Seminar *Coalgebraic Logic*
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Summary

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1 Executive Summary

The seminar dealt with recent developments in the emerging area of coalgebraic logic and was the first Dagstuhl seminar on that topic. Coalgebraic logic is a branch of logic which studies coalgebras as models of systems and their logics. It can be seen as generalising and extending the classical theory of modal logic to more general models of systems than labelled transition systems. Traditionally, modal logics find their use when reasoning about behavioural and temporal properties of computation and communication, whereas coalgebras give a uniform account for a large class of different systems.

The seminar discussed foundational topics in a particular branch of logic, so problems which command a direct application in an industrial context were outside the seminar's scope. We expect, however, that specification methods related to coalgebraic logics will enter fields like model checking and other areas of industrial interest, once the mathematical foundations in this area are firmer and better understood.

1.1 Background

The following glossary puts coalgebraic logic in its larger context.

Modal logic is a field with roots in philosophical logic and mathematics. As applied to computer science it has become central in order to reason about the behavioural and temporal properties of computing and communicating systems, as well as to model properties of agents such as knowledge, obligations, and permissions. Two of the reasons for the success of modal logic are the following. First, many modal logics are —despite their remarkable expressive power— decidable and, therefore, amenable to automated reasoning and verification. Second, Kripke's relational semantics of modal logic turned out to be amazingly flexible, both in terms of providing techniques

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to prove properties of modal logics and in terms of allowing the different applications of modal logic to artificial intelligence, software agents, etc.

Coalgebra is a more recent area. Following on from Aczel's seminal work on non-well founded set theory, coalgebra has been developed into a general theory of systems. The basic idea is that coalgebras are given with respect to a parameter F. Different choices of F yield, for example, the Kripke frames and models of modal logic, the labelled transition systems of process algebra, the deterministic automata of formal language theory, or the Markov chains used in statistics. Rutten showed that, in analogy with universal algebra, a theory of systems, called universal coalgebra, can be built uniformly in the parameter F, simultaneously covering the above and other examples. Crucial notions such as behavioural equivalence (observational equivalence, bisimilarity), final semantics and coinduction find their natural place here.

Coalgebraic logic combines coalgebra and modal logic to study *logics of systems* uniformly in the parameter F. Given the plethora of different transition systems and their ad hoc logics, such a uniform theory is clearly desirable. *Uniformity* means that results on, for example, completeness, expressivity, finite model property and complexity of satisfiability can be established at once for all functors (possibly satisfying some, usually mild, conditions). Additionally, there is also a concern for *modularity*: Typically, a parameter F is composed of basic features (such as input, output, non-determinism, probability). Modularity then means that the syntax/proof systems/algorithms for the logic of F are obtained compositionally from the syntax/proof systems/algorithms for the logics of the basic features.

1.2 Structuring the Seminar

When we planned the seminar, we envisaged six broad topics. We indicate which of the talks fall under which topic.

1. Category Theoretic Aspects of Coalgebraic Logic Although much is already known, the category theoretic foundations of coalgebra are still being extended, as in the talks by Adámek, Gumm, Petrişan, Velebil.

2. Probabilistic Transition Systems Some of the most important examples of coalgebras are different variants of Markov transition systems. Their theory was explored in the talks by Desharnais, Doberkat, Schubert, Sokolova, Zhou.

3. Stone Duality Simplifying the picture, coalgebraic logic can be understood as algebraic logic for coalgebras, where the relationship between algebras and coalgebras is given by Stone type dualities, the perspective taken in the presentations by Bezhanishvili, Gehrke, Vosmaer. This point of view is also closely related to the presentations by Petrisan, Velebil, listed under 2.

4. Coalgebraic Logic, Automata Theory, Fixed Point Logics Different aspects of coalgebraic logic were discussed in the following talks: Bilkova, Palmigiano (proof theory), Kissig (the logic of accepted languages), Leal (comparison of different logics), Schröder (correspondence theory), Venema (fixed point logics). Fixed point logics also appeared in the talks of Cîrstea and Kupke listed under 6.

5. Coalgebraic Logic for Structural Operational Semantics The following talks discussed various applications of coalgebraic techniques to process algebra and program semantics: Bonsangue, Ciancia, Gadducci, Hansen, Klin, Levy, Milius, Staton. The talks of Bonsangue, Hansen, and Milius were also closely related to coalgebra automata.

6. Applied Coalgebraic Logic Although the theoretical foundations of coalgebraic techniques are still being developed, they do give rise to new algorithms, eg for checking the satisfiability of various modal logics. Three different examples were presented in the talks of Cîrstea, Kupke, Pattinson.

