & Lindahl (2000) and Lindahl & Odelstad (2000). In Lindahl & Odelstad (2003), citizenship is treated from the point of view of organic wholes.

in 1944-1945 by Anders Wedberg and Per-Olof Ekelöf, and in 1951 Alf Ross published his well-known essay on "Tû-Tû".[9] In this debate, an often used example is the concept of ownership. Ross represents a set of legal rules concerning ownership (denoted O) in essentially the following way, where F_i expresses a possible legal ground and C_j a legal consequence of O.

$$\left.\begin{array}{l} F_1 \rightarrow \\ F_2 \rightarrow \\ F_3 \rightarrow \\ \vdots \\ F_p \rightarrow \end{array}\right\} \; O \rightarrow \; \left\{\begin{array}{l} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_n \end{array}\right.$$

Ross himself comments on this scheme in the following way:

> "O" (ownership) merely stands for the systematic connection that F_1 as well as F_2, F_3,...,F_p entail the totality of legal consequences C_1, C_2, C_3,..,C_n. As a technique of presentation this is expressed then by stating in one series of rules the facts that "create ownership" and in another series the legal consequences that "ownership" entails. (Ross 1956-57, p. 820.)

Note that the rules that "create ownership" can be expressed by one rule: $F_1 \vee ... \vee F_p \longrightarrow O$.[10] And the rules describing what 'ownership' entail can also be condensed to one rule: $O \longrightarrow C_1 \wedge ... \wedge C_n$. So an equivalent way of representing the legal rules concerning ownership according to Ross is the following scheme:

$$F_1 \vee ... \vee F_p \longrightarrow O \longrightarrow C_1 \wedge ... \wedge C_n$$

Whereas $F_1, ..., F_p$ can be called grounds and $C_1, ..., C_n$ consequences of O, $F_1 \vee ... \vee F_p$ is *the* ground of O and $C_1 \wedge ... \wedge C_n$ *the* consequence of O.

Note that the rule $F_i \rightarrow O$ is a way of introducing O into the discourse, and appropriately we can call such a rule an *introduction rule* of O. In harmony with this, the rule $O \longrightarrow C_j$ can be called an *elimination rule* of O, since in a sense such a rule can eliminate O from the discourse. Analogous to the use of the phrases 'the ground' and 'the consequence' we can say that

$$F_1 \vee ... \vee F_p \longrightarrow O$$

is *the* introduction rule of O and

$$O \longrightarrow C_1 \wedge ... \wedge C_n$$

[9] For a more detailed analysis of the early Scandinavian debate see Lindahl & Odelstad (1999a) section 1.2.

[10] $\longrightarrow$ is a consequence relation.

is *the* elimination rule of O.[11]

In Wedberg (1951), three different methods for treating the concept of 'ownership' are discussed. The first and second of these methods aim at a definition of ownership in terms of grounds and consequences respectively. Wedberg's third method treats ownership as a 'vehicle of inference'. According to Wedberg this means that ownership is a tool for inferring statements of legal consequences from statements of legal facts, and, therefore, ownership is undefined. Obviously, Wedberg's third method for treating ownership is close to Ross's view.

We will return to the question of defining intermediate concepts in relation to regarding them as vehicles of inferences. As a point of departure for further discussions and refinements, we regard intermediate concepts as characterized by their grounds and consequences. The characterization of the concept citizenship, c, thus has the following form:

$$g_1\mathcal{R}c, g_2\mathcal{R}c, ..., c\mathcal{R}h_1, c\mathcal{R}h_2, ...$$

For the view of intermediate concepts adopted in this essay, the discussion in legal philosophy has been an important source of inspiration. But there are of course also other theories that have influenced this research. The following quotation from Lindahl & Odelstad (1999a) emphasizes this, where "the ideas mentioned above" are the ideas of Wedberg and Ross.

> In the theory of language of Michael Dummett, there are features with some resemblance to the ideas mentioned above. According to Dummett, the meaning of an expression is determined, on one hand by the condition for correctly uttering it, and on the other hand by what the uttering of the expression commits the speaker to. Therefore, the meaning of a statement is identified in part by the conditions from which it can be inferred and in part by what can be inferred from the statement. In the case of utterances of sentences composed by the connectives "and", "or" etc., this is given by what are called introduction and elimination rules in Gentzen's system of natural deduction. (Lindahl & Odelstad, 1999a, p. 165.)

Introduction and elimination rules are discussed further in Lindahl & Odelstad (2008a).

The analysis of the concept of 'intermediary' involves complicated questions of meaning and is therefore a philosophically loaded topic. The formal theory of intervenients which is presented in Lindahl & Odelstad (2008a) and (2008b) is intended as a means for a thorough analysis of the concept of an intermediary.

An interesting issue in the discussion of intermediaries is the negation of an intermediate concept. Suppose that a_1 is the ground of the intermediary m and that a_2 is the consequence of m. Let m' be the negation of m, i.e. not-m. Is m' an intermediate concept? If the answer is yes, what can be said about

[11] Introduction and elimination rules are discussed in Lindahl & Odelstad (2008a) with reference to Gentzen.

its grounds and consequences? This question, which is discussed in Lindahl & Odelstad (2008a), is complicated, especially if we turn to open intermediaries.

4.2 Open intermediaries

The concept 'work of equal value' is an essential concept in the Swedish Equal Opportunities Act. The following quotation demonstrates this (emphasis added here):

> Employers and employees shall cooperate in pursuing active efforts to promote equality in working life. They shall strive in particular to prevent and eliminate differences in pay and in other conditions of employment between women and men performing *work* that may be considered equal or *of equal value*. They shall also promote equal opportunities for wage growth for women and men.
>
> Work is to be considered *equal in value* to other work if, based on an overall assessment of the nature of the work and the requirements imposed on the worker, it may be deemed to be of similar value. Assessments of work requirements shall take into account criteria such as knowledge and skills, responsibility and effort. When the nature of the work is assessed, particular regard shall be taken of the working conditions.

The concept 'work of equal value' is an intermediary with—using the Janus-metaphor—one face looking at the nature of and requirements for the work and the other face looking at efforts to promote equality in working life, especially equal pay for equal work. The law does not supply us with a complete set of introduction rules for the concept. Instead it mentions some criteria that equality of value depends on, viz. knowledge and skills, responsibility and effort. However, one can extract the following uncontroversial introduction rule: if x and y are work that requires the same degree of knowledge, skills, responsibility and effort, then x and y are work of equal value. We can express this in a formalised style as follows:

$$x \sim_1 y \,\&\, x \sim_2 y \,\&\, x \sim_3 y \,\&\, x \sim_4 y \,\&\, x \sim_5 y \;\longrightarrow\; x \sim_v y$$

where

$\sim_1$ is the relation 'equal knowledge'
$\sim_2$ is the relation 'equal skills'
$\sim_3$ is the relation 'equal responsibility'
$\sim_4$ is the relation 'equal effort'
$\sim_5$ is the relation 'equal working conditions'
$\sim_v$ is the relation 'equal value'

Note that the equality relations $\sim_1$,$\sim_2$,$\sim_3$,$\sim_4$ and $\sim_5$ are here regarded as conditions and we can therefore apply Boolean operations on the equality relations, for example construct conjunctions of them. One of the grounds of $\sim_v$ is thus the condition

$$\sim_1 \wedge \sim_2 \wedge \sim_3 \wedge \sim_4 \wedge \sim_5 \,.$$

But it is also possible that work x and y are of equal value even if they are not equal with respect to the requirements knowledge, skills, responsibility, effort and working condition. We can imagine a situation such that x requires more knowledge than y, and y more responsibility than x but that these two differences balance out. But to turn this observation into an introduction rule is often not possible. The applicability of the concept work of equal value in a certain case must therefore be based on judgments of what holds in the actual case. And even if the law does not state detailed rules for these judgments it gives guidelines, for example in terms of what are possible inputs in such judgments or what factors or circumstances must be taken into account.

The grounds of the concept 'work of equal value' is thus only partially determined by the law in the form of introduction rules. The application of the concept in special cases deserves interpretative decisions based on the role and function of the concept in the law. We call such intermediaries *ground-open*. Concepts such that the consequences are only partially determined by elimination rules are called *consequence-open*.

Open intermediaries are further discussed in Lindahl & Odelstad (2008a). For a detailed discussion of the concept work of equal value, see Odelstad (2008a).

4.3 Intermediaries in normative systems

A normative system is only in rather special cases a two-sorted implicative conceptual system, i.e. a system of grounds and a system of consequences. Instead, normative systems often contain also many intermediate concepts. In more complex normative systems, for example legal systems, there are usually more than one system of intermediaries, and these systems often form a kind of network, where between intermediaries of two different sorts there are intermediaries of a third sort.[12] Note that a rule can simultaneously be an introduction rule for one concept and an elimination rule for another.

Intermediaries do not only exist in normative systems but in many other *msic*-systems. This is discussed in Lindahl & Odelstad (1999a) p. 178.

4.4 A remark on related work

The Scandinavian discussion of intermediate concepts has had a crucial influence on the theory of *msic*-systems put forth in this essay. The following works have been of special significance: Wedberg (1951), Ross (1951), Halldén (1978) and Lindahl (1985). Hedenius (1941) does not consider intermediate concepts but Hedenius' discussion about spurious and genuine norms is of great interest in this context. The works on introduction and elimination rules in logic and philosophy

[12] In Lindahl & Odelstad (2008b), this is illustrated as Figure 1. There the lines between different nodes represent sets of introduction or elimination rules.

