Extending the Rackoff technique to Affine nets

Rémi Bonnet*, **Alain Finkel**†, **and M. Praveen**‡

LSV, INRIA, CNRS, ENS Cachan

Abstract

We study the possibility of extending the Rackoff technique to Affine nets, which are Petri nets extended with affine functions. The Rackoff technique has been used for establishing EXPSPACE upper bounds for the coverability and boundedness problems for Petri nets. We show that this technique can be extended to strongly increasing Affine nets, obtaining better upper bounds compared to known results. The possible copies between places of a strongly increasing Affine net make this extension non-trivial. One cannot expect similar results for the entire class of Affine nets since coverability is Ackermann-hard and boundedness is undecidable. Moreover, it can be proved that model checking a logic expressing generalized coverability properties is undecidable for strongly increasing Affine nets, while it is known to be EXPSPACE-complete for Petri nets.

1998 ACM Subject Classification F.2.2 Nonnumerical Algorithms and Problems, D.2.4 Software/Program Verification, F.4.1 Mathematical Logic

Keywords and phrases Complexity of VASS, Affine nets, Rackoff technique, model checking, coverability, boundedness.

Digital Object Identifier 10.4230/LIPIcs.FSTTCS.2012.301

1 Introduction

Context Petri nets are infinite state models and have been used for modelling and verifying properties of concurrent systems. Various extensions of Petri nets that increase the power of transitions have been studied, for example Reset/Transfer (Petri) nets [8], Self-Modifying nets [21] and Petri nets with inhibitory arcs [17]. In [9], Well Structured nets are defined as another extension where transitions can be any non-decreasing function. The same paper also defines Affine Well Structured nets (shortly: Affine nets) that can be seen as the affine restriction of Well Structured nets, or as a restriction of the Self-Modifying nets of [21] to matrices with only non-negative integers.

While reachability is decidable for Petri Nets [11, 15, 13], it is undecidable for extensions with at least two extended transitions like Double/Reset/Transfer/Zero-test arcs [8]. However, it remains decidable for Petri Nets with one such extended arc [2] or even with hierarchical zero-tests [17]. The framework of Well-Structured Transition Systems [10] provides the decidability of coverability, termination and boundedness for Petri nets and some of its monotonic extensions [8, 9]. However, boundedness is undecidable for Reset nets (and hence for Affine nets) [8]. Complexity results on Petri nets extensions are scarce, two notable results being that coverability is Ackermann-complete for Reset nets [20, 19] (while reachability

* This author is supported by the french Agence Nationale de la Recherche, REACHARD (grant ANR-11-BS02-001)

† This author is supported by the french Agence Nationale de la Recherche, REACHARD (grant ANR-11-BS02-001) and by the "Chaire DIGITEO"

‡ This author is supported by an ERCIM "Alain Bensoussan" Fellowship Programme. This programme is supported by the Marie Curie Co-funding of Regional, National and International Programmes (COFUND) of the European Commission.

32nd Int'l Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS 2012).
Editors: D. D'Souza, J. Radhakrishnan, and K. Telikepalli; pp. 301–312
LIPICS Leibniz International Proceedings in Informatics
Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

and boundedness are undecidable) and boundedness is Expspace-complete for a subclass of (strongly increasing) Affine nets [7].

Other extensions of Petri nets increase the state space of the transition system. These are for example branching Vector Addition Systems [6], ν-Petri nets [18], or Data nets [12] (equivalent to Timed Petri nets [3]). As ν-Petri nets and Data nets subsume Reset nets, boundedness and reachability are undecidable, and coverability is Ackermann-hard. On the other hand, while reachability is an (important) open problem for branching Vector Addition Systems, coverability and boundedness are known to be AltExpspace-complete by a proof that uses the Rackoff technique [6].

Finally, we note that some recent papers [5, 1] have extended the Rackoff technique to show Expspace upper bounds for the model-checking of some logics (that generalizes the notion of coverability and boundedness) for Petri nets.

Our contribution The goal is to exhibit a class of extensions of Petri nets for which the Rackoff technique can be extended in order to give an Expspace upper bound for coverability and boundedness. We do not look at extensions that change state spaces, as the complexity of coverability and boundedness for those is known to be either AltExpspace-complete (branching Vector Addition Systems) or Ackermann-hard (ν-Petri nets, Data nets). Moreover, as this technique relies heavily on the monotonicity of Petri Nets, it is natural to consider only monotonic extensions. The largest classes of such extensions are Affine nets and Well Structured nets, that both include most of the usually studied Petri net extensions. As we know that coverability and boundedness are respectively Ackermann-hard and undecidable for Reset nets, we must forbid resets in order for our generalization to work. This is done by disallowing any 0 in the diagonal of the matrices associated with the functions of Affine nets, yielding again the class of *strongly increasing Affine nets*, as defined in [9], that are equivalent to the Post-Self-Modifying nets (PSM) defined by Valk in [21]. This class is interesting because it strictly subsumes Petri nets. For example, PSM can recognize the language of palindromes, which Petri nets can not. More generally, all recursively enumerable languages are recognized by PSM [21], while boundedness (and other properties) is still decidable [21].