Further topics Moss gave a presentation on new developments on the logic of recursion, which is one of the oldest topics in coalgebraic logic going back to the book *Vicious Circles* by Barwise and Moss (1996). New perspectives for coalgebraic logic where opened by the talks by Abramsky and Jacobs (quantum systems), and Pavlovic (security).

2 List of Abstracts

Coalgebras, Chu Spaces, and Representations of Physical Systems

Samson Abramsky, University of Oxford

We highlight two limitations of the standard coalgebraic framework: it does not accommodate contravariance, and is too rigid to allow e.g. physical symmetries to be represented. We introduce a fibrational structure on coalgebras in which contravariance is represented by indexing. We use this structure to give a universal semantics for quantum systems based on a final coalgebra construction. We characterize equality in this semantics as projective equivalence. We also define an analogous indexed structure for Chu spaces, and use this to obtain a novel categorical description of the category of Chu spaces. We use the indexed structures of Chu spaces and coalgebras over a common base to define a truncation functor from coalgebras to Chu spaces. We use this truncation functor to lift the full and faithful representation of the groupoid of physical symmetries on Hilbert spaces into Chu spaces, obtained in our previous work, to the coalgebraic semantics.

Terminal Coalgebras in Many-Sorted Sets

Jiri Adamek, TU Braunschweig

For endofunctors of many-sorted sets the iterative construction of terminal coalgebras is proved to converge whenever a terminal coalgebra exists. The existence of a fixed point is, by Lambek's Lemma, a necessary condition; we prove that in case of one sort the existence of two successor fixed points is sufficient for terminal coalgebras.
As demonstrated by James Worell the number of steps needed for the finite power-set functor is $\omega + \omega$. In contrast, the initial algebra construction takes, for any endofunctor of many-sorted sets, a cardinal number of steps.

Model theory of descriptive frames

Nick Bezhanishvili, Imperial College, London

Descriptive general frames are duals of modal algebras and, thus, provide completeness for normal modal logics. From the coalgebraic perspective, descriptive frames can be represented as coalgebras for the Vietoris functor on the category of Stone spaces. In this talk I will discuss some model theoretic aspects of descriptive frames. In particular, I will show that many results of classical model theory fail on descriptive frames.
This is joint work with Ian Hodkinson.

Universal property of nabla modality

Marta Bilkova, Charles University, Prague

The coalgebraic cover modality $\nabla_{\mathcal{T}}$ for a finitary standard and weak-pullbacks preserving endofunctor $\mathcal{T}$ of Set fully captures the modal logic corresponding to $\mathcal{T}$-coalgebras. We show that, if in addition the functor $\mathcal{T}$ is finitely presentable in the category of finitary endofunctors of Set, $\nabla_{\mathcal{T}}$ is a relative left adjoint.
Namely, for $\nabla_{\mathcal{T}}$ as a monotone map from the $\mathcal{T}$-lifted preorder $\mathcal{T}\mathcal{L}$ of all modal formulas to the preorder of all modal formulas $\mathcal{L}$ there exists a monotone map g from $\mathcal{L}$ to $\mathcal{P}_\omega\mathcal{T}\mathcal{L}$ such that

$$\nabla_{\mathcal{T}}\alpha \leq b \text{ holds iff } \alpha \, \overline{\mathcal{T}}(\leq) \, \gamma \text{ for some } \gamma \text{ in } g(b).$$

Such monotone maps were called $\mathcal{O}$-adjoints in [2] and are an instance of a weakened representability notion relative to a doctrine, see [1].
This is joint work with Alessandra Palmigiano, Jiri Velebil, and Yde Venema.

References

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[2] L. Santocanale, Y. Venema: Completeness for Flat Modal Fixpoint Logics, submitted (December 2008).

An algebraic approach to coalgebras

Marcello M. Bonsangue, Leiden University

Kleene's Theorem gives a fundamental correspondence between regular expressions and deterministic finite automata (DFAs): each regular expression denotes a language that can be recognized by a DFA and, vice-versa, the language accepted by a DFA can be specified by a regular expression. Languages denoted by regular expressions are called regular. Two regular expressions are called (language) equivalent if they denote the same regular language. Kozen in 1991 provided regular expressions with sound and complete algebraic laws that permit formal reasoning about equivalences between expressions.
Coalgebras provide a general framework for the study of dynamical systems such as DFAs. In this presentation we show the application of the above program to coalgebras over a large class of functors $F : \mathbf{Set} \to \mathbf{Set}$. We incrementally introduce a set of expressions for finite deterministic, non-deterministic and quantitative coalgebras and proved an analogue of Kleene's Theorem: each expression denotes the behavior of a finite coalgebra and, conversely, the behavior of a finite coalgebra can be specified by an expression. We also provide a sound and complete axiomatization for these coalgebraic calculi, with the property that two expressions are provably equivalent if and only if they are bisimilar.
Example of coalgebraic calculi we consider include labeled transition systems, weighted automata and probabilistic systems.