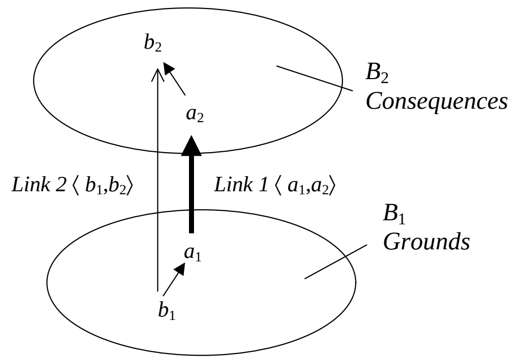

Fig. 3. Norm $\langle a_1, a_2 \rangle$ is narrower than norm $\langle b_1, b_2 \rangle$.

of mathematics by Gentzen, Dummett and Prawitz have, as emphasized above, also influenced this work. (See Gentzen 1934, Dummett 1973 and Prawitz 1977).

There are similarities between Richard Hare's prescriptivism and the view of intermediaries developed in the work that Lindahl and I have conducted. In Lindahl & Odelstad (1999a), there is a reference to Hare (1989), but the relation between open intermediate concepts and prescriptivism ought to be investigated in more detail.

I have been influenced by P.W. Bridgman's operationalistic approach to concept formation and it seems to me that operationalism and the ideas about intermediate concepts fit well together in roughly the following manner: If a predicative concept is neither purely normative nor operationally definable, consider if it is an intermediate concept. To develop this dictum in detail is not, however, within the scope of the present essay.

5 Implicative closeness between strata

One important problem area in the study of *msic*-systems is the "closeness" between different strata. Some of the ideas regarding this topic will be informally described in this section.

Consider the norms (links) from the system B_1 of grounds to the system B_2 of consequences. One norm can be "narrower" than another, which is illustrated in Figure 3.[13] Suppose that $\langle a_1, a_2 \rangle$ and $\langle b_1, b_2 \rangle$ are norms from the system of grounds B_1 to the system of consequences B_2.

Figure 3 illustrates that $\langle a_1, a_2 \rangle$ is narrower than $\langle b_1, b_2 \rangle$. We can say alternatively that $\langle a_1, a_2 \rangle$ "lies between" b_1 and b_2. We define the relation 'at least as narrow as', expressed by $\trianglelefteq$, in the following way:

$$\langle a_1, a_2 \rangle \trianglelefteq \langle b_1, b_2 \rangle \text{ if and only if } b_1 \mathcal{R} a_1 \text{ and } a_2 \mathcal{R} b_2.$$

[13] See Lindahl & Odelstad (2003) p. 84.

It is easy to see that if $\mathcal{R}$ is a quasi-ordering, i.e. transitive and reflexive, then $\trianglelefteq$ is also a quasi-ordering.

A norm that is *maximally narrow* is *minimal* with respect to the relation 'at least as narrow as'. Hence, a norm $\langle a_1, a_2 \rangle$ is maximally narrow if there is no norm in the system that is strictly narrower than $\langle a_1, a_2 \rangle$, i.e. if $\langle a_1, a_2 \rangle$ is a minimal element with respect to 'at least as narrow as'. In a normative system, the set of norms that are maximally narrow play a crucial role. Given certain requirements of a well-formed normative system, all the other norms of the system are determined by its maximally narrow norms and, therefore, any change of such a system implies a change of at least one maximally narrow norm. This is discussed in Odelstad & Lindahl (2002), Lindahl & Odelstad (2003) and (2008a).

The idea behind intermediaries is that they are intermediate between different strata of concepts and offer narrow links between the strata. It is important to notice that the intermediaries between two strata constitute a stratum itself. The introduction rules of the intermediaries are links from the "bottom stratum" to the "intermediate stratum" and the elimination rules of the intermediaries are links from the "intermediate stratum" to the "top stratum". The introduction rule and the elimination rule of an intermediary constitute narrow links, since the introduction rule determines the weakest ground of the intermediary and the elimination rule the strongest consequence. Intermediate concepts are thus studied in terms of how narrow they are the structure of grounds and the structure of consequences. Generally, the "implicative closeness" between strata is analysed using concepts as minimal joining, weakest ground and strongest consequence. Figure 4 illustrates the two last mentioned notions: a_1 is a weakest ground of m if $b_1 \mathcal{R} m$ implies $a_1 \mathcal{R} b_1$. And a_2 is a strongest consequence of m if $m \mathcal{R} b_2$ implies $a_2 \mathcal{R} b_2$. As a preliminary approximation we can say that the introduction rule of an intermediary states its weakest ground and the elimination rule states its strongest consequence. In Lindahl & Odelstad (2008b), this is discussed in more detail and a rudimentary typology of intermediate concepts is established.

6 Deontic consequences

Let us for a moment return to the simple picture of a normative system consisting of a system of grounds and a system of consequences. The consequences are normative conditions. So far, what we have said about normative conditions is just that they can be constructed by applying a deontic operation to descriptive conditions. There is an extensive literature on deontic operations and it is not intended to enter this discussion here. In this essay, the combination of deontic and action logic developed by Stig Kanger will be used, especially the theory of normative positions created by Kanger and Lindahl.

6.1 Deontic logic with the action operator Do

Kanger exploited the possibilities of combining the deontic operator Shall with the binary action operator Do. The operation Do means that one sees to it

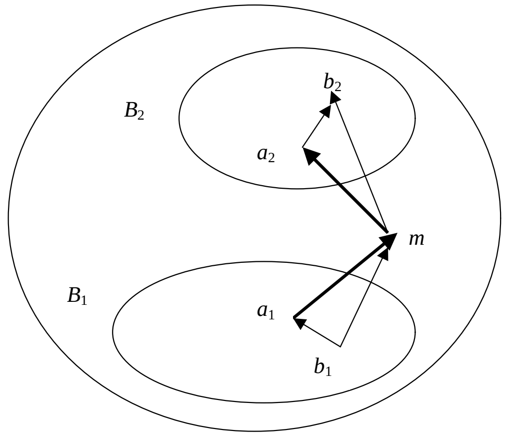

Fig. 4. m is an intermediate concept between B_1 and B_2 with weakest ground a_1 and strongest consequence a_2.

that something is the case (see Kanger, 1957). To be more exact, Shall $\mathrm{Do}(x, q)$ means that it shall be that x sees to it that q, while for example $\neg$Shall $\mathrm{Do}(y, \neg q)$ means that it is not the case that it shall be that y sees to it that not q. The combination of the deontic operator Shall with the action operator Do and the negation operation $\neg$ gives us a powerful language for expressing purely normative sentences. Kanger emphasized the possibilities of external and internal negation of sentences where these operators are combined. Using combinations of deontic and action operators, we can formulate norms in a more effective way. A conditional norm may for example have the following form: 'If p then it shall be the case that x sees to it that q', which thus can be written as

$$p \to \text{Shall Do}(x, q).$$

In such norms, p is often a state of affairs which is about x and y, while q is a state of affair which deals with y, i.e. p can be seen as predicate with x and y as variables while q is a predicate with y as the only variable. Hence, a conditional norm can have the following form:

$$p(x, y) \to \text{Shall Do}(x, \neg q(y)).$$

A concrete example of a norm which has this form is as follows. Suppose that $p(x, y)$ means that x owns y and y is a dog while $q(y)$ means that y fouls public places. The norm above then says that the owner of a dog shall see to it that the dog does not foul in public places.

Note that the sentence May $\mathrm{Do}(x, q)$ can be defined in terms of the operators Shall and Do in the following way:

$$\text{May Do}(x, q) \text{ if and only if } \neg\text{Shall } \neg\text{Do}(x, q).$$

It is worth noting that conditional norms have some similarities with production rules. According to Luger (2002) p. 171, a production rule is

> a *condition-action* pair and defines a single chunk of problem-solving knowledge. The *condition part* of the rule is a pattern that determines when that rule may be applied to a problem instance. The *action part* defines the associated problem-solving step.

The antecedent (or ground) in a norm corresponds to the condition part in a production rule, and the consequent (or consequence) in a norm corresponds to the action part. A production rule thus has the logical form

$p \to \text{Do } q$

or perhaps better

$p \to \text{Shall Do } q$.

6.2 Normative positions

In 1913, the American jurist Wesley Newcomb Hohfeld published a work in philosophy of law which has been very influential. It carries the title *Fundamental Legal Conceptions as Applied in Judicial Reasoning* and contains a characterization of eight fundamental legal notions, which were meant to serve as fundamental elements in the analysis of more complex legal relations. Inspired by Hohfeld's work, Kanger developed a theory of normative positions using the deontic-action-language. Kanger's theory of normative positions was originally expressed as a theory of types of rights. He emphasized that the term 'right' has various meanings. For example, if Mrs. x has lent 100 dollars to Mr. y, then x has a right of the simple type Claim against y that she gets back the money she has lent to y. Let

$q_1(x, y)$: x gets back the money x has lent to y.

The type of right Claim with regard to $q_1(x, y)$ is defined in the following way:

$\text{Claim}(x, y, q_1(x, y))$ if and only if $\text{Shall Do}(y, q_1(x, y))$.

This means that y shall see to it that x gets back the money she lent y. Further, Mrs. x has probably a right of type Immunity to walk outside Mr. y's shop. Let

$q_2(x, y)$: x walks outside y's shop

Immunity with regard to q_2 is defined as follows:

$\text{Immunity}(x, y, q_2(x, y))$ if and only if $\text{Shall } \neg\text{Do}(y, \neg q_2(x, y))$.

Hence, it shall be the case that y does not see to it that x does not walk outside y's shop. (These examples are taken from Lindahl, 1994, p. 891-892.)