While the complexity of the reachability problem for Petri nets is unknown, the complexity of coverability and boundedness has been shown to be Expspace-complete (lower bound of $\textsc{Space}(\mathcal{O}(2^{c\sqrt{n}}))$ by Lipton [14] and $\textsc{Space}(\mathcal{O}(2^{cn\log n}))$ upper bound by Rackoff [16], where n is the size of the net). In [7], the boundedness problem is shown to be in $\textsc{Space}(\mathcal{O}(2^{cn^2\log n}))$ for Post-Self-Modifying nets: the proof associates a standard Petri net that weakly simulates the original Post-Self-Modifying net and then applies the Rackoff theorem [16] as a black box (Expspace upper bound for coverability could also be shown by the same construction).

We give two results: (1) We extend the Rackoff technique to work directly on strongly increasing Affine nets, improving the upper bounds for coverability and boundedness (from $\textsc{Space}(\mathcal{O}(2^{cn^2\log n}))$ to $\textsc{Space}(\mathcal{O}(2^{cn\log n}))$). (2) We state the limit of strongly increasing Affine nets by proving that model checking a fragment of CTL (which can express generalizations of boundedness and various other problems) is undecidable for strongly increasing Affine nets, while it is Expspace-complete for Petri nets [1].

Following are the three main difficulties in extending the Rackoff technique to strongly increasing Affine nets.

1. Showing upper bounds for the lengths of sequences certifying coverability or unboundedness is not enough — short sequences can give rise to large numbers.
2. We can no longer rely on ignoring places that go above some value. The effect of a

transition on a place will depend on the exact value at other places.
3. The effect of firing a sequence of transitions can not be determined by its Parikh image.
To overcome the first difficulty, we define transition systems that abstract the real ones, where markings from short sequences of transitions will have either small numbers or ω. This also overcomes the second difficulty, since ignored places will have the value ω in the abstract transition systems. If such a place affects another place, the affected place will also get the value ω. To overcome the third difficulty, we classify the set of places according to the way they affect each other. Places that have high values and interfere among themselves will always be unbounded so that they can be "ignored" (implemented by introducing another abstraction), leaving behind places that are amenable to analysis by Petri net techniques. Since this technique depends on the observation that places interfering with one another are unbounded, it can not be used for problems that require more precise answers than unboundedness. Model checking the fragment of CTL mentioned above does require such precise answers and turns out to be undecidable for strongly increasing Affine nets.

The following table summarises the complexity of various problems on Petri nets and strongly increasing Affine nets, with the contributions of this paper in bold. Abbreviations used in the table: SIAN for Strongly increasing Affine nets, MC(eiPrCTL) for model checking eventually increasing Presburger CTL, $\textsc{Sp}(2^{c\sqrt{n}} : 2^{cn\log n})$ for $\textsc{Space}(2^{c\sqrt{n}})$ lower bound and $\textsc{Space}(2^{cn\log n})$ upper bound.

	Petri nets	SIAN	Affine nets
Reachability	Decidable [15, 11]	Undecidable [8]	Undecidable [8]
Coverability	$\textsc{Sp}(2^{c\sqrt{n}} : 2^{cn\log n})$ [14, 16]	$\textsc{Sp}(2^{c\sqrt{n}} : \mathbf{2^{cn\log n}})$	Ackermann-hard [20]
Boundedness	$\textsc{Sp}(2^{c\sqrt{n}} : 2^{cn\log n})$ [14, 16]	$\textsc{Sp}(2^{c\sqrt{n}} : \mathbf{2^{cn\log n}})$	Undecidable [8]
MC(eiPrCTL)	EXPSPACE-complete [1]	**Undecidable**	Undecidable [8]

Due to space constraints, proofs have been skipped in the following, as are the details of model checking eventually increasing Presburger CTL. All missing details and proofs can be found in the full version of this paper [4].

2 Preliminaries

Let $\mathbb{Z}$ be the set of integers, $\mathbb{N}$ be the set of non-negative integers and $\mathbb{N}^+$ be the set of positive integers. For any set P, $card(P)$ is the cardinality of P.

A *transition system* $\mathcal{S} = (S, \rightarrow)$ is a set S endowed with a transition relation "$\rightarrow$", i.e., with a binary relation on the set S. We write $s \xrightarrow{+} t$ to mean that there exist $r \in \mathbb{N}^+$ and a sequence of states $s_0 = s, s_1, \ldots, s_r = t$ such that $s_0 \rightarrow s_1 \rightarrow \cdots \rightarrow s_r$. We write $s \xrightarrow{*} t$ to mean that $s \xrightarrow{+} t$ or $s = t$. A state $t \in S$ of a transition system $\mathcal{S} = (S, \rightarrow)$ is *reachable* from a state s if $s \xrightarrow{*} t$. The *reachability set* of $\mathcal{S}$ from the state s_0 is denoted by $RS(\mathcal{S}, s_0)$ and is defined to be the set of states reachable from s_0.

