Iterative Methods in Combinatorial Optimization

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Abstract

In these lectures, I will describe a simple iterative method that supplies new proofs of integrality of linear characterizations of various basic problems in combinatorial optimization, and also allows adaptations to design approximation algorithms for NP-hard variants of these problems involving extra "degree-like" budget constraints. It is inspired by Jain's iterative rounding method for designing approximation algorithms for survivable network design problems, and augmented with a relaxation idea in the work of Lau, Naor, Salvatipour and Singh in their work on designing the approximation algorithm for its degree bounded version. Its application was further refined in recent work of Bansal, Khandekar and Nagarajan on degree-bounded directed network design.

I will begin by reviewing the background material on LP relaxations and their solvability and properties of extreme point or vertex solutions to such problems. I will then introduce the basic framework of the method using the assignment problem, and show its application by re-deriving the approximation results of Shmoys and Tardos for the generalized assignment problem.

I will then discuss linear characterizations for the spanning tree polyhedron in undirected graphs and give a new proof of integrality using an iterative method. I will then illustrate an application to approximating the degree-bounded version of the undirected problem, by proving the results of Goemans and Lau & Singh.

I will continue with showing how these methods for spanning trees simplify and generalize to showing linear descriptions of maximum weight matroid bases and also maximum weight sets that are independent in two different matroids. This also leads to good additive approximation algorithms for a bounded degree version of the matroid basis problem.

I will close with applications of the iterative method by revisiting Jain's original proof for the SNDP and giving a new proof that unifies its treatment with that for the Symmetric TSP polyhedron (describing joint work with Nagarajan and Singh). I will also outline the versatility of the method by pointing out the other problems for which the method has been applied, summarizing the discussion in a recent monograph I have co-authored on this topic with Lap Chi Lau and Mohit Singh (published by Cambridge University Press, 2011).

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