Kanger's work was considerably improved and extended into a formal theory of normative positions in Lindahl (1977). Lindahl developed three systems of types of normative positions. The simplest one is the system of one-agent types of normative position, and only this system is used in this essay. The one-agent types are constructed in the following way. Let $\pm\alpha$ stand for either of α or $\neg\alpha$. Starting from the scheme $\pm\text{May}\pm\text{Do}(x, \pm q)$, where $\pm$ stands for the two alternatives of affirmation or negation, a list is made of all maximal and consistent conjunctions, 'maxiconjunctions', such that each conjunct satisfies the scheme.[14] Maximality means that if we add any further conjunct, satisfying the scheme, then this new conjunct either is inconsistent with the original conjunction or redundant. Note that the expression $\neg\text{Do}(x,q)\& \neg\text{Do}(x,\neg q)$ expresses x's passivity with regard to q. Here this expression is abbreviated as $\text{Pass}(x,q)$. By this procedure, the following list of seven maxiconjunctions is obtained, which are denoted $\mathbf{T}_1(x,q),\ldots,\mathbf{T}_7(x,q)$, see Lindahl (1977), p. 92.

$\mathbf{T}_1(x,q)$: $\text{MayDo}(x,q)$ & $\text{MayPass}(x,q)$ & $\text{MayDo}(x,\neg q)$.
$\mathbf{T}_2(x,q)$: $\text{MayDo}(x,q)$ & $\text{MayPass}(x,q)$ & $\neg\text{MayDo}(x,\neg q)$.
$\mathbf{T}_3(x,q)$: $\text{MayDo}(x,q)$ & $\neg\text{MayPass}(x,q)$ & $\text{MayDo}(x,\neg q)$.
$\mathbf{T}_4(x,q)$: $\neg\text{MayDo}(x,q)$ & $\text{MayPass}(x,q)$ & $\text{MayDo}(x,\neg q)$.
$\mathbf{T}_5(x,q)$: $\text{MayDo}(x,q)$ & $\neg\text{MayPass}(x,q)$ & $\neg\text{MayDo}(x,\neg q)$.
$\mathbf{T}_6(x,q)$: $\neg\text{MayDo}(x,q)$ & $\text{MayPass}(x,q)$ & $\neg\text{MayDo}(x,\neg q)$.
$\mathbf{T}_7(x,q)$: $\neg\text{MayDo}(x,q)$ & $\neg\text{MayPass}(x,q)$ & $\text{MayDo}(x,\neg q)$.

$\mathbf{T}_1,\ldots,\mathbf{T}_7$ are called the types of one-agent positions.[15] Given the underlying logic, the one-agent types are mutually disjoint and their union is exhaustive. i.e. constitute a partition. Note that $\neg\text{MayDo}\ (x,q)$ & $\neg$ MayPass (x,q) & $\neg\text{MayDo}(x,\neg q)$ is logically false, according to the logic of Shall and May.

It is easy to see that the last three types can more concisely be described as follows:

$\mathbf{T}_5(x,q)$: Shall $\text{Do}(x,q)$.
$\mathbf{T}_6(x,q)$: Shall $\text{Pass}(x,q)$.
$\mathbf{T}_7(x,q)$: Shall $\text{Do}(x,\neg q)$.
Note that the following "symmetry principles" hold (Lindahl, 1977, p. 92):
$\mathbf{T}_1(x,q)$ if and only if $\mathbf{T}_1(x,\neg q)$
$\mathbf{T}_2(x,q)$ if and only if $\mathbf{T}_4(x,\neg q)$
$\mathbf{T}_3(x,q)$ if and only if $\mathbf{T}_3(x,\neg q)$
$\mathbf{T}_5(x,q)$ if and only if $\mathbf{T}_7(x,\neg q)$
$\mathbf{T}_6(x,q)$ if and only if $\mathbf{T}_6(x,\neg q)$

In Lindahl & Odelstad (2004) and Odelstad & Boman (2004) the one-agent-types in the Kanger-Lindahl theory of normative positions are used as operators on descriptive conditions to get deontic conditions. As a simple example, suppose that r is a unary condition. Then $T_i r$ (with $1 \leq i \leq 7$) is the binary condition such that

$$T_i r(y,x) \text{ iff } \mathbf{T}_i(x, r(y)),$$

where $\mathbf{T}_i(x,r(y))$ is the ith formula of one-agent normative positions. Note that for example $\mathbf{T}_3(x,r(y))$ means

[14] The notion of 'maxiconjunction' was introduced in Makinson (1986), p. 405f.

[15] Formally, a "type" $\mathbf{T}_i$ (1≤i≤7) of one-agent positions refers to the set of all ordered pairs $\langle x,q\rangle$ such that $\mathbf{T}_i(x,q)$.

$$\text{MayDo}(x, r(y)) \ \& \ \neg\text{MayPass}(x, r(y)) \ \& \ \text{MayDo}(x, \neg r(y)).$$

T_i is called a one-agent position-operator. If $\langle p, T_i r\rangle$ is a norm, then from $p(x_1, x_2)$ we can, by using the norm, infer $T_i r(x_1, x_2)$ and thus also $\mathbf{T}_i(x_2, r(x_1))$, which means that, with regard to the state of affairs $r(x_1)$, x_2 has a normative position of type $\mathbf{T}_i$.

The theory of normative positions was developed during the 60s and 70s, primarily as an analytical tool to be used in jurisprudence and political science. The Kanger-Lindahl theory of normative positions was applied to problems in computer science in the 90s, see Jones & Sergot (1993) and (1996), Sergot (1999) and (2001), Krogh (1995) and Krogh & Herrestad (1999).

6.3 Normative systems as *msic*-systems

Conceiving of normative systems as *msic*-systems is a kind of representation of normative systems. What characterizes the subclass of normative systems among *msic*-systems in general are their cognitive features. A normative system consists of one stratum of descriptive grounds and another stratum of normative consequences and eventually one or more strata of intermediaries. Furthermore, a normative system contains links or joinings between the strata. Note that the final consequences are expressed in terms of normative conditions, for example constructed by applying deontic operations to descriptive conditions. Thus, representing normative systems in this way puts the emphasis on concepts and not on propositions.

7 The algebraic approach to *msic*-systems

The study of the structure of *msic*-systems, especially the implicative closeness between different strata, is one of the main goals of a series of papers co-authored together with Lars Lindahl (see References for details).[16] As tools for this endeavour, algebraic concepts and theories are used. In this section, two of the structures that play a crucial role as such tools will be described briefly. But first a preliminary remark.

7.1 Set-theoretical predicates

A common way of characterizing formal theories in mathematics is described by Suppes as follows:

[16] See especially Lindahl & Odelstad (2003), (2004), (2008a) and (2008b). Technical results in our papers include a characterization of an *msic*-system in terms of the most narrow joinings between different strata, characterization of the structure of the most narrow joinings between two strata, conditions for the extendability of intermediate concepts, and finally, a specification of the conditions such that the Boolean operations on intermediate concepts will result in intermediate concepts and characterization of most narrow joinings in terms of weakest grounds and strongest consequences.

> The kernel of the procedure for axiomatizing theories within set theory may be described very briefly: to axiomatize a theory is to define a predicate in terms of notions of set theory. A predicate so defined is called a *set-theoretical* predicate. (Suppes, 1957, p. 249.)

A simple example of a set-theoretical predicate is 'to be a quasi-ordering':

Definition 1. *Let A be a set and R a binary relation on A. The relational structure $\langle A, R\rangle$ is a* quasi-ordering *if for all a, b, c in A, the following axioms are satisfied:*
(1) aRa (reflexivity)
(2) If aRb and bRc, then aRc (transitivity).

'To be a quasi-ordering' is a predicate, which is true or false of relational structures. This set-theoretical predicate characterizes an axiomatized theory, the theory of quasi-orderings, and a model of that theory is a structure satisfying the predicate 'to be a quasi-ordering'.

Two set-theoretical predicates which play a crucial role in the *msic*-theory will now be presented.

7.2 Boolean quasi-orderings and joining systems

Definition 2. *The relational structure $\langle B, \wedge, ', R\rangle$ is a* Boolean quasi-ordering (Bqo) *if $\langle B, \wedge, '\rangle$ is a Boolean algebra, R is a quasi-ordering, $\bot$ is the zero element, $\top$ is the unit element and R satisfies the additional requirements:*
(1) aRb and aRc implies $aR(b \wedge c)$,
(2) aRb implies $b'Ra'$,
(3) $(a \wedge b)Ra$,
(4) not $\top R \bot$.

Boolean algebras are well-known structures with many applications. A Boolean quasi-ordering is a quasi-ordering defined on a Boolean algebra in such a way that it determines a new Boolean algebra related to the first one in a special way. This is explained in more detail in Lindahl & Odelstad (2004). The definition of a Boolean joining system, which follows below, presupposes the definition of a Boolean quasi-ordering. Many normative systems can be represented as Boolean joining systems or combinations of two or more such systems. First a reminder of a notion discussed earlier:

Definition 3. *The* narrowness-relation determined by *the quasi-orderings $\langle B_1, R_1\rangle$ and $\langle B_2, R_2\rangle$ is the binary relation $\trianglelefteq$ on $B_1 \times B_2$ such that $\langle a_1, a_2\rangle \trianglelefteq \langle b_1, b_2\rangle$ if and only if $b_1 R_1 a_1$ and $a_2 R_2 b_2$.*

Note that $\trianglelefteq$ is a quasi-ordering on $B_1 \times B_2$.