Let P be a finite non-empty set of *places* with $card(P) = m \in \mathbb{N}^+$ and let $\langle p_1, \ldots, p_m\rangle$ be an arbitrary but fixed order on the set of places. A function $M : P \rightarrow \mathbb{N}$ is called a *marking*. We denote by $\mathbf{0}$ the marking such that $\mathbf{0}(p) = 0$ for all $p \in P$. Given a subset $Q \subseteq P$ and markings M_1, M_2, we write $M_1 =_Q M_2$ (resp. $M_1 \geq_Q M_2$) if $M_1(p) = M_2(p)$ (resp. $M_1(p) \geq M_2(p)$) for all $p \in Q$. We write $M_1 > M_2$ if $M_1 \geq M_2$ and $M_1 \neq M_2$. We denote by *Id* the identity matrix, whose dimension will be clear from context. We denote by $\mathbf{A}_1 \geq \mathbf{A}_2$, where $\mathbf{A}_1, \mathbf{A}_2 \in \mathbb{N}^{m\times m}$, the condition that $\mathbf{A}_1(p_1, p_2) \geq \mathbf{A}_2(p_1, p_2)$ for all $p_1, p_2 \in P$. We consider *(positive) affine functions* from $\mathbb{N}^m$ into $\mathbb{N}^m$ defined by $f(M) = \mathbf{A}M + \mathbf{B}$, where $\mathbf{A}$ is a (positive) matrix in $\mathbb{N}^{m\times m}$ and $\mathbf{B}$ is a vector in $\mathbb{Z}^m$. It can be verified that for every affine function $f(M) = \mathbf{A}M + \mathbf{B}$ with an upward closed domain (i.e., $domf \subseteq \mathbb{N}^P$ such that $M_1 \in domf$ and $M_1 \leq M_2$ imply $M_2 \in domf$), there exists a *finite* set of vectors $\{\mathbf{C}_1, \ldots, \mathbf{C}_k\} \subseteq \mathbb{Z}^m$ such that $domf = \cup_{1 \leq i \leq k}\{M \in \mathbb{N}^m \mid \mathbf{A}M + \mathbf{B} \geq \mathbf{0}$ and $M + \mathbf{C}_i \geq \mathbf{0}\}$. With $\mathbf{A} \in \mathbb{N}^{m \times m}$ and $\mathbf{B}, \mathbf{C} \in \mathbb{Z}^m$, we denote by $f \triangleq (\mathbf{A}, \mathbf{B}, \mathbf{C})$ the affine function such that $f(M) = \mathbf{A}M + \mathbf{B}$ and $domf = \{M \in \mathbb{N}^m \mid \mathbf{A}M + \mathbf{B} \geq \mathbf{0}$ and $M + \mathbf{C} \geq \mathbf{0}\}$. In terms of Petri nets, the vector $\mathbf{C}$ restricts the markings to which the transition can be applied. For example, if the transition should not subtract anything from a place p but should only be applicable to markings M with $M(p) \geq 1$, we can set $\mathbf{A}(p,p) = 1, \mathbf{B}(p) = 0$ and $\mathbf{C}(p) = -1$. In the following, we just write $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ if the name f is not important.

▶ **Definition 1.** An *Affine net* $\mathcal{N}$ (of dimension m) is a tuple $\mathcal{N} = (m, F)$ where $m \in \mathbb{N}^+$ and F is a finite set of affine functions with upward closed domains in $\mathbb{N}^m$.

The application of the transition function f to M_1 resulting in M_2 is denoted by $M_1 \xrightarrow{f} M_2$. The associated *Affine transition system* $\mathcal{S}_\mathcal{N} = (S, \xrightarrow{F})$ is naturally defined by $S = \mathbb{N}^P$ and $M_1 \xrightarrow{f} M_2$ iff $M_1 \in domf$ and $f(M_1) = M_2$. If there is a sequence $\sigma = f_1 f_2 \cdots f_r$ of transition functions such that $M \xrightarrow{f_1} M_1 \xrightarrow{f_2} \cdots\cdots \xrightarrow{f_r} M_r$, we denote it by $M \xrightarrow{\sigma} M_r$. The markings $M, M_1, \ldots, M_r$ are called *intermediate markings arising while firing σ from M*. We say a sequence σ of transition functions is *enabled at a marking* M if $M \xrightarrow{\sigma} M'$ for some marking M'. We denote the length of σ by $|\sigma|$. We denote the set of transition functions of $\mathcal{N}$ occurring in σ by $alph(\sigma)$. A sequence σ' is called a *sub-sequence* of σ if σ' can be obtained from σ by removing some transition functions.

▶ **Definition 2.** An affine function $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ is *strongly increasing* [9, Section 2.2] if $\mathbf{A} \geq Id$. An Affine net $\mathcal{N} = (m, F)$ is strongly increasing if each of its functions is strongly increasing.

Note that if $M_1 \xrightarrow{f} M_2$ and $M_1' > M_1$, then the fact that f is strongly increasing implies that $M_1' \xrightarrow{f} M_2'$ for some $M_2' > M_2$ and for every $p \in P$, $M_1'(p) > M_1(p)$ implies $M_2'(p) > M_2(p)$.