Families: an efficient categorical model of computation with resources

Vincenzo Ciancia, Universidad Complutense de Madrid

An important concern in programming language semantics is to find fully abstract models, where all the semantically equivalent programs are identified; for the so called interactive systems, labelled transition systems (LTS) equipped with bisimilarity provide a solution in simpler cases. Often, the semantics of a programming language has to deal with resource allocation. For example, in the π-calculus, mobility is achieved by allocating fresh communication channels. The LTS semantics is not completely satisfactory, since the definition of bisimulation is non-standard, due to the necessity of matching allocation on both sides of the bisimulation game. This problem is elegantly addressed by coalgebras over some presheaf category, that is, a functor category $\mathsf{Set}^{\mathsf{C}}$. Each program is equipped with a type representing the available resources. Allocation functors can be combined with polynomials to take into account resource generation in the standard (coalgebraic) bisimulation. Varying the index category C, resources can have a rich structure. A well known case is when C is the category I of finite sets and injective relabellings; here the modelled resources are pure names.
The presheaf approach makes it difficult to implement finite state methods such as partition refinement and model checking, since programs with the same shape, that only differ for the identity of their resources, are not identified. In particular, this is a problem for finite memory programs that allocate some resources and discard other ones in a loop, giving rise to infinite models. In the case of pure names, the alternative approach of named sets allows one to specify coalgebras with local names, featuring an implicit garbage collection machinery. Pullback-preserving presheaves in $\mathbf{Set}^{\mathsf{I}}$ and named sets are linked by a categorical equivalence.

In this talk, we discuss the advantages and the open problems in extending the equivalence result to other index categories, giving rise to the categorical model of families. Locality of interfaces leads to constructions that are typical when handling resources in computing. For example, the categorical product is made up of triples consisting of a pair of elements, and a binding between their local resources. We propose families as an alternative model for the semantics of resource-aware programming languages, and explain how the categorical equivalence defines a framework where both approaches can be used simultaneously, for specification purposes on the side of presheaves, and implementation of finite state methods on the other side.

Path-Based Coalgebraic Temporal Logics

Corina Cîrstea, University of Southampton

We give a general coalgebraic account of the notions of infinite trace and infinite execution in state-based, dynamical systems, by extending the generic theory of finite traces and executions developed by Hasuo and coauthors [1]. The systems we consider are modelled as coalgebras of endofunctors obtained as the composition of a computational type (e.g. nondeterministic or stochastic) with a general transition type. This generalises existing work by Jacobs [2] that only accounts for a nondeterministic computational type. We subsequently introduce path-based temporal (including fixpoint) logics for coalgebras of such endofunctors, whose semantics is based upon the notion of infinite execution. Our approach instantiates to both nondeterministic and stochastic computations, yielding, in particular, path-based fixpoint logics in the style of CTL* for nondeterministic systems, as well as generalisations of the logic PCTL for probabilistic systems.

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A Demonic Approach to Information in Probabilistic Systems

Josee Desharnais, Université Laval, Québec

We establish a Stone-type duality between specifications and infLMPs. An infLMP is a probabilistic process whose transitions satisfy super-additivity instead of additivity. Interestingly, its simple structure can encode a mix of probabilistic and non-deterministic behaviors.
We take the view of analysing probabilistic transition systems through events, or sets of states, as is usually done in probability theory. It is well known that bisimulation for probabilistic processes without non determinism is characterized by a simple logic : two states are bisimilar if and only if they satisfy exactly the same formulas of that logic. Since formulas can be seen as sets of states, they are ideal candidates for events: any LMP can be associated to a morphism from the logical formulas to its sigma-algebra of states, where the image of a formula is the set of states that satisfy it. We are interested in the converse: can a probabilistic process be defined by the set of its logical properties only? We can abstract the sigma-algebra of states as a σ-complete Boolean algebra and we can ask the question: when is it the case that a given function $\hat{\mu}$ mapping propositions of the logic to elements of an arbitrary (σ-complete) Boolean algebra A correspond to some LMP whose σ-algebra is isomorphic to A and whose semantics accords with $\hat{\mu}$? We give a response to this question. opening the way to working with probabilistic processes in an abstract way, that is, without any explicit mention of the state space, manipulating properties only. In other words, this opens the way to a Stone-type duality theory for these processes.
Our duality shows that an infLMP can be considered as a demonic representative of a system's information. Moreover, it carries forward a view where states are less important, and events, or

properties, become the main characters, as it should be in probability theory. Along the way, we show that bisimulation and simulation are naturally interpreted in this setting, and we exhibit the interesting relationship between infLMPs and the usual probabilistic modal logics.
Most of this work has been published in Concur 2009.
This is joint work with François Laviolette and Amèlie Turgeon.