Definition 4. *A* Boolean joining system (Bjs) *is an ordered triple $\langle \mathcal{B}_1, \mathcal{B}_2, J\rangle$ such that $\mathcal{B}_1 = \langle B_1, \wedge, ', R_1\rangle$ and $\mathcal{B}_2 = \langle B_2, \wedge, ', R_2\rangle$ are Bqo's and $J \subseteq B_1 \times B_2$, and the following requirements are satisfied:*

(1) for all $b_1, c_1 \in B_1$ *and* $b_2, c_2 \in B_2$, $\langle b_1, b_2\rangle \in J$ *and* $\langle b_1, b_2\rangle \trianglelefteq \langle c_1, c_2\rangle$ *implies* $\langle c_1, c_2\rangle \in J$,
(2) for any $C_1 \subseteq B_1$ *and* $b_2 \in B_2$, *if* $\langle c_1, b_2\rangle \in J$ *for all* $c_1 \in C_1$, *then* $\langle a_1, b_2\rangle \in J$ *for all* $a_1 \in lub_{R_1}C_1$,
(3) for any $C_2 \subseteq B_2$ *and* $b_1 \in B_1$, *if* $\langle b_1, c_2\rangle \in J$ *for all* $c_2 \in C_2$, *then* $\langle b_1, a_2\rangle \in J$ *for all* $a_2 \in glb_{R_2}C_2$.

A norm can, as has been pointed out above, in many contexts be regarded as consisting of two objects, a ground condition and a consequence condition standing in an implicative relation to each other. The ground belongs to one Boolean quasi-ordering and the consequence to another. Therefore, we can view a normative system as a set of joinings of a Boolean quasi-ordering of grounds to a Boolean quasi-ordering of consequences, where $\wedge$ and $'$ are Boolean operations on the conditions. A normative system $\mathcal{N}$ can therefore be represented as a Boolean joining system $\langle \mathcal{B}_1, \mathcal{B}_2, J\rangle$ where $\mathcal{B}_1 = \langle B_1, \wedge, ', R_1\rangle$ is a Boolean quasi-ordering of ground-conditions, $\mathcal{B}_2 = \langle B_2, \wedge, ', R_2\rangle$ a Boolean quasi-ordering of consequence-conditions and the set J, where $J \subseteq B_1 \times B_2$, is the set of norms. Note that the implicative relation in the system $\mathcal{N}$ is represented in the different parts of the system by the relations R_1, R_2 and J respectively.

It is worth noting that there is a difference in notational conventions between the definition of a *Bqo* and the definition of a *Bjs*. In a *Bqo*, if the relation R holds between a and b this is written aRb. If in a *Bjs* J holds between a_1 and a_2 this is written $\langle a_1, a_2\rangle \in J$. The reason is that in the intended models of *Bjs*'s, the elements in J are treated as objects in a way that does not hold for the elements in R. In a representation of a *Bjs* as a normative system, $\langle a_1, a_2\rangle \in J$ means that the norm $\langle a_1, a_2\rangle$ holds in the system, and the elements in J are subject to comparison with respect to, for example, narrowness.

Given the narrowness relation $\trianglelefteq$ one can determine the set of minimal elements of J, $\min J$, with respect to $\trianglelefteq$. Under fairly general conditions, the set $\min J$ characterizes J in the following way:

$$\langle a_1, a_2\rangle \in J \;\text{ iff }\; \exists \langle b_1, b_2\rangle \in \min J : \langle b_1, b_2\rangle \trianglelefteq \langle a_1, a_2\rangle\,.$$

Given certain general presuppositions, one can choose a subset C of $\min J$ from which $\min J$ can be inferred and which therefore also determines J. We call such a set C a *base of minimal elements* of J. In many contexts, the elements in C can be represented by intermediate concepts. An intermediary is determined by the condition that constitute its maximally narrow ground and the condition that constitutes its maximally narrow consequence. See Lindahl & Odelstad (2008a) and (2008b) for further details.

7.3 Models and variations of the algebraic theories

As has been emphasized in earlier sections with normative systems as a key example, one approach to the representation of *msic*-systems is by regarding concepts as conditions subject to Boolean operations and with an implicative relation defined on these conditions. A *Bqo* or a *Bjs* with domains of conditions is called a *condition implication structure*, abbreviated *cis*. A special kind of *cis*-representation of a normative system is the *npcis*-representation of normative conditions. In an *npcis,* a normative condition is constructed by applying the one-agent position-operators to descriptive conditions (see Lindahl & Odelstad, 2004).

There are some limitations of the *cis*-representation of *msic*. One problem is the formation of conjunctions and disjunctions of conditions of different arity. How this can be handled is discussed in Lindahl & Odelstad (2004) section 3. Another weakness of the *cis*-representation is that new conditions can only be constructed out of given conditions by Boolean operations. As a consequence, it is, for example, not possible to define within a *cis* the condition 'to be the grandfather of' in terms of the conditions 'to be the father of' and 'to be the mother of'. Note that if we want 'grandfather' to be a condition in our *cis* we can of course include it as a primitive condition.

With reference to the limitations mentioned above, it might be held that the *cis*-representation is too simple to be suitable for an overall representation of an actual legal system or a complex *msic*-systems of some other kind. Nevertheless, the *cis*-representation is sufficiently rich to permit a detailed study of a number of issues pertaining especially to intermediate concepts in a legal system.[17] The *cis*-representation can in a sense be viewed as an "idealized model" for studying different phenomena in *msic*-systems. When judging the usefulness of the *cis*-representation it is worth noting the following: Even if there are a number of difficulties when it comes to a detailed representation of norms as joinings in a Boolean joining system, it may be the case that these difficulties do not appear when the objective in view is rather to construct an artificial normative system regulating an artificial multiagent-system.

Condition implication structures are not the only kind of models of Boolean joining systems that are interesting as representations of *msic*-systems. It is easy to see that we can construct a *Bqo* out of a first order theory Σ. Consider the structure $\langle B, \wedge, ', R\rangle$ where $\langle B, \wedge, '\rangle$ is the Lindenbaum algebra of the predicate calculus. Let R be the quasi-ordering on B determined by the Lindenbaum algebra of Σ. Then $\langle B, \wedge, ', R\rangle$ is a Boolean quasi-ordering.

Boolean joining systems are obviously based on the notion of a Boolean algebra. However, it is possible to define an analogous kind of systems based on lattices. Such a system $\langle \mathcal{L}_1, \mathcal{L}_2, J\rangle$ consists of the latticed quasi-orderings $\mathcal{L}_1 = \langle L_1, \wedge, \vee, R_1\rangle$ and $\mathcal{L}_2 = \langle L_2, \wedge, \vee, R_2\rangle$ and the set J of joinings between them and can be called a *latticed joining system,* abbreviated *Ljs.* A large fraction of the formal result proved for *Bjs*'s will hold also for *Ljs*'s, roughly because the complement operation in the Boolean algebras does not play a role in the proofs. There are models of the theory of *Ljs* that can be interesting representations of *msic*-systems. This holds, for example, when the concepts in the *msic*-systems

[17] Cf. Lindahl & Odelstad (2006b), where it is suggested that a representation based on cylindric algebras would be more appropriate than a representation based on Boolean algebras.

are not conditions but instead for instance aspects or equality relations for aspects.

7.4 The formal representation of *msic*-systems

The formal theory of *msic*-systems is to a large extent a question of representation. The algebraic framework for the representation of *msic*-systems in the work that Lindahl and I have conducted has gone through different "stages" and I will outline and discuss these stages here.

Stage 1: Lattice-representation In Lindahl & Odelstad (1996) and (1999a), an *msic*-system is represented as a lattice $\langle L, \leq \rangle$ of conditions extended with a quasi-ordering ρ. The lattice operations represent conjunction and disjunction respectively. Negation is not included for purely pragmatic reasons; in the first version of the theory we preferred to simplify the matter but still be able to express our main ideas about intermediaries. The partial ordering $\leq$ in the lattice represents "logical implication" and the quasi-ordering ρ represent implications in a more general sense. The relation between the partial ordering $\leq$ and the quasi-ordering ρ is such that the partial ordering $\leq_\rho$ generated from ρ by the formation of equivalence classes is a lattice and $\leq$ is a subset of $\leq_\rho$. $\langle L_\rho, \leq_\rho \rangle$ is the quotient algebra of $\langle L_\rho, \leq_\rho \rangle$ with the respect to the indifference part of ρ. A two-sorted conceptual system is represented as a system of two sublattices $\langle L_1, \leq_1 \rangle$ and $\langle L_2, \leq_2 \rangle$ of $\langle L, \leq \rangle$ and the set $\{\langle x_1, x_2 \rangle \in L_1 \times L_2 \mid x_1 \leq_\rho x_2\}$ of joinings between the sublattices.

Stage 2: *Bqo*-representation In Odelstad & Lindahl (1998), the formal framework for representing *msic*-systems is modified in some respects:
(1) We incorporate the operation of negation and suppose that the conditions constitute a Boolean algebra $\langle B, \wedge, ' \rangle$.
(2) We do not make a transition to the quotient algebra of $\langle B, \wedge, ' \rangle$ with respect to the indifference part of ρ. Instead we construct the Boolean quasi-ordering $\langle B, \wedge, ', \rho \rangle$. The reason is that we want to distinguish between two conditions even if they are indifferent with respect to ρ. See Lindahl & Odelstad (2004) section 2.1.
(3) We make a clearer separation between the algebraic theories and the models used for the representation of *msic*-systems. In stage 1, we regarded the lattice operations $\wedge$ and $\vee$ as representing conjunction and disjunction of conditions, since we only had one intended model in view. A *Bqo* of conditions is one kind of *Bqo*-model which can be used for representing *msic*-systems and we do not exclude the possibility that there can be other kinds of models.

Note that an *msic-system* is represented as a system of substructures of $\langle B, \wedge, ', \rho \rangle$, called fragments, and the set of joinings between them. The formal tools for the representation of *msic*-systems based on the *Bqo*-theory is further developed in Odelstad & Lindahl (2000), Lindahl and Odelstad (2000) and (2004). This *Bqo*-representation is used in Odelstad & Boman (2004) and Lindahl & Odelstad (2003).