▶ **Definition 3.** Given an Affine net $\mathcal{N}$ with an initial marking M_{init} and a target marking M_{cov}, the *coverability problem* is to determine if there exists a marking $M \in RS(\mathcal{S}_\mathcal{N}, M_{init})$ such that $M \geq M_{cov}$. The *boundedness problem* is to determine if there exists a number $B \in \mathbb{N}$ such that for all markings $M \in RS(\mathcal{S}_\mathcal{N}, M_{init})$, $M(p) \leq B$ for all $p \in P$.

For an Affine net $\mathcal{N}$, $R_\mathcal{N}$ will denote the maximum absolute value of any entry in $\mathbf{A}$, $\mathbf{B}$ or $\mathbf{C}$ for any transition function $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ of $\mathcal{N}$. When $\mathcal{N}$ is clear from context, we skip the subscript $\mathcal{N}$ and write R. We also write *function* instead of *transition function* when it is clear from context that it is a transition function in an Affine net. The *size* of $\mathcal{N}$ with initial marking M_{init} is defined to be $(card(F)(m^2 + m)\log R + m \log\|M_{init}\|_\infty)$, where $\|M_{init}\|_\infty$ is the maximum entry in M_{init}. If $\mathbf{A} = Id$ for each function $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ of $\mathcal{N}$, then $\mathcal{N}$ is a Petri net.

In Affine nets, markings cannot decrease too much.

▶ Proposition 4. If $M_1 \xrightarrow{\sigma} M_2$, then $M_2(p) \geq M_1(p) - R|\sigma|$ for all $p \in P$.

3 Value Abstracted Semantics

In Affine nets, a short sequence of functions can generate markings with large values. Beyond some value, it is not necessary to store the exact values of a marking to decide coverability and boundedness. Let $\mathbb{N}_\omega = \mathbb{N} \cup \{\omega\}$ and $\mathbb{Z}_\omega = \mathbb{Z} \cup \{\omega\}$ where addition, multiplication and order are as usual with the extra definition of $\omega \times 0 = 0 \times \omega = 0$. To avoid using excessive memory space to store large values of markings, we introduce extended markings $W : P \to \mathbb{N}_\omega$. The domains of transitions functions are extended to include extended markings: for a function $f = (\mathbf{A}, \mathbf{B}, \mathbf{C})$ and an extended marking W, we have $W \in dom f$ iff $W + \mathbf{C} \geq \mathbf{0}$ and $\mathbf{A}W + \mathbf{B} \geq \mathbf{0}$. The result W' of applying f to $W \in dom f$ is given by $W' = \mathbf{A}W + \mathbf{B}$, denoted by $W \xrightarrow{f} W'$.

For an extended marking $W : P \to \mathbb{N}_\omega$, let $\omega(W) = \{p \in P \mid W(p) = \omega\}$ and $\overline{\omega(W)} = P \setminus \omega(W)$. For a function $t : \{0, \ldots, m\} \to \mathbb{N}$ (which will be used to denote thresholds) and an extended marking W, we define $[W]_{t\to\omega}$ and $[W]_{\omega\to t}$ by:

$$([W]_{t\to\omega})(p) = \begin{cases} W(p) & \text{if } W(p) < t(card(\overline{\omega(W)})), \\ \omega & \text{otherwise.} \end{cases}$$

$$([W]_{\omega\to t})(p) = \begin{cases} W(p) & \text{if } W(p) \in \mathbb{N}, \\ t(card(\overline{\omega(W)}) + 1) & \text{otherwise.} \end{cases}$$

The threshold function t gives the threshold beyond which numbers can be abstracted. In the extended marking $[W]_{t\to\omega}$, values beyond the threshold given by t are abstracted by ω. In the marking $[W]_{\omega\to t}$, abstraction is reversed by replacing ω with the corresponding threshold value.

▶ **Definition 5.** Let $t : \{0, \ldots, m\} \to \mathbb{N}$ be a threshold function and $\mathcal{N}$ be a strongly increasing Affine net. The associated *t-transition system* $\mathcal{S}_{\mathcal{N},t} = (S_t, \xrightarrow{F}_t)$ is defined by $S_t = \mathbb{N}_\omega^P$ and $W_1 \xrightarrow{(\mathbf{A},\mathbf{B},\mathbf{C})}_t W_2$ iff $W_1 \geq \mathbf{C}$ and $W_2 = [(\mathbf{A}W_1 + \mathbf{B})]_{t\to\omega} \in \mathbb{N}_\omega^P$. We write $W_0 \xrightarrow{\sigma}_t W_r$ if $\sigma = f_1 \cdots f_r$ and $W_{i-1} \xrightarrow{f_i}_t W_i$ for each i between 1 and r. The extended markings $W_0, \ldots, W_r$ are called *intermediate extended markings in the run* $W_0 \xrightarrow{\sigma}_t W_r$.

Note that for any $W_1 \xrightarrow{f}_t W_2$, $\omega(W_2) \supseteq \omega(W_1)$. In the t-transition system, a place having the value ω will retain it after the application of any function. The following propositions establish some relationships between t-transition systems and natural transition systems.