Negating Congruences for Stochastic Relations

Ernst-Erich Doberkat, TU Dortmund

Given an equivalence relation ρ, we investigate modelling the negation of ρ through an equivalence relation ρ' so that $\rho \wedge \rho'$ is the identity, and ρ together with ρ' is comprehensive, i.e., $\rho \vee \rho'$ covers the entire space. This looks very much like complementation, but the lattice of equivalence relations is complemented only in very rare special cases, in particular under the constraints we impose. We are interested in congruences for stochastic relations, so we investigate from the starting point above under which conditions a congruence can be negated, ending up with a variety of constructions that all are more or less intended to characterize non-congruent objects.
Since congruences are based on countably generated equivalence relations, and these relations are in a one-to-one correspondence with countably generated σ-algebras, we briefly study these σ-algebras first. We can then carry results for these σ-algebras over to the space of equivalence relations. This is the technical starting point. Techniques developed for factoring stochastic relations based on those investigated in Universal Algebra help then to bring forth the desired results.
The results are applied to morphisms, a simple version von Hennessy-Milner logic serves as an illustration.

On a Presheaf Semantics for the Calculus of Explicit Fusions

Fabio Gadducci, University of Pisa

Name-passing calculi are specification languages for concurrent systems, considered as structured entities interacting via some kind of synchronisation mechanism. One of the main challenges for such languages has been represented by the development of adequate denotational semantics. Only recently the use of presheaf categories proved fruitful for providing fully abstract models to those calculi with a symmetric communication mechanism, like the π-calculus. The index categories which have been successfully employed for such languages are based on (injective) name relabellings, resulting e.g. in the presheaf category $\mathsf{Set}^{\mathbb{I}}$, for $\mathbb{I}$ the category of (finite) sets and injective functions.
In this talk we consider a calculus based on a different synchronisation mechanism: the *calculus of explicit fusions*, where process communication relies on an underlying store of name equalities. We propose to model both its syntax and semantics using the presheaf category $\mathsf{Set}^{\mathbb{E}}$: each object of $\mathbb{E}$ is an equivalence relation over a (finite) set of names and, analogously to $\mathbb{I}$, morphisms preserve names, but equivalence classes can be merged, thus obtaining semantical fusion of names without loosing any syntactical name. Besides recasting coalgebraically the standard semantics for explicit fusions, the so-called *inside-outside bisimulation*, we furthermore investigate the connections between $\mathsf{Set}^{\mathbb{I}}$ and $\mathsf{Set}^{\mathbb{E}}$, trying to highlight some correspondences between the languages themselves.

Extended Stone duality and canonical extensions

Mai Gehrke, Radboud University of Nijmegen

Stone duality provides the dual equivalence between algebras and spaces which is central in the relationship between modal algebras and Kripke semantics as well as the central mechanism for the relationship between specification of program logics and denotational semantics for these logics as layed out in Abramsky's Domain Theory in Logical Form. In seeking to extend the scope of this duality, one may ask for natural dualities for various related structures or for extended Stone or Priestley type dualities. The latter corresponds to the coalgebraic picture and an algebraic tool for studying the scope and limits of extended Stone and Priestley dualities is provided by canonical extensions. This talk will highlight a few of the recent developments in this area that I expect are pertinent to work in coalgebraic logic.

Coalgebraic types

H. Peter Gumm, University of Marburg

The coalgebraic structure of Set-coalgebras is determined by properties of their type functors. In particular, preservation of certain limits is reflected by structural properties of the corresponding coalgebras. After a survey of old and new results in this direction, we consider decompositions of Set-functors which preserve the relevant properties. We shall develop an intuitive framework, which sheds some new light on Set-functors, on their role as coalgebraic types and on the semantics of coalgebraic modalities.

Bialgebras of Bitstream Arithmetic and Mealy Machines

Helle Hvid Hansen, TU Eindhoven

Brzozowski [4] showed how to construct deterministic automata from regular expressions by defining an automaton structure on the set of regular expressions. Rutten showed in [6] how to construct Mealy machines from arithmetic bitstream specifications by defining a Mealy machine structure on the set of arithmetic bitstream expressions. More recently, Bonchi, Bonsangue, Rutten and Silva [2, 3] showed that the same ideas can be applied in a much more general setting of coalgebras for inductively defined functors and generalised regular expressions.
We refer to the abovementioned constructions as examples of coalgebraic synthesis, since they all rely on the fact that it is possible to inductively define coalgebraic structure on a set of algebraic specifications. Such interaction between syntax (algebra) and behaviour (coalgebra) can often be captured by a bialgebra for a distributive law [1, 7]). Indeed, Jacobs showed in [5] that Brzozowski's construction gives rise to a bialgebra on the set of regular expressions, and that the well known relationships between regular expressions, deterministic automata and formal languages can be viewed in a bialgebraic framework.
In this talk we show that also Rutten's construction of Mealy machines from arithmetic specifications can be described in terms of a bialgebra on the set of arithmetic expressions. In particular, we show that the stream differential equations used to define the coalgebraic behaviour of expressions give rise to a GSOS law between the arithmetic signature functor and the Mealy functor. This talk presents work in progress. We hope to be able to extend the bialgebraic picture to the work of [2, 3], and to find more general principles and methods relevant to coalgebraic synthes