Stage 3: *Bjs*-representation In the *Bqo*-representation of *msic*-systems, strata of concepts of different sorts are represented as fragments of the basic *Bqo* $\langle B, \wedge, ', \rho \rangle$. Hence, B contains conditions of different sorts. But B contains also Boolean combinations of concepts of different sorts, i.e. compound concepts of a "mixed sort". In many contexts, however, concepts of such mixed sorts are not of any interest and make the situation unnecessarily complicated. To avoid this complication, a *Bjs* can be a useful tool for representations. A two-sorted conceptual system is then represented as a *Bjs* $\langle \mathcal{B}_1, \mathcal{B}_2, J \rangle$ consisting of two *Bqo*'s $\mathcal{B}_1$ and $\mathcal{B}_2$ together with the set of joinings J between them. The *Bqo*'s $\mathcal{B}_1$ and $\mathcal{B}_2$ are not necessarily fragments of one *Bqo*. The axioms of a *Bjs* are such that two fragments of a *Bqo* and the joinings between them constitute a *Bjs*.

However, if one wants to study *msic*-systems containing conditions of several different sorts, this would involve a number of *Bjs*'s related to each other in a complicated way. It may then be useful to have as a background a Boolean algebra $\langle B, \wedge, ' \rangle$ representing the "language" of the *msic*-system and a binary relation ρ representing the non-logical (for example normative) content of the system. The sets of joinings between different strata of concepts will then be contained in ρ. A *msic*-system may therefore appropriately be represented as a *supplemented Boolean algebra*, abbreviated *sBa*, $\langle B, \wedge, ', \rho \rangle$ with *Bjs*'s lying within it. This is the approach in Lindahl & Odelstad (2008a) and (2008b).

7.5 Non-Boolean joining systems

In this section, two examples of joining systems consisting of concepts but not constituting *Bjs* will be outlined briefly.

Joining systems of equality-relations In this essay I have focused on *msic*-systems where the concepts are conditions subject to the Boolean operations. But there are kinds of conditions that do not constitute Boolean algebras. One example is equality-relations. The term 'equality-relation' here refer to a relation of equality with respect to some aspect α, and it is presupposed in this context that an equality-relation is always an equivalence-relation, i.e. a reflexive, transitive and symmetric relation. Let A be a non-empty set and let $E(A)$ be the set of equivalence relations on A. Define the binary relation $\leq$ on $E(A)$ in the following way: For all $\varepsilon_1, \varepsilon_2 \in E(A)$

$$\varepsilon_1 \leq \varepsilon_2 \text{ iff } x\varepsilon_1 y \text{ implies } x\varepsilon_2 y. \tag{1}$$

The reader should be reminded of the following well-known fact. $\mathcal{E}(A) = \langle E(A), \leq \rangle$ is a complete lattice. Note that the negation ε' of an equivalence relation $\varepsilon \in E(A)$ is not an equivalence relation, i.e. $\varepsilon' \notin E(A)$. Let $\mathcal{E}_1 = \langle E_1, \leq_1 \rangle$ and $\mathcal{E}_2 = \langle E_2, \leq_2 \rangle$ be disjoint complete sublattices of $\mathcal{E}(A)$ and consider $\langle \mathcal{E}_1, \mathcal{E}_2, J \rangle$ where $J = \leq / (E_1(A) \times E_2(A))$. Given some general conditions $\langle \mathcal{E}_1, \mathcal{E}_2, J \rangle$ is a joining system. We have here an example of a joining system which consists of conditions but they do not constitute a Boolean algebra.

A Boolean quasi-ordering is a Boolean algebra extended with a quasi-ordering satisfying certain conditions. We can define an analogous structure based on a lattice instead of a Boolean algebra. Let $E(A)$ and $\leq$ be as above and let $\langle E(A), \wedge, \vee\rangle$ be the lattice $\langle E(A), \leq\rangle$ expressed in terms of operations instead of a partial ordering, i.e. $\varepsilon_1 \wedge \varepsilon_2 = \inf\{\varepsilon_1, \varepsilon_2\}$ and $\varepsilon_1 \vee \varepsilon_2 = \sup\{\varepsilon_1, \varepsilon_2\}$. Suppose that R is a quasi-ordering on $E(A)$ such that

(1) aRb and aRc implies $aR(b \wedge c)$.
(2) aRc and bRc implies $(a \vee b)Rc$.
(3) $(a \wedge b)Ra$.
(4) $aR(a \vee b)$.

Then $\langle E(A), \wedge, \vee, R\rangle$ is called a *latticed quasi-ordering*. The transition to the quotient algebra of $\langle E(A), \wedge, \vee\rangle$ with respect to the indifference part of R will result in a lattice. (Cf. Lindahl & Odelstad, 1999a, p. 171.) The *msic*-systems consisting of equality-relations can often be represented as latticed quasi-orderings, and this also holds for *msic*-systems consisting of aspects.

Joining systems of aspects As pointed out above, this essay has focused on *msic*-systems where the concepts are conditions. But there are other kinds of concepts, for example aspects, in many disciplines called attributes. As examples of aspects let me mention a few: area, temperature, age, loudness and archeological value. It is a common view of aspects that they can, in some way or another, be represented as relational structures. In Odelstad (1992), a theory of aspects, where aspects are represented by systems of relationals, is set out. A relational is a function with sets as arguments and structures as values. On sets of systems of relationals, several quasi-orderings can be defined but here only one example will be given.

Let $\mathrm{Rels}\,\mathcal{D}$ denote the set of systems of relationals whose range of definition is the family $\mathcal{D}$ of sets. This means that for all $\Re \in \mathrm{Rels}\,\mathcal{D}$ it holds that $\Re = \langle\rho_i\rangle_{i\in I}$ for some set I and for all $A \in \mathcal{D}$, $\rho_i(A) \subseteq A^{\nu_i}$ where ν_i is the arity of the relational ρ_i. Hence, $\Re(A) = \langle A, \rho_i\rangle_{i\in I}$. Let $\Im(\Re(A), \Re(B))$ denote the set of isomorphisms from $\Re(A)$ to $\Re(B)$. We can define a relation *sub* on $\mathrm{Rels}\,\mathcal{D}$ in the following way: If $\Re_1, \Re_2 \in \mathrm{Rels}\,\mathcal{D}$ then

$$\Re_2 \; sub \; \Re_1 \quad \text{iff} \quad \text{for all } A, B \in \mathcal{D} : \Im(\Re_2(A), \Re_2(B)) \supseteq \Im(\Re_1(A), \Re_1(B)). \quad (2)$$

It is obvious that *sub* is a quasi-ordering on $\mathrm{Rels}\,\mathcal{D}$. It follows from Odelstad (1992) that $\langle \mathrm{Rels}\,\mathcal{D}, sub\rangle$ is a complete quasi-lattice and it is therefore possible that there are joining systems lying within $\langle \mathrm{Rels}\,\mathcal{D}, sub\rangle$.[18] The relational

[18] If $\langle A, R\rangle$ is a quasi-ordering such that $\mathrm{lub}_R\{a, b\} \neq \varnothing$ and $\mathrm{glb}_R\{a, b\} \neq \varnothing$ for all $a, b \in A$, then $\langle A, R\rangle$ will be called a *quasi-lattice.* If $\mathrm{lub}_R X \neq \varnothing$ and $\mathrm{glb}_R X \neq \varnothing$ for all $X \subseteq A$, then a quasi-ordering $\langle A, R\rangle$ is a *complete quasi-lattice.*

Suppose that $\langle A, R\rangle$ is a quasi-lattice, Q the equality-part of R and A_Q is the set of Q-equivalence classes generated by elements of A. Then $\langle A_Q, \rho\rangle$, where $[a]_Q\, \rho\, [b]_Q$ iff aRb, is a lattice. If $\langle A, R\rangle$ is a complete quasi-lattice then $\langle A_Q, \rho\rangle$ is a complete lattice.

systems in $\langle \mathrm{Rels}\,\mathcal{D}, sub\rangle$ can be of different sorts and it is a meaningful question if they form joining systems or even latticed joining systems. Note that in $\langle \mathrm{Rels}\,\mathcal{D}, sub\rangle$ the implicative relation *sub* is not implication in the usual sense but expresses a kind of dependence relation.

7.6 A remark on input-output logic

In a series of papers, Makinson and van der Torre have developed a highly interesting theory called input-output logic, see for example Makinson and van der Torre (2000) and (2003). One striking similarity between input-output logic and the theory of *msic*-systems is that norms are represented as ordered pairs. This observation raises the question if there are some deep similarities between input-output logic and *msic*-theory. However, let me first state some obvious differences between the two theories. While *msic*-systems are by definition at least two-sorted, this does not holds for input-output logic. A common feature of the study of *msic*-systems reported here is the implicative closeness between strata of different sorts in an *msic*-system. An analogous study does not seem to have been carried out for input-output logic. The strata of an *msic*-system of conditions are Boolean structures (*Bqo's* to be more precise), but the strata of *msic*-systems of other kinds need not be Boolean structures; instead, they can for example be lattice-like structures. In input-output logic, the set of inputs constitute a Boolean algebra and the same holds for the set of outputs.

The following remark sheds some light on the relation between input-output logic and the theory of *msic*-systems. (Knowledge of input-output logic is presupposed.) Suppose that $\langle \mathcal{B}_1, \mathcal{B}_2, J\rangle$ is a *Bjs* where $\mathcal{B}_1 = \langle B_1, \wedge, ', R_1\rangle$ and $\mathcal{B}_2 = \langle B_2, \wedge, ', R_2\rangle$. Makinson and van der Torre state a number of rules for the output operators they define. Translated to a *Bjs* these rules are as follows:

Strengthening Input: From $\langle a_1, a_2\rangle \in J$ to $\langle b_1, a_2\rangle \in J$ whenever $b_1 R_1 a_2$.
Follows from condition (1) of a *Bjs*.
Conjoining Input: From $\langle a_1, a_2\rangle \in J$ and $\langle a_1, b_2\rangle \in J$ to $\langle a_1, a_2 \wedge b_2\rangle \in J$.
Follows from condition (3) of a *Bjs*.
Weakening Output: From $\langle a_1, a_2\rangle \in J$ to $\langle a_1, b_2\rangle \in J$ whenever $a_2 R_2 b_2$.
Follows from condition (1) of a *Bjs*.
Disjoining Input: From $\langle a_1, a_2\rangle \in J$ and $\langle b_1, a_2\rangle \in J$ to $\langle a_1 \vee b_1, a_2\rangle \in J$.
Follows from condition (2) of a *Bjs*.