▶ Proposition 6. Let $W_1 \xrightarrow{\sigma}_t W_2$, $card(\overline{\omega(W_2)}) = card(\overline{\omega(W_1)}) < m$ and $t(card(\overline{\omega(W_1)}) + 1) \geq R|\sigma| + x$ for some $x \in \mathbb{N}$. Then $[W_1]_{\omega\to t} \xrightarrow{\sigma} M_2$ such that $M_2 =_{\overline{\omega(W_2)}} W_2$ and $M_2(p) \geq x$ for all $p \in \omega(W_2)$.

A routine induction on $|\sigma|$ allows to prove the following:

▶ Proposition 7. If $M_1 \xrightarrow{\sigma} M_2$, then $M_1' \xrightarrow{\sigma}_t W_2 \geq M_2$ for any $M_1' \geq M_1$.

4 Coverability

In this section, we give a $\textsc{Space}(\mathcal{O}(2^{c_2 n \log n}))$ upper bound for the coverability problem in strongly increasing Affine nets, for some constant c_2. Let $R' = \max(\{M_{cov}(p) \mid p \in P\} \cup \{R\})$, where M_{cov} is the marking to be covered. In the rest of this section, we fix a strongly increasing Affine net $\mathcal{N} = (m, F)$ with an initial marking M_{init} and the marking to be covered M_{cov}. The set of places is $P = \{p_1, \ldots, p_m\}$.

We briefly recall the Rackoff technique for the $\textsc{Expspace}$ upper bound for the coverability problem in Petri nets. The idea is to define a function $\ell : \mathbb{N} \to \mathbb{N}$ and prove that for a Petri net with m places, coverable markings can be covered with sequences of transitions of length at most $\ell(m)$. This is done by induction on the number of places. In a Petri net with $i + 1$

places, suppose $M_{init} \xrightarrow{\sigma} M' \geq M_{cov}$ and M is the first intermediate marking where one of the values is more than $R\ell(i) + R' - 1$ (this is the intuition behind the definition of the threshold function t_1 below). If there is no such marking, all intermediate markings have small values and it is easy to bound the length of σ by $(R\ell(i) + R')^{i+1}$. Otherwise, let $\sigma = \sigma_1\sigma_2$ such that $M_{init} \xrightarrow{\sigma_1} M \xrightarrow{\sigma_2} M' \geq M_{cov}$. The length of σ_1 is bounded by $(R\ell(i)+R')^{i+1}$. Temporarily forgetting the existence of place p (where $M(p) \geq R\ell(i)+R'$), we conclude by induction hypothesis that starting from M, M_{cov} can be covered (in all places except p) with a sequence σ_2' of length at most $\ell(i)$. Since $M(p) \geq R\ell(i) + R'$ and σ_2' reduces the value in p by at most $R\ell(i)$, σ_2' in fact covers all places, including p. Hence, $\sigma_1\sigma_2'$ covers M_{cov} from M_{init} and its length is at most $(R\ell(i) + R')^{i+1} + \ell(i) + 1$. This is the intuition behind the definition of the length function ℓ_1 below. The counterpart of "temporarily forgetting p" is assigning it the value ω.

▶ **Definition 8.** The functions $\ell_1, t_1 : \mathbb{N} \to \mathbb{N}$ are as follows.

$$t_1(0) = 0 \qquad \ell_1(0) = 0$$
$$t_1(i+1) = R\ell_1(i) + R' \qquad \ell_1(i+1) = (t_1(i+1))^{i+1} + \ell_1(i) + 1$$

▶ **Definition 9.** A *covering sequence enabled at M* is a sequence σ of functions such that $M \xrightarrow{\sigma} M'$ and $M' \geq M_{cov}$. A *t_1-covering sequence enabled at W* is a sequence σ of functions such that $W \xrightarrow{\sigma}_{t_1} W'$ and $W' \geq M_{cov}$.

The following lemma shows that even after abstracting values that are above the ones given by the threshold function t_1, there is still enough information to check coverability.

▶ **Lemma 10.** *If a t_1-covering sequence σ is enabled at W, then M_{cov} is coverable from $[W]_{\omega \to t_1}$.*

▶ **Lemma 11.** *If there is a covering sequence σ enabled at M_{init}, there is a t_1-covering sequence σ' enabled at M_{init} such that $|\sigma'| \leq \ell_1(m)$ (recall that $m = card(P)$).*

▶ **Lemma 12.** *For all $i \in \mathbb{N}$, $\ell_1(i) \leq (6RR')^{(i+1)!}$.*

▶ **Theorem 13.** *For some constant c_1, the coverability problem for strongly increasing Affine nets is in* $\text{NSPACE}(\mathcal{O}(2^{c_1 m \log m}(\log R' + \log\|M_{init}\|_\infty)))$.

Taking $n = (card(F)(m^2 + m) \log R + m \log\|M_{init}\|_\infty) + m \log\|M_{cov}\|_\infty$ as the size of the input to the coverability problem, we can infer from the above theorem an upper bound of $\text{SPACE}(\mathcal{O}(2^{c_2 n \log n}))$.

5 Boundedness

In this section, we give a $\text{SPACE}(\mathcal{O}(2^{c_4 n \log n}))$ upper bound for the boundedness problem in strongly increasing Affine nets, for some constant c_4. In the rest of this section, we fix a strongly increasing Affine net $\mathcal{N} = (m, F)$ with an initial marking M_{init}. The set of places is $P = \{p_1, \ldots, p_m\}$.