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Convexity and Duality

Bart Jacobs, Radboud University, Nijmegen

This talks describes convex sets categorically, namely as algebras of a distribution monad. It is shown that convex sets occur in two dual adjunctions, namely one with preframes via the Boolean truth values $\{0, 1\}$ as dualising object, and one with effect algebras via the (real) unit interval $[0, 1]$ as dualising object. These effect algebras are of interest in the foundations of quantum mechanics.

Generic Trace Logics

Christian Kissig, University of Leicester

Finite trace semantics is well-understood as a suitable semantics for non-deterministic, probabilistic, or graded transition systems. In their recent works on Generic Trace Theory have Jacobs and collaborations proposed a uniform definition of finite trace semantics for Set-based coalgebras with a branching structure given in terms of a monad, in addition to the transition structure from a Set-functor.

In Generic Trace Theory, finite trace semantics is defined as a final coalgebra semantics obtained by induction along the initial sequence. The construction relies on the limit-colimit-coincidence of Smyth and Plotkin, and requires the monad to be such that the Kleisli-category can be locally directed completely ordered.

We lift several of the assumptions made in generic trace theory, and obtain a slightly more general definition of finite trace semantics which works also in the restricted settings of finite non-deterministic and finitely graded branching.

Finite trace semantics induces finite trace equivalence. We propose a coalgebraic logic which is invariant under finite trace equivalence, and, with additional assumptions made, finitely expressive and complete. We need to assume that the monad is commutative. We obtain generic trace logics through an adjunction on the category of algebras for the branching monad, in the spirit of coalgebraic modal logics of Pattinson and Schröder.

Modal logic and bialgebras

Bartek Klin, University of Cambridge

A coalgebraic treatment of Structural Operational Semantics (SOS) is provided by certain natural transformations: distributive laws of endofunctors (modelling process syntax) on other endofunctors (modelling system behaviour). Models of such laws are bialgebras, which can be seen as coalgebras for behaviour functors lifted to categories of algebras for syntax functors.

On the other hand, coalgebraic modal logic is defined by natural transformations that link system behaviour with logical syntax along a (contravariant) adjunction between two categories. These natural transformations combine behaviour and syntax in single composite endofunctors on the slice category of the adjunction, and models of logics are coalgebras for these lifted endofunctors.

In the framework of logical distributive laws, these two aspects are combined in a study of bialgebras on slice categories of adjunctions. In more elementary terms, SOS specifications are combined

with modal logics for processes in a well structured manner that ensures compositionality of logical equivalence relations with respect to process syntax. Technically, this requires a suitable notion of logical behaviour, and an SOS-like distributive law where logical formulas play the role of processes.
In this talk, I briefly describe the basics of the logical distributive law approach, cast in the setting of slice categories of adjunctions. I also say how it relates to modal logic decomposition techniques used in the SOS community to prove compositionality results.

Tableaux for coalgebraic fixpoint logics

Clemens Kupke, Imperial College, London

The satisfiability problem of the modal mu-calculus is decidable in exponential time. This well-known fact can be proven using (possibly) non-wellfounded tableaux. In my talk I will demonstrate how the tableaux-based proof can be generalised to a proof of ExpTime-decidability of a family of coalgebraic fixpoint logics, ie., coalgebraic logics extended with least and greatest fixpoint operators. Our decidability result yields, as concrete applications, previously unknown complexity bounds for the probabilistic mu-calculus and for an extension of coalition logic with fixpoints.
This is joint work with Corina Cîrstea and Dirk Pattinson.

Predicate liftings and Nabla together at last!

Raul Leal, University of Amsterdam

In this talk we show how to develop a coalgebraic modal language which is obtained from a friendly interaction of predicate liftings and Moss's nabla modality. The main technical tool for this marriage is that of presentations of functors by operations and equations. Some of the features of our language are: it is an equational language and it has a sound and complete axiomatization for any weak pullback preserving set functor.
This is joint work with Alexander Kurz.