There are three conditions on a joining space in a Boolean joining system. The comparison with input-output logic above shows that it could be of interest to define weaker kinds of systems characterized by, for example, condition (1) and (3).

8 Applications of *msic*-systems in agent theory

8.1 Introduction

The applications of the theory of *msic*-systems in computer science can follow different paths. One path goes through the representation of normative systems as *msic*-systems and the applications of normative systems in computer science. Along a related path, the focus is on intermediate concepts, which are important in normative systems but also in other kinds of systems, for example in knowledge representation systems. A third path is the use of conceptual structures in fields like the Semantic Web and information extraction. Here a few comments on the use of *msic*-systems in the theory of artificial agents will be made, where the *msic*-systems will mainly represent normative systems.

8.2 Agent *oeconomicus norma*

Within economic theory the consumer's behaviour has traditionally been described as determined by a utility function. During the last three decades there has been a growing interest among researchers in how norms (for example rules of law) pose restrictions on the behaviour induced by the utility function. The behaviour of the consumers or other economic agents, according to this model, is the result of the interplay between optimization of the utility function and restrictions due to norms. We may perhaps speak of *norm-regulated Homo oeconomicus*. It has also been suggested that a model of this kind could be used for regulating the behaviour of artificial agents. We can perhaps call this model *Agent oeconomicus norma.* The role that norms will have in regulating the behavior of agents is, according to this model, to delimit the autonomy of the agents. Metaphorically one can say that the norms define the scope (*Spielraum*) for an agent. The agent chooses the act it likes best within the scope determined by the norms.

Norm-regulation of agents presupposes a precise and significant representation of norms and normative systems. As was explained in previous sections, a norm is here represented as an implicative sentence where the antecedent is a descriptive condition stating the circumstances of an agent, and the consequent is a condition expressing the normative or deontic position that the agent has with respect to a state of affairs. Hence, from the norms of the system will follow a deontic structure over possible state of affairs implying that some states may be permissible while the rest are non-permissible. The "wish" or "desire" of an agent is represented as a preference structure over possible states or situations. The agent chooses an act which leads to one of the permissible states that it prefers the most.

In Odelstad & Boman (2004), the ideas outlined above were developed using the typology of normative (deontic) positions developed by Kanger and Lindahl and the algebraic representation of normative systems that Lindahl and I have developed. The aim of Odelstad & Boman (2004) was to present a model of how norms can be used to regulate the behaviour of multiagent-systems on the assumption that the role of norms is to define the *Spielraum* for an agent.[19] An abstract architecture was defined in terms of a set-theoretical predicates and a Mas (a multiagent-system) having this architecture is called a norm-regulated Dalmas.[20] One of the results in Odelstad & Boman (2004) was a scheme for how normative positions will restrict the set of actions that the agents are permitted to choose from.

8.3 Normative positions regulating actions

A Dalmas is an ordered 7-tuple $\langle \Omega, S, A, \mathcal{A}, \Delta, \Pi, \Gamma \rangle$ containing

- an agent set Ω ($\omega, \varkappa, \omega_1, ...$ elements in Ω),
- a state or phase space S ($r, s, s_1, ...$ elements in S),
- an action set A such that for all $a \in A$, $a : \Omega \times S \to S$ such that $a(\omega, r) = s$ means that if the agent ω performs the act a in state r, then the result will be state s ($a, b, a_1, ...$ elements in A),
- a function $\mathcal{A} : \Omega \times S \to \wp(A)$ where $\wp(A)$ is the power set of A; $\mathcal{A}(\omega, s)$ is the set of acts accessible (feasible) for agent ω in state s,
- a deontic structure-operator $\Delta : \Omega \times S \to \mathcal{D}$ where $\mathcal{D}$ is a set of deontic structures of the same type with subsets of A as domains and $\Delta(\omega, s)$ is ω's deontic structure on $\mathcal{A}(\omega, s)$ in state s,
- a preference structure-operator $\Pi : \Omega \times S \to \mathcal{P}$ where $\mathcal{P}$ is a set of preference structures of the same type with subsets of A as domains and $\Pi(\omega, s)$ is ω's preference structure on $\mathcal{A}(\omega, s)$ in state s,
- a choice-set function $\Gamma : \Omega \times S \to \wp(A)$ where $\Gamma(\omega, s)$ is the set of actions for ω to choose from in state s.

Note that in the definition the Cartesian product $\Omega \times S$ motivates the introduction of a name for the elements in $\Omega \times S$. Let $\mathfrak{D}$ be a Dalmas. A *situation* for the system $\mathfrak{D}$ is determined by the agent to move ω and the state s. A situation is represented by an ordered pair $\langle \omega, s \rangle$. The set of situations for $\mathfrak{D}$ is thus $\Omega \times S$.

The idea behind a norm-regulated Dalmas is roughly the following: What is permissible for an agent to do in a situation $\langle \omega, s \rangle$ is determined by a normative system $\mathcal{N}$. This idea can be explicated in the following way. Let

$$T_i d(\omega_1, ..., \omega_\nu, \omega; \omega, s) \tag{3}$$

mean that in the situation where it is the agent ω's turn to draw and the state of the system is s, ω has the normative position of type $\mathbf{T}_i$ with regard to the state of affairs $d(\omega_1, ..., \omega_\nu)$.

Prohibited$_{\omega,s}(a)$ means that in the situation where it is ω's turn to draw and the state of the system is s, ω is prohibited to execute the act a.

[19] For the use of the term 'Spielraum' in this context, see Lindahl (1977) and Lindahl (2005).

[20] The term Dalmas is chosen since the architecture is constructed for the application of *d*eontic-*a*ction *l*ogic.

The following seven principles establish connections between the condition $T_i d$ and the predicate Prohibited (see Odelstad & Boman, 2004, p. 160f.):

1. From $T_1 d(\omega_1, ..., \omega_\nu, \omega; \omega, s)$ follows no restriction on the acts.
2. From $T_2 d(\omega_1, ..., \omega_\nu, \omega; \omega, s)$ follows that
   if $d(\omega_1, ..., \omega_\nu; s)$ and $\neg d(\omega_1, ..., \omega_\nu; a(\omega, s))$ then $\text{Prohibited}_{\omega,s}(a)$.
3. From $T_3 d(\omega_1, ..., \omega_\nu \omega; \omega, s)$ follows that
   if $[d(\omega_1, ..., \omega_\nu; s)$ iff $d(\omega_1, ..., \omega_\nu; a(\omega, s))]$ then $\text{Prohibited}_{\omega,s}(a)$.
4. From $T_4 d(\omega_1, ..., \omega_\nu, \omega; \omega, s)$ follows that
   if $\neg d(\omega_1, ..., \omega_\nu; s)$ and $d(\omega_1, ..., \omega_\nu; a(\omega, s))$ then $\text{Prohibited}_{\omega,s}(a)$.
5. From $T_5 d(\omega_1, ..., \omega_\nu, \omega; \omega, s)$ follows that
   if $\neg d(\omega_1, ..., \omega_\nu; a(\omega, s))$ then $\text{Prohibited}_{\omega,s}(a)$.
6. From $T_6 d(\omega_1, ..., \omega_\nu, \omega; \omega, s)$ follows that
   if not $[d(\omega_1, ..., \omega_\nu; s)$ iff $d(\omega_1, ..., \omega_\nu; a(\omega, s)]$ then $\text{Prohibited}_{\omega,s}(a)$.
7. From $T_7 d(\omega_1, ..., \omega_\nu, \omega; \omega, s)$ follows that
   if $d(\omega_1, ..., \omega_\nu; a(\omega, s))$ then $\text{Prohibited}_{\omega,s}(a)$.

These principles can be used to define a deontic structure-operator Δ such that to each agent ω in a state s is assigned the set of feasible acts a that are not eliminated as $\text{Prohibited}_{\omega,s}(a)$ according to the rules (1)-(7) above. Since

$\text{Prohibited}_{\omega,s}(a)$ is equivalent to $\neg\text{Permissible}_{\omega,s}(a)$.

it follows that

$$\Delta(\omega, s) = \{\text{Permissible}_{\omega,s}(a) : a \in A\}.$$

Note that at the outset, all feasible acts are permissible. The basic idea is that we eliminate elements from the set of permissible acts for ω in s using the norms and sentences expressing what holds for the agents with respect to grounds in the norms.

The method used for representing norms in an architecture for norm-regulated Mas can be of importance for the effectiveness of the architecture. Here a few examples of what can be regarded as desiderata for a norm-representation method are mentioned.

1. The system of norms is depicted in a lucid, concise and effective way.
2. Changes and extensions of the normative system are easily described.
3. The normative system can be divided in different parts which can be changed independently.
4. The multi-agent system can by itself change the normative system wholly or partially.

The last item in the list may deserve a comment. It is often difficult to predict the effect of a normative system for a Mas or the effect of a change of norms. It is therefore desirable that the Mas can by itself evaluate the effect of the normative system and compare the result with other normative systems that it changes to. The result can be a kind of evolution of normative systems obtained by machine learning.

In Odelstad & Boman (2004), the *npcis*-model was used for representing normative systems, which resulted in an opportunity to test some aspects of this kind of representation in the area of multiagent-systems.