▶ **Definition 14.** A *self-covering pair enabled at M* is a pair (σ_1, σ_2) of sequences of functions such that $M \xrightarrow{\sigma_1} M_1 \xrightarrow{\sigma_2} M_2$ and $M_2 > M_1$.

Since all transition functions are strongly increasing and their domains are upward closed, we can infer from the above definition that $M_i \xrightarrow{\sigma_2} M_{i+1}$ and $M_{i+1} > M_i$ for all $i \in \mathbb{N}^+$. Hence, if a self covering pair is enabled at M_{init}, then $\mathcal{N}$ is unbounded. Conversely, if $\mathcal{N}$ is unbounded, infinitely many distinct markings can be reached from M_{init}. Since there are only finitely many transition functions, König's lemma implies that there is an infinite sequence of functions enabled at M_{init} such that all intermediate markings are distinct. We infer from Dickson's lemma that there are two markings M_1, M_2 along this sequence such that M_2 is after M_1 and $M_2 > M_1$. Let $M_{init} \xrightarrow{\sigma_1} M_1 \xrightarrow{\sigma_2} M_2$. By Definition 14, (σ_1, σ_2) is a self-covering pair enabled at M_{init}.

The Rackoff technique again defines a length function $\ell' : \mathbb{N} \to \mathbb{N}$ and shows that if a Petri net with m places is unbounded, there is a self-covering pair of total length at most $\ell'(m)$. As an example, let us consider giving an upper bound for $\ell'(2)$. Consider the sequence of markings shown below, produced by a self-covering pair. Let σ_2 be the portion occurring

$$\begin{array}{lccccc}
p_1: & 1 & & 100 & & \left[\begin{array}{ccc} 90 & & 190 \\ 140 & & 280 \\ 100 & & 90 \\ 200 & \cdots & 190 \\ 1 & & 1 \\ 3 & & 3 \end{array}\right] & & 180 \\
p_2: & 2 & & 150 & & & & 270 \\
p_3: & 1 & & 100 & & & & 100 \\
p_4: & 3 & \cdots & 200 & \cdots & & \cdots & 200 \\
p_5: & 0 & & 2 & & & & 3 \\
p_6: & 1 & & 4 & & & & 4
\end{array}$$

after the first marking where p_1 has the value 100. Since p_5, p_6 have low values throught, we would like to abstract the remaining places and reduce the length of σ_2 to get an upper bound on $\ell'(2)$. We denote $\{p_5, p_6\}$ by $P^{\sigma_2}_{<\omega}$. In the block of intermediate markings shown above enclosed in [ ], the first and last markings are identical when projected to p_5 and p_6. Since this block does not change p_5 and p_6, we can remove this block, provided that after removal, the abstracted places p_1, p_2, p_3, p_4 will still have values at least $100, 150, 100, 200$ respectively. To decide whether this is the case, the effect of the block on p_1, p_2, p_3, p_4 is calculated in a Petri net by simply summing up the effect of each transition in the block. In a strongly increasing Affine net, this is however not possible since the effect of the block depends not only on the transitions in it, but also on the values in the marking at the beginning of the block. In addition, affine functions can copy the value of one place to another one.

If some transition copies the value of some place among p_1, p_2, p_3, p_4 into p_5 or p_6, a large value will result in p_5 or p_6, so that they too can be abstracted, letting us use induction hypothesis to deal with the remaining fewer number of non-abstracted places. To deal with the other case, we assume that no transition in σ_2 does this kind of copying (σ_2 isolates $\{p_5, p_6\}$ from $\{p_1, p_2, p_3, p_4\}$). Next, suppose a function $f = (\mathbf{A}, \mathbf{B}, \mathbf{C})$ occurs inside the block, where $\mathbf{A}$ and $\mathbf{B}$ are as shown in Fig. 1 in the next page. Let the rows and columns of $\mathbf{A}$ correspond to $p_1, p_2, \ldots, p_6$ in that order. The function f doubles the value in p_2 and copies the value of p_3 to p_1, but isolates $\{p_3, p_4\}$ from $\{p_1, p_2, p_3, p_4\}$. In the following, we will say f "crosses" $P^{\sigma_2}_{\times} = \{p_1, p_2\}$ and isolates $P^{\sigma_2}_{is} = \{p_3, p_4\}$ from $P \setminus P^{\sigma_2}_{<\omega} = \{p_1, p_2, p_3, p_4\}$. Since p_1, p_2 had large values to begin with, they will have even larger values after crossed by f. We will see that this will result in crossed places becoming unbounded, and so we can forget the exact effect of a block on such places, and just remember that they are crossed by some function. The forgetting part is done in Definition 16 by simplifying the matrix $\mathbf{A}$, and the remembering part is done by setting the corresponding entry in $\mathbf{B}$ to 1. To decide whether a block can be removed or not, it remains to compute the effect of $P^{\sigma_2}_{<\omega} = \{p_5, p_6\}$ on $P^{\sigma_2}_{is} = \{p_3, p_4\}$. This can be achieved since we know the exact values in p_5, p_6. This is formalised in the proof of Lemma 20.

