Tree Automata, (Dis-)Equality Constraints and Term Rewriting: What's New?

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Abstract

Connections between Tree Automata and Term Rewriting are now well known. Whereas tree automata can be viewed as a subclass of ground rewrite systems, tree automata are successfully used as decision tools in rewriting theory. Furthermore, applications, including rewriting theory, have influenced the definition of new classes of tree automata. In this talk, we will first present a short and not exhaustive reminder of some fruitful applications of tree automata in rewriting theory. Then, we will focus on extensions of tree automata, specially tree automata with local or/and global (dis-)equality constraints: we will emphasize new results, compare different extensions, and sketch some applications.

This talk will be strongly inspired by recent works of several researchers, specially but not exclusively: Emmanuel Filiot, Guillem Godoy, Florent Jacquemard, Jean-Marc Talbot, Camille Vacher.

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Tree automata can be used in numerous ways as decision tools in rewriting theory. An ideal way is to encode the reducibility relation by a recognizable relation. Of course, recognizability of the reducibility relation is a very strong requirement which limits this approach to very restricted subclasses of term rewriting systems. A weaker requirement is that the rewrite relation preserves recognizability. This approach is a key point in reachability analysis and tree regular model checking. More generally, encoding set of descendants of terms and possibly sets of normal forms by tree automata provide canonical techniques to obtain decidability results and a lot of work has been done to characterize subclasses of term rewriting systems having "good recognizability properties". Even powerful, these methods remain restricted. To enhance their power, numerous works have been developed, e.g. in reachability analysis, by using abstract interpretation or over-approximations. Other works have recently focused on using rewrite strategies. An other approach is the use of extended tree automata. E.g., equational tree automata have been proposed to handle equational theories and several extensions have been defined to take into account associativity. This talk will focus on new results about extensions of tree automata with equality and disequality constraints.

From Local Constraints . . .

A typical example of language which is not recognized by a finite tree automaton is the set of ground instances of a non-linear term. This implies of course that the set of ground normal forms of a t.r.s. is not necessarily recognizable. An other point is that images by a

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non-linear morphism of a recognizable tree language are not necessarily recognizable. E.g. if a morphism associates $h(x)$ with $f(x,x)$ and a with b, the image by this morphism of $h^*(a)$ is the non-recognizable set of well-balanced terms over $\{f, b\}$. So, extending tree automata to handle non-linearity is very natural and in the early 80's, M. Dauchet and J. Mongy have proposed a new class for this purpose, by enriching tree automata rules by equality constraints. E.g. if a rule is associated with the constraint $1.1 = 1.2$, it can be applied at position p in t only if the subterms at positions $p.1.1$ and $p.1.2$ are equal. A more general class can be defined by allowing also disequality constraints. Unfortunately, emptiness is undecidable even when only equality constraints are allowed. Several restrictions of this class have so been studied (see e.g. a survey in [2]). Let us remind two of them, which are of special interest for term rewriting. The first one, the class of automata with constraints between brothers, restricts equality and disequality tests to sibling positions. This class has good decidability and closure properties. E.g., this class allows to represent normal forms for left-shallow t.r.s. -but not for general ones - and it helped recently for providing new decidability results for normalization. The second one, the class of reduction automata, bounds, roughly speaking, the number of equality constraints. This class has provided the decidability of the reducibility theory and as a corollary a (new) way of deciding ground reducibility. Recently, a strong work by Godoy & alt. [4] has given some new emphasis on these classes. Indeed, it defines some new subclasses having good properties. This enables them to decide whether the homomorphic image of a tree recognizable language is recognizable. As a corollary, they get a new simple proof of decidability of recognizability of the set of normal forms of a t.r.s..

. . . to Global Ones

A new approach has been recently proposed : adding constraints to perform (dis-)equality tests, but globally. The idea is to enrich the automaton by two relations, $=, \neq$, over the states. Roughly speaking, the run will be correct if the subterms associated with two "equal" (resp. "not equal") states are equal (resp. different). E.g. this approach enables to check that all the subterms rooted by a f are equal or to encode that every identifier is different. This approach has led to (almost) simultaneous definitions of classes by different researchers to different purposes: rigid tree automata [5], tree automata with global equality and disequality tests (TAGED) [3], tree automata with global constraints [1]. The second part of this talk will give an overview of these classes and sketch their links with other classes.

References – A very incomplete list of references the talk will rely upon

1 Luis Barguñó, Carles Creus, Guillem Godoy, Florent Jacquemard, and Camille Vacher. The emptiness problem for tree automata with global constraints. In *Proceedings of the 25th Annual IEEE Symposium on Logic in Computer Science, LICS*, pages 263–272. IEEE Computer Society, 2010.

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