8.4 Prolog implementation of norm-regulated DALMAS

In Hjelmblom (2008), an implementation in Prolog of the theory of a norm-regulated DALMAS is presented. The algebraic theory is instrumentalized through an executable logic program. Important issues in the transition from a set-theoretical description to a Prolog implementation are discussed. Results include a general-level Prolog implementation, which may be freely used to implement specific systems.

The Prolog implementation gives a procedural semantics to the algebraic theory, see Lloyd (1987). Running the Prolog program has not only pedagogical value, but can aid understanding of the implications of changing parts of the underlying theory. The fact that the Prolog program runs without notably long response time also testifies, albeit informally, to the acceptable computational complexity of the canonical model. Any domain-specific model created with Hjelmblom's Prolog implementation can have its computational complexity analysed more formally through algorithmic analysis if necessary (see Purdom Jr. & Brown, 1985).

8.5 Norms and forest cleaning

Forest management treatments presuppose, in a state of incomplete information, principles for choosing those trees that ought to be taken away and those that shall be left standing. In this section, which is a report on a work in progress carried out in cooperation with Ulla Ahonen-Jonnarth, the question is raised whether those principles can be structured as a combination of a normative system and a utility function. Of special interest is the possibility to evaluate the efficiency of the normative system and the utility function and, furthermore, suggest improvements of them.[21]

In the forest industry there is an increasing interest in the automation of forest management treatments, perhaps with the ultimate goal that autonomous robots will be able to do a substantial part of such work. But before robots of this kind can be constructed many difficult problems must be solved, for example how the robots will perceive the environment and how they will transport themselves. But there are also decision-making problems involved. Three important kinds of forest management treatments are cleaning, thinning and harvesting, and they all require methods or principles for making decisions about which trees shall be removed and which will be left standing. Such "remove-decisions" must be made on-line with information based only on the robot's nearest vicinity and about that part of the stand already cleared. The treatment cannot be evaluated

[21] This section is based on Ahonen-Jonnarth & Odelstad (2005), Ahonen-Jonnarth & Odelstad (2006) and Odelstad (2007).

until the actual stand is completely cleared. Testing and evaluating principles for remove-decisions by field experiments is expensive and time-consuming. It is therefore an interesting question wether evaluating experiments could be made *in silico*, i.e. through simulation.

In Ahonen-Jonnarth & Odelstad (2005), a platform for simulation of young forest stands is presented. Given field data of a special type of young forest, for example a 10-year-old, somewhat damp, spruce forest at 200 meters above sea level in the middle of Sweden, it is possible to simulate different stands of this type of forest. Field data of a few different types of young forests has so far been used for simulation. As a base for the simulation of different stands of the same forest type, it is of course also possible to use man-made, artificial data, or to assign values to the parameters that govern the simulation.

One of the goals of our present work on automation of forest cleaning is to formulate different principles for making the remove-decisions, test the principles in simulated forests of different types and evaluate and compare the results. We are especially interested in the possibility that, given a method for evaluating the result of cleaning, the system can improve the decision-making principles and even suggest new ones on the basis of machine learning. How the principles for the remove-decisions ought to be formally represented seems to be a complicated question. One possibility we want to investigate is to use norm-regulated DALMAS as the architecture for a cleaning agent. At this preliminary stage, a cleaning agent is regarded as “a solitary being” and, hence, a cleaning DALMAS is a one-agent-system (thus more correctly a DALOAS), but we will here regard a one-agent-system as a degenerated MAS. But at a later stage, more then one agent may be involved, for example can ‘nature’ be regarded as an agent or can individual trees be regarded as agents. The last mentioned alternative is especially interesting if the growth of a forest stand is incorporated in the simulation.

A DALMAS can achieve the cleaning-decisions for a stand p in the following way. The stand is divided into n different areas. A state for the system is the stand with i areas cleaned, where $1 \leq i \leq n$, and a specification of what area to clean next. The initial state is the stand with 0 areas cleaned and the final state is the state with n areas cleaned. Let each area be denoted by a unique number between 1 and n, and let S_i be the ith state. C_i denotes the set of cleaned areas and U_i the set of uncleaned areas in S_i. Thus, $C_i \cup U_i = \{1, 2, \ldots, n\}$ and $C_i \cap U_i = \emptyset$. C_i contains i numbers and U_i contains $n-i$ numbers. $S_i = \langle C_i, U_i, j \rangle$ where j is the area which will be cleaned next, i.e. $j \in U_i$ and $S_{i+1} = \langle C_i \cup \{j\}, U_i \backslash \{j\}, k \rangle$ for some $k \in U_i \backslash \{j\}$.

A few examples of possible norms regulating a cleaning DALMAS are given below:

(a) If there is only one undamaged tree in the area to be cleaned with a diameter within the desirable range, then this tree shall be saved.

(b) If there is at least one undamaged tree in the area to be cleaned with a diameter within the desirable range, then a damaged tree with a diameter below the desirable range may be taken away.

(c) If, in the area to be cleaned, a tree t is damaged and is closer than 0.5m to an undamaged tree with a diameter within the desirable range and with distances to other undamaged trees larger than 0.5m, then t may not be saved.

In many situations, the norms of a DALMAS do not determine the action to be taken in each state, but utility considerations are also necessary. Given a utility function we can search for the optimal way of cleaning the actual area, on the assumption that the cleaning satisfies the given norms.

For the possibility of using norms in the automation of forest cleaning in the way outlined above, it may be an important issue whether the cleaning system can optimize the system of norms regulating its remove-decisions. This is a special case of a more general problem: Suppose that $\mathfrak{D}$ is a DALMAS, where the agents cooperate to solve a problem. Which normative system will lead to the most effective behavior of the system? It is desirable that $\mathfrak{D}$ itself could determine the optimal normative system for the task in question. Given a set of grounds and a set of consequences, which together constitute the vocabulary of the system, $\mathfrak{D}$ can test all possible sets of minimal norms (in many cases satisfying certain constraints, for example represented by intermediaries). If there is a function for evaluating the result of a run of $\mathfrak{D}$, then different normative systems can be compared and the best system can be chosen. A change of vocabulary corresponds to a "mutation" among normative systems and can lead to dramatic changes in the effectiveness. Note that, in principle, the evaluation function can be very complicated, for example it can be multi-dimensional.

ACKNOWLEDGEMENT
Heartfelt thanks to Lars Lindahl, Magnus Boman, Ulla Ahonen-Jonnarth and Magnus Hjelmblom for fruitful cooperations and stimulating discussions. Financial support was given by Gävle University, the KK-foundation and the Swedish Research Council.

9 References

Ahonen-Jonnarth, U, Odelstad, J. (2005) Simulation of cleaning of young forest stands. *Reports from Creativ Media Lab,* 2005:2. University of Gävle.

Ahonen-Jonnarth, U. & Odelstad, J. (2006) Evaluation of Simulations with Conflicting Goals with Application to Cleaning of Young Forest Stands. *Proceedings of ISC 2006* (Fourth Annual International Industrial Simulation Conference), Palermo, Italy, June 5-7, 2006.

Alchourrón, C.E. & Bulygin, E. (1971) *Normative Systems.* Springer, Wien.

Dummett, M. (1973) *Frege: Philosophy of Language*, Duckworth, London.

Einstein, A. (1936) Physics and Reality. *The Journal of the Franklin Institute*, Vol. 221, NO 3, March 1936. Reprinted in A. Einstein: *Ideas and opinions*. Souvenir Press 1973.

Einstein, A. (1949) Reply to Criticisms. In P. A. Schilpp (ed.) *Albert Einstein: philosopher-scientist.* Evanston, Ill. : Library of Living Philosophers.

Einstein, A. (1973) *Ideas and opinions*. Souvenir Press.

Ekelöf, P.-O. (1945) Juridisk slutledning och terminologi, *Tidsskrift for Rettsvitenskap,* **58**, pp. 211 ff.

Gentzen, G. (1934). Untersuchungen über das logische Schließen, I. *Mathematische Zeitschrift,* **39**, 176-210.

Halldén, S. (1978) Teckenrelationen och språkreglernas juridik. In *En Filosofibok Tillägnad Anders Wedberg*. Stockholm.

Hare, R. (1989) *Essays in Ethical Theory*. Oxford University Press, Oxford.

Heath, P.L. (1967) Concept. In P. Edwards (ed.) *The Encyclopedia of Philosophy*, Macmillan, New York.

Hedenius, I. (1941) *Om rätt och moral.* Tidens förlag, Stockholm. Second ed. Wahlström & Widstrand, Stockholm, 1965.

Hjelmblom, M. (2008). *Deontic Action-Logic Multi-Agent Systems in Prolog.* Thesis work for degree of Master of Science in Computing Science, Uppsala University, FoU-rapport Nr 30, University of Gävle.

Hohfeld, W.N. (1923) *Fundamental Legal Conceptions as Applied in Judicial Reasoning and Other Legal Essays* (ed. W.W. Cook), Yale University Press, New Haven.

Jones, A.J.I. & Sergot, M.J. (1993) On the Characterisation of Law and Computer Systems: The Normative Systems Perspective. In J-J.Ch. Meyer & R.J. Wieringa, (eds.) *Deontic Logic in Computer Science: Normative System Specification.* Wiley.

Jones, A.J.I. & Sergot, M.J.(1996) A Formal Characterisation of Institutionalised Power. *Journal of the IGPL* 4 (3):429-445. Reprinted in Valdés, E.G. et al.(eds.) *Normative Systems in Legal and Moral Theory. Festschrift for Carlos E. Alchourrón and Eugenio Bulygin.* Duncker & Humboldt, Berlin, 1997, pp. 349-367.