▶ **Definition 15.** Let $f = (\mathbf{A}, \mathbf{B}, \mathbf{C})$ be a function.

- f *multiplies* a place p if $\mathbf{A}(p, p) \geq 2$.
- f *copies* p' *to* p if $\mathbf{A}(p, p') \geq 1$ for any two places $p \neq p'$.
- f *isolates* Q_1 *from* Q_2, where $Q_1, Q_2 \subseteq P$, if $\mathbf{A}(p, p') = 0$ for all $p \in Q_1$ and $p' \in Q_2 \setminus \{p\}$.

Although the sets $P^{\sigma_2}_{<\omega}$, $P^{\sigma_2}_{\times}$ and $P^{\sigma_2}_{is}$ are determined by σ_2, we avoid heavy notation in the following definition and instead use an arbitrary partition of P into $P_{<\omega}$, $P_\times$ and P_{is}.

▶ **Definition 16.** Let $\rho = (P_{<\omega}, P_\times, P_{is})$ be a triple such that the sets $P_{<\omega}$, $P_\times$ and P_{is} partition the set of places P. To each function $f = (\mathbf{A}, \mathbf{B}, \mathbf{C})$, we associate another function $f[\rho] = (\mathbf{A}[\rho], \mathbf{B}[\rho], \mathbf{C})$, where $\mathbf{A}[\rho]$ and $\mathbf{B}[\rho]$ are as follows.

$$(\mathbf{B}[\rho])(p) = \begin{cases} \mathbf{B}(p) & \text{if } p \notin P_\times \\ 1 & \text{if } f \text{ multiplies } p \in P_\times \\ 1 & \text{if } f \text{ copies } p' \text{ to } p \in P_\times, \\ & p' \in (P_\times \cup P_{is} \setminus \{p\}) \\ 0 & \text{otherwise} \end{cases} \qquad (\mathbf{A}[\rho])(p,p') = \begin{cases} \mathbf{A}(p,p') & \text{if } p \notin P_\times \\ 1 & \text{if } p \in P_\times \\ & \text{and } p = p' \\ 0 & \text{otherwise} \end{cases}$$

Associated with the triple ρ is the ρ-*transition system* $\mathcal{S}_{\mathcal{N},\rho} = (S_\rho, \longrightarrow_\rho)$, defined by $S_\rho = \mathbb{N}^P_\omega$ and $W_1 \xrightarrow{(\mathbf{A},\mathbf{B},\mathbf{C})}_\rho W_2$ iff $W_1 + \mathbf{C} \geq \mathbf{0}$ and $W_2 = \mathbf{A}[\rho] W_1 + \mathbf{B}[\rho] \in \mathbb{N}^P_\omega$. We write $W_0 \xrightarrow{\sigma}_\rho W_r$ if $\sigma = f_1 \cdots f_r$ and $W_{i-1} \xrightarrow{f_i}_\rho W_i$ for each i between 1 and r. We write $\sigma[\rho]$ to denote the sequence of functions obtained from σ by replacing each function f of σ by $f[\rho]$.

For any extended marking W, if $W(p) \in \mathbb{N}$ for all $p \in P$ (i.e., if W is a marking), then applying any function to W in the ρ-transition system will result in another marking (new ω values are not introduced). In the example given before Definition 15, partition the set of places into $P_\times = \{p_1, p_2\}$, $P_{is} = \{p_3, p_4\}$ and $P_{<\omega} = \{p_5, p_6\}$. The corresponding matrices $\mathbf{A}[\rho]$ and $\mathbf{B}[\rho]$ defining $f[\rho]$ are as shown below.

Figure 1 Examples of transition functions

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 2 \\ 0 \\ -1 \end{bmatrix} \quad \mathbf{A}[\rho] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \quad \mathbf{B}[\rho] = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$$

We obtain $\mathbf{A}[\rho]$ from $\mathbf{A}$ by replacing the first two rows by the corresponding rows of the identity matrix. The fact that $\{p_1, p_2\}$ is the set of places "crossed" by f is instead indicated by setting $\mathbf{B}[\rho](p_1) = \mathbf{B}[\rho](p_2) = 1$. The other entries of $\mathbf{B}[\rho]$ are not changed. Places with high values and crossed by some function will always be unbounded and this technique can not be applied for problems that need more precise answers. For example, eventually increasing existential Presburger CTL (eiPrECTL$_\geq$(U)), introduced in [1], can express the presence of sequences along which the value of one place grows unboundedly while another place remains bounded. Model checking eiPrECTL$_\geq$(U) is shown to be EXPSPACE-complete [1] for Petri nets by extending the Rackoff technique. We show in the full version that model checking eiPrECTL$_\geq$(U) is undecidable for strongly increasing Affine nets.

▶ **Definition 17.** Let $\rho = (P_{<\omega}, P_\times, P_{is})$ be a triple such that the sets $P_{<\omega}$, $P_\times$ and P_{is} partition the set of places P. A sequence of functions σ is a ρ-*pumping sequence enabled at a marking* M_0 if

1. $M_0 \xrightarrow{\sigma}_\rho M_1$ and $M_1 > M_0$,

2. for all places $p \in P_\times$, some function $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ in σ has $\mathbf{B}[\rho](p) = 1$,
3. each function in σ isolates $P_{is} \cup P_{<\omega}$ from $P_\times \cup P_{is}$ and
4. no function in σ multiplies p for any place $p \in P_{is}$.