Similarity quotients as final coalgebras

Paul Blain Levy, University of Birmingham

Nodes of transition systems modulo similarity form a final coalgebra for a suitable endofunctor on the category of posets. Conversely, given a final coalgebra, two nodes are similar when their anamorphic images are related.
These results generalize to an algebraic framework that includes as special cases

- upper and lower similarity for transition systems with divergence
- bisimilarity
- nested similarity.

In the framework developed by Thijs, binary composition of relations must be preserved by relational lifting, but in our framework, it is only laxly preserved. This more liberal condition is what allows nested similarity to be an example.

A Sound and Complete Expression Calculus for Linear Systems

Stefan Milius, TU Braunschweig

Regular expressions are a well-known tool to specify the behaviour of finite sequential automata. Kleene's classical theorem states that the semantics of every finite automaton can be expressed by a regular expression. Furthermore, Kleene algebras provide a sound and complete calculus for the behavioural equivalence of finite automata.
Recently, Bonsangue, Rutten and Silva provided an analog of these classical results pertaining to coalgebras for endofunctors of the category of sets. From every Kripke polynomial functor one can derive a calculus of expressions that is sound and complete with respect to the behavioral equivalence of finite coalgebras.
Based on these ideas I will present a sound and complete expression calculus for finite dimensional linear systems. These systems are coalgebras for the functor $H = R \times -$ on the category of real vector spaces, where R are the reals. Finite dimensional linear systems are equivalently presented by finite stream circuits. So the calculus allows one to reason about the equivalence of finite closed stream circuits.
The main technical result of my talk is that expressions modulo the laws of the calculus form the final locally finite dimensional coalgebra for H. This gives a new syntactic characterization of the coalgebra of rational streams known from Rutten's stream calculus.
Time permitting, I will show that, more generally the final locally finite (dimensional) coalgebra is, equivalently, the initial iterative algebra for a set (vector space) endofunctor.

Coalgebra and the Logic of Recursion

Lawrence S. Moss, Indiana University, Bloomington

One source of coalgebras in theoretical computer science begins with the idea that coalgebras are generalized transition systems, with the final coalgebra carried by all possible behaviors of all possible transition systems. As such, the final coalgebra may be taken as the 'codomain of semantics', with the domain a set of terms of some kind or other. This leads to a research program of constructing languages for interesting 'transition trajectories'; typically the terms of these languages are finite objects but the behaviors are infinite. Part of this program involves logics for the equivalence of terms. This is where the logic of recursion (first proposed by Yiannis Moschovakis) enters, since it is a general logic of fixed point terms.

Proof systems for coalgebraic logic

Alessandra Palmigiano, University of Amsterdam

The research we will report on takes its move on an alternative presentation of classical and positive modal logic where the coalgebraic modalities ∇ and Δ are taken as primitive. Analogously to the duality between $\Box$ and $\Diamond$, Δ can be defined as the dual of ∇, but also, interestingly, in the following, negation-free way:

$$\Delta\alpha = \begin{cases} \nabla\varnothing \vee \bigvee\{\nabla\{a\} \mid a \in \alpha\} \vee \nabla\{\bigwedge\alpha, \top\} & \text{if } \alpha \neq \varnothing \\ \nabla\{\top\} & \text{if } \alpha = \varnothing \end{cases} \tag{1}$$

In earlier work, we introduced a one-sided Gentzen system for an expansion of the Boolean propositional language with the nabla operator only; this system was shown to be sound, complete w.r.t. the class of all Kripke models, and cut-free. Moreover, we defined a sound and complete two-sided Gentzen system for the positive fragment of the same language, the cut rule of which was shown not to be eliminable. The main feature of both Gentzen systems was their being generalizable to the coalgebraic setting where F is an arbitrary weak pullback-preserving Set-endofunctor.

Here will report on an improved two-sided sequent calculus for the $\nabla\Delta$-based coalgebraic logic over a standard and weak-pullback preserving functor F: this sequent calculus is sound, complete, invertible and *cut free*. Moreover, sound, complete, invertible and cut-free variants of this system include a simplified two-sided version for the powerset functor and both a one-sided and a two-sided versions for the Δ-free fragment of the language.