Kanger, S. (1957) *New Foundations for Ethical Theory.* Part 1. Stockholm. Reprinted in R. Hilpinen (ed.) *Deontic Logic: Introductory and Systematic Readings*, pp. 36-58. Dordrecht, 1971.

Krogh, C. (1995) The Rights of Agents. *Intelligent Agents II,* IJCAI'95 Workshop (ATAL), Springer.

Krogh, C. & Herrestad, H. (1999) Hohfeld in Cyberspace and Other Applications of Normative Reasoning in Agent Technology", *Artificial Intelligence and Law* 7: 81-96.

Lindahl, L. (1968) Om tysta löften inom civilrätten. In *Sanning, Dikt, Tro: Till Ingemar Hedenius.* Bonniers, Stockholm.

Lindahl, L. (1977) *Position and Change. A Study in Law and Logic.* Reidel, Dordrecht.

Lindahl, L. (1985) Definitioner, begreppsanalys och mellanbegrepp i juridiken. In *Rationalitet och Empiri i Rättsvetenskapen,* pp. 37-52. Juridiska Fakultetens i Stockholm skriftserien, nr 6. Stockholm.

Lindahl, L. (1994) Stig Kanger's Theory of Rights. In D. Prawitz, B. Skyrms & D. Westerståhl (eds.) *Logic, Methodology and Philosophy of Science IX,* pp. 889-911 Elsevier.

Lindahl, L. (1997) Norms, Meaning Postulates, and Legal Predicates. In Valdés et al. (eds) *Normative Systems in Legal and Moral Theory. Festschrift for Carlos E. Alchourrón and Eugenio Bulygin,* pp. 293-307. Duncker & Humblot, Berlin.

Lindahl, L. (2000) Deskription och normativitet: De juridiska begreppens Janusansikte. In Numhauser-Henning, A. (ed.) *Normativa perspektiv. Festskrift till Anna Christensen.* Lund.

Lindahl, L. (2003) Operative and Justificatory Grounds in Legal Argumentation. In K. Segerberg & R. Sliwinsky (eds.) *Logic, law, morality: thirteen essays in practical philosophy in honour of Lennart Åqvist,* pp. 111-126. Uppsala philosophical studies 51, Department of Philosophy, Uppsala.

Lindahl, L. (2004) Deduction and Justification in the Law. The Role of Legal Terms and Concepts. *Ratio Juris* **17**: 182 - 202.

Lindahl, L. (2005) Hohfeld Relations and Spielraum for Action. In C. Dahlman (ed.) *Studier i rättsekonomi: festskrift till Ingemar Ståhl,* pp. 121-150. Studentlitteratur, Lund.

Lindahl, L. & Odelstad, J. (1996) Grounds and consequences in conceptual systems. In S. Lindström, R. Sliwinski och J. Österberg (eds.) *Odds and Ends. Philosophical Essays Dedicated to Wlodek Rabinowicz.* Uppsala Philosophical Studies 45, Department of Philosophy, Uppsala.

Lindahl, L. & Odelstad, J. (1999a) Intermediate Concepts as Couplings of Conceptual Structures. In H. Prakken, & P. McNamara (ed.*) Norms, Logics and Information Systems. New Studies on Deontic Logic and Computer Science.* IOS Press, Amsterdam.

Lindahl, L. & Odelstad, J. (1999b). Normative systems: core and amplifications. In R. Sliwinski (ed.) *Philosophical Crumbs. Essays Dedicated to Ann-Mari Henschen-Dahlquist.* Uppsala Philosophical Studies 49. Department of Philosophy, Uppsala.

Lindahl, L. & Odelstad, J. (2000). An algebraic analysis of normative systems. *Ratio Juris* 13: 261-278.

Lindahl, L. & Odelstad, J. (2003) Normative Systems and Their Revision: An Algebraic Approach. *Artificial Intelligence and Law,* 11: 81-104.

Lindahl, L. & Odelstad, J. (2004) Normative Positions within an Algebraic Approach to Normative Systems. *Journal Of Applied Logic,* 2: 63-91.

Lindahl, L. & Odelstad, J. (2006a). Intermediate Concepts in Normative Systems. In L. Goble & J-J.Ch. Meyer (eds.) *Deontic Logic and Artificial Normative Systems.* (DEON 2006). LNAI 4048, pp. 187-200. Springer-Verlag.

Lindahl, L. & Odelstad, J. (2006b). Open and Closed Intermediaries in Normative Systems. In T.M. van Engers (ed.) *Legal Knowledge and Information Systems.* (Jurix 2006). IOS Press, Amsterdam.

Lindahl, L. & Odelstad, J. (2008a) Intermediaries and Intervenients in Normative Systems. *Journal of Applied Logic* 6: 229-250.

Lindahl, L. & Odelstad, J. (2008b) Strata of Intervenient Concepts in Normative Systems. In R. van der Meyden & L. van der Torre (eds.): *DEON 2008,* LNAI 5076, pp. 203-217. Springer-Verlag, Berlin Heidelberg.

Lloyd J.W. (1987) *Foundations of Logic Programming.* Second, Extended Edition. Springer-Verlag.

Luger, G.F. (2002) *Artificial Intelligence. Structures and Strategies for Complex Problem Solving,* 4th ed. Addison-Wesly.

Makinson, D. (1986) On the Formal Representation of Right Relations. *Journal of Philosophical Logic,* **15**, 403-425.

Makinson, D. & van der Torre, L. (2000). Input-output Logics. *Journal of Philosophical Logic,* **29**, 383-408.

Makinson, D. & van der Torre, L. (2003) What is input/output logic? In *Foundations of the Formal Sciences II: Applications of Mathematical Logic in Philosophy and Linguistics,* In: *Trends in Logic*, vol. 17, Kluwer, Dordrecht.

Nagel, E. (1961) *The Structure of Science.* Routledge, London.

Negnevitsky, M. (2005) *Artificial Intelligence. A Guide to Intelligent systems.* Second Edition. Addison-Wesley.

Odelstad, J. (1989) *Om den metodologiska subjektivismen.* Lecture delivered in the higher seminar of theoretical philosophy, Uppsala University, April 1989. (Unpublished manuscript in Swedish.)

Odelstad, J. (1992) *Invariance and Structural Dependence.* Lecture notes in Economics and Mathematical Systems 380. Springer-Verlag.

Odelstad, J. (2002a) Norms for Multi-Agent Systems - The Representation Problem. In J. Bubenko, J. & B. Wangler (eds) *Promote IT 2002.* Proceedings of the Second Conference for the Promotion of Research in IT. Skövde.

Odelstad, J. (2002b) *Artificial Agents and Norms.* Department of Computer and Systems Sciences, SU & KTH, Masters' series, No. 02-80-DSV-SU, Stockholm.

Odelstad, J. (2002c) *Intresseavvägning. En beslutsfilosofisk studie med tillämpning på planering.* Thales, Stockholm.

Odelstad, J. (2003) An Abstract Architecture for Norm-Regulated Agents. In *Promote IT 2003.* Proceedings of the Third Conference for the Promotion of Research in IT, Visby.

Odelstad, J. (2007) Agents, Norms and Forest Cleaning. In G. Boella, L. van der Torre & H. Verhagen (eds.): *Normative Multi-Agent Systems,* Dagstuhl Seminar Proceedings 07122, ISSN 1862 – 4405.
URL http://drops.dagstuhl.de/portals/index.php?semnr=07122.

Odelstad, J. (2008a) Likvärdigt arbete - en logisk och rättsfilosofisk analys. ('Work of equal value–a logical and philosophical analysis', in Swedish, 40 pp.) To be published.

Odelstad, J. (2008b) *Many-Sorted Implicative Conceptual Systems.* DSV Report series No. 08-012. Royal Institute of Technology, Stockholm.

Odelstad, J. & Boman, M. (2004) Algebras for Agent Norm-Regulation. *Annals of Mathematics and Artificial Intelligence,* 42: 141-166.

Odelstad, J. & Lindahl, L. (1998) *Conceptual Structures as Boolean Orderings.* In L. Lindahl, J. Odelstad & R. Sliwinski (eds.) *Not Without Cause. Philosophical Essays Dedicated to Paul Needham.* Uppsala Philosophical Studies 48, Department of Philosophy, Uppsala.

Odelstad, J. & Lindahl, L. (2000) Normative Systems Represented by Boolean Quasi-Orderings. *Nordic Journal of Philosophical Logic,* **5**, 161-174.

Odelstad, J. & Lindahl, L. (2002) The Role of Connections as Minimal Norms in Normative Systems. In T. Bench-Capon, A. Daskalopulu & R. Winkels (eds.) *Legal Knowledge and Information Systems.* (Jurix 2002) IOS Press, Amsterdam.

Olsson, J. (2006) *Normsystem för Wastecollectors-systemet.* Candidate Thesis, University of Gävle.

Purdom Jr., P.W. & Brown, C.A. (1985) *The Analysis of Algorithms.* Holt, Rinehart & Winston, London.

Ross, A. (1951) Tû-Tû. In O.A. Borum & K. Illum (eds.) *Festskrift til Henry Ussing.* København: Juristforbundet. (English translation as Ross (1956-57)).

Ross, A. (1956-57) Tû-Tû. *Harvard Law Review,* **70**, 812-825. (English translation of Ross (1951)).

Sergot, M. (1999) Normative Positions. In P. McNamara & H. Prakken (eds.): *Norms, Logics and Information Systems,* pp. 289-308. IOS Press, Amsterdam.

Sergot, M. (2001) A Computational Theory of Normative Positions *ACM Transactions on Computational Logic,* **2** , 581-622.

Suppes, P. (1957) *Introduction to Logic.* Van Nostrand, New York.

von Wright, G.H. (1951) Deontic Logic, *Mind,* **60**, 1-15.

Wedberg, A. (1951). Some problems in the logical analysis of legal science. *Theoria,* **17**, 246-275.