Optimal Tableau Algorithms for Coalgebraic Logics

Dirk Pattinson, Imperial College, London

Tableau methods are one of the main techniques that underly auto mated reasoning for modal logics, and are implemented in an ever growing number of tools. Despite the fact that tableau algorithms work extremely well in practice, they often do not meet the known complexity bounds for the logics in question. Recently, it has been shown that optimality can be obtained for some logics while retaining practicality by using a technique called ìglobal cachingî. Here, we show that global caching is applicable to all logics that can be equipped with coalgebraic semantics, for example, classical modal logic, graded modal logic, probabilistic modal logic and coalition logic. In particular, the coalgebraic approach also covers logics that combine these various features. We thus show that global caching is a widely applicable technique and also provide foundations for optimal tableau algorithms that uniformly apply to a large class of modal logics.
Technically, we give a sound and complete tableau calculus for coal gebraic modal logics in the presence of global assumptions, and obtain an EXPTIME upper bound by translating the satisfiability problem to reachability games. Based on the completness of the tableau calculus, we then introduce two concrete algorithms to decide satisfiability. Both algorithms are proved correct coinductively, and can be seen to gener alise ancestor equality blocking, and global caching, respectively. Apart from giving a coinductive reconstruction of ancestor equality blocking and global caching, this showcases the wide applicability of both and demon strates that automated reasoning with coalgebraic logics in the presence of global assumptions is also in practice not (much) harder than for modal logics with an underlying relational semantics.

Probable security

Dusko Pavlovic, University of Oxford

Modern cryptography is a realm of complex constructions with Probabilistic Polynomial-Time Turing (PPT) machines, viewed up to observational equivalence modulo negligible functions. Modern authentication protocols often involve Bayesian reasoning about heterogenous channels. Formal security proofs sometimes turn out to be as unreliable as the systems that they prove secure. I shall show how categorical and coalgebraic tools help with these problems.

Applications of universal-algebraic functors

Daniela Petrisan, University of Leicester

We consider the notion of 'universal-algebraic' functor. These are functors on many-sorted varieties that preserve sifted colimits. Universal algebra stems from the doctrine of finite products, and sifted colimits are precisely those colimits that commute in Set with finite products. Universal-algebraic functors on many-sorted varieties can be presented by operations and equations. We exploit the flexibility provided by this notion and discuss several applications.
First, it can be used to study universal algebra over the category Nom of nominal sets. The signatures for algebras over Nom are given by functors having presentations by operations and equations. We prove an HSP-like theorem for algebras over nominal sets, using completely standard universal algebra and the fact that Nom is a full reflective subcategory of a many-sorted variety. Since our notion of signature is quite general, the equational logic obtained in our setting

is more expressive than the nominal algebra logic of Gabbay and Mathijssen, or the nominal equational logic of Clouston and Pitts. However we can isolate a 'uniform' fragment of our logic that corresponds to these logics, thus we give a new way of comparing the two different approaches.
Second, we give a categorical approach to algebraic semantics of first-order logic. We show how to obtain algebraic models of first-order logic as algebras for a functor on a many-sorted variety $\mathsf{BA}^{\mathsf{fbb}_+}$ satisfying some additional equations. These algebras are equivalent to the polyadic algebras of Halmos, and we can dualise them to obtain a coalgebraic semantics of first-order logic.
Third, this notion links the uniform treatment of logics for coalgebras of an arbitrary type T with concrete syntax and proof systems. Analysing the many-sorted case is essential for achieving modular completeness proofs for coalgebraic logics.

Coalgebraic Correspondence Theory

Lutz Schröder, DFKI Bremen

Classically, modal logic is seen as a fragment of first-order logic, determined by a standard translation of modal formulas into first-order formulas that captures Kripke semantics. This fragment was characterized by van Benthem as containing precisely the bisimulation invariant formulas; it was later shown by Rosen that this characterization remains true over finite models, although the tools used in the original proof include model theoretic facts such as compactness and saturation that break down over finite models. We refer to the former type of results (characterization over all models) as van-Benthem-type results, and to results over finite models as Rosen-type results.
The design of a correspondence language for coalgebraic modal logic is a delicate problem, since one needs to include a sort for neighbourhoods to represent predicate liftings, avoiding at the same time the full expressive power of monadic second order logic, whose bisimulation invariant fragment includes non-modal formulas. We design such a language and prove a generic Rosen-type result stating that every first-order formula which is invariant under behavioural equivalence is equivalent to possibly infinitary but bounded-rank modal formula, and hence to a finitary modal formula in case the modal similarity type is finite.

Expressivity of coalgebraic logic over measurable spaces

Christoph Schubert, TU Dortmund

We study the relationship between logical and behavioral equivalence for coalgebras on general measurable spaces. Modal logics are interpreted using predicate liftings. Prominent examples of these coalgebras include stochastic relations and labelled Markov transition systems. The corresponding Hennessy–Milner type logics are a prime example for coalgebraic modal logics.
It is shown that the notions of logical und behavioral equivalence coincide for coalgebras for a wide class of functors. Moreover, we present some results on compositionality of the logics. Throughout, we establish our results for general measurable spaces without relying on topological assumptions.

Exemplaric Expressivity of Modal Logics

Ana Sokolova, Universität Salzburg

This work investigates expressivity of modal logics for transition systems, multitransition systems, Markov chains, and Markov processes, as coalgebras of the powerset, finitely supported multiset, finitely supported distribution, and measure functor, respectively. Expressivity means that logically indistinguishable states, satisfying the same formulas, are behaviourally indistinguishable. The investigation is based on the framework of dual adjunctions between spaces and logics and focuses on a crucial injectivity property. The approach is generic both in the choice of systems and modalities, and in the choice of a "base logic". Most of these expressivity results are already known, but the applicability of the uniform setting of dual adjunctions to these particular examples is what constitutes the contribution of the paper.

Semantics and modal logics for process languages with names and substitutions

Sam Staton, University of Cambridge

Structural operational semantics for modern process languages (like p-calculi) involves features such as variable binding and substitution. These are not found in basic first-order calculi (like Milner's Pure CCS). In [3, 4], I have shown that, if we are careful, we can understand the semantics of the more elaborate modern calculi as follows: first, move from the category of sets to a different category, such as a presheaf category; secondly, reinterpret the old results for first-order calculi in these different categories. Recent work in coalgebraic modal logic (e.g. [1, 2]) has also shown that this approach is helpful. Selected references:

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Sifted Colimits and Presentations of Functors

Jiri Velebil, Czech Technical University, Prague

Logics for T-coalgebras are obtained by giving a equational presentation of the "dual" of T. In the setting of enriched categories we prove that endofunctors on finitary (enriched) varieties which preserve sifted colimits have an equational presentation in the enriched sense.
We discuss applications of this result to categories of nominal sets and to varieties enriched over posets. This is a work in progress, jointly with Alexander Kurz.

Coalgebra automata (towards a universal theory of automata)

Yde Venema, University of Amsterdam

Automata operating on infinite objects provide an invaluable tool for the specification and verification of the ongoing behavior of infinite systems. Coalgebra automata generalize the well-known automata that operate on specific types of infinite structures such as words/streams, trees, graphs or transition systems. The motivation underlying the introduction of coalgebra automata is to gain a deeper understanding of this branch of automata theory by studying properties of automata in a uniform manner, parametric in the type of the recognized structures. Coalgebraic automata theory thus contributes to Universal Coalgebra as a mathematical theory of state-based evolving systems.
In the talk we introduce parity automata that correspond to coalgebraic modal fixpoint logic based on predicate liftings. More specifically, we define the notion of a Λ-automaton, where Λ is a set of predicate liftings for a given functor T, and the notion of acceptance for such an automaton, which is defined in terms of a parity acceptance game. The main result that we discuss states that a Λ-automaton accepts a pointed coalgebra iff it accepts a finite coalgebra which is obtained from the automaton itself by some effective construction. This result corresponds to a general bounded model property for coalgebraic modal fixpoint logics. Time permitting we discuss some complexity issues, and relate our results to work by Cirstea, Kupke and Pattinson.

The Vietoris construction from the perspective of (co-)algebraic modal logic

Yde Venema, University of Amsterdam

In this talk we discuss some applications in (point-free) topology of geometric coalgebraic logic, based on a language with finitary conjunctions, infinitary disjunctions, and the finitary version of Moss' coalgebraic modality ∇.
In the first part of the talk we give a presentation of the Vietoris construction on compact Hausdorff spaces, based on the notion of Egli-Milner relation lifting. We then move on to the dual, algebraic construction, involving a geometric modality (ie, the nabla associated with the power set functor). This construction corresponds to Johnstone's Vietoris locales.
In the last part of the talk we discuss how to generalize this approach. We define, given a endofunctor T on the category of sets with functions, a construction V_T on locales (point-free topologies) that specializes to Johnstone's Vietoris functor in case we take for T the power set functor. We prove that this construction preserves the property of regularity if T preserves weak pullbacks, and conjecture (hopefully prove) that in case T maps finite sets to finite sets, then V_T preserves compactness as well. The latter result would imply that V_T is an endofunctor on the category of compact Hausdorf spaces, which would pave the way for a generalization of the Vietoris construction on compact Hausdorff *spaces*.

A Guided Maximal Consistent Extension Theorem for Probability Logic

Chunlai Zhou, Tsinghua University, Beijing

In literature, different deductive systems are developed for probability logics. But, for formulas, they provide esstentially equivalent definitions of consistency. In this talk, we present a guided maximally consistent extension theorem which says that any probability assignment to formulas in a finite local language satisfying some constraints specified by probability formulas is consistent in probability logics. Moreover, we employ this theorem to show two interesting results:

(1) The satisfiability of a probability formula is equivalent to the solvability of the corresponding system of linear inequalities through a certain translation based on atoms not on Hintikka sets;

(2) the Countably Additivity Rule in Goldblatt 2009 is necessary for his deductive construction of final coalgebras for functors over Meas.

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