Next we develop formalisations needed to show that if there are ρ-pumping sequences, there are short ones. Suppose $W_1 \xrightarrow{\sigma}_t W_2$ and $\overline{\omega(W_1)} = \overline{\omega(W_2)}$. Then each function in σ isolates $\overline{\omega(W_1)}$ from $\omega(W_1)$ (otherwise, we could not have $\overline{\omega(W_1)} = \overline{\omega(W_2)}$). The following proposition establishes a relationship between t-transition systems and ρ-transition systems.

▶ Proposition 18. Let t be a threshold function, $W_1 \xrightarrow{\sigma}_t W_2$ and $\overline{\omega(W_1)} = \overline{\omega(W_2)}$. Let $\rho = (\overline{\omega(W_1)}, P_\times, P_{is})$. If $[W_1]_{\omega \to t} \xrightarrow{\sigma}_\rho M_2$, then $M(p) < t(card(\overline{\omega(W_1)}))$ for every intermediate marking M arising while firing $\sigma[\rho]$ from $[W_1]_{\omega \to t}$ and every $p \in \overline{\omega(W_1)}$.

The following definition of loops will be used only in the proof of Lemma 20.

▶ **Definition 19.** Suppose W_1 is an extended marking such that $\overline{\omega(W_1)} = P_{<\omega}$ and σ is a sequence such that all functions in $\sigma[\rho]$ isolate $P_{<\omega}$ from $P \setminus P_{<\omega}$. Suppose σ can be decomposed as $\sigma = \pi_1 \pi \pi_2$ and $W_1 \xrightarrow{\pi_1}_\rho L \xrightarrow{\pi}_\rho L \xrightarrow{\pi_2}_\rho W_2$. The pair (π, L) is a *$P_{<\omega}$-loop* if all extended markings (except the last one) arising while firing $\pi[\rho]$ from L are distinct from one another.

▶ **Lemma 20.** *There exists a constant d such that for any strongly increasing Affine net $\mathcal{N}$ and for every ρ-pumping sequence σ enabled at some marking M_0, there exists a ρ-pumping sequence σ' enabled at M_0' such that $|\sigma'| \le (2eR)^{dm^3}$, where:*

- *$\rho = (P_{<\omega}, P_\times, P_{is})$ is a triple such that $P_{<\omega}$, $P_\times$ and P_{is} partition the set of places P,*
- *$e = 1 + \max\{M(p) \mid p \in P_{<\omega}$ and M is an intermediate marking occurring while firing σ from $M_0\}$ and*
- *M_0' is any marking such that $M_0' =_{P_{<\omega}} M_0$ and $M_0'(p) \ge R|\sigma'|$ for all $p \in P_{is} \cup P_\times$.*

Proof. Let $M_0 \xrightarrow{\sigma}_\rho M_k$. Removing all $P_{<\omega}$-loops from σ may not result in a ρ-pumping sequence. As in the Rackoff technique, we use the existence of small solutions to linear Diophantine equations to show that a small number of loops can be retained to get a shorter ρ-pumping sequence. Some intermediate steps of this proof deal with sub-sequences of σ that may not be enabled at M_0'. They will however be enabled at the extended marking W_0 where $W_0 =_{P_{<\omega}} M_0$ and $W_0(p) = \omega$ for all $p \in P_\times \cup P_{is}$. The proof is organised into the following steps.

Step 1: We first associate a vector with each sub-sequence of σ to measure the effect of the sub-sequence on $P_\times \cup P_{is}$.

Step 2: Next we remove some $P_{<\omega}$-loops from σ to obtain σ'' such that for every intermediate extended marking W arising while firing $\sigma[\rho]$ from W_0, W also arises while firing $\sigma''[\rho]$ from W_0.

Step 3: The sequence σ'' obtained above is not necessarily a ρ-pumping sequence. With the help of the vectors defined in step 1, we formulate a set of linear Diophantine equations to encode the fact that the effects of σ'' and the $P_{<\omega}$-loops that were removed combine to give the effect of a ρ-pumping sequence. This step uses the fact that in ρ-transition systems, $\mathbf{B}[\rho](p)$ is set to 1 for each place $p \in P_\times$, indicating that p is “crossed” by some function.

Step 4: Then we use the result about the existence of small solutions to linear Diophantine equations to infer that only a small number of $P_{<\omega}$-loops need to be retained to ensure that the essential properties of pumping sequences (as encoded by the Diophantine equations) are satisfied. We use this to construct a sequence σ' that meets the length constraint of the lemma.

Step 5: Finally, we prove that σ' is a ρ-pumping sequence enabled at M'_0.

Step 1 is where this proof differs substantially from the ideas used by Rackoff in [16]. We give details of Step 1 here. The missing details are in the full version.

Step 1: Suppose $\pi \in alph(\sigma)^*$ is a sequence consisting of functions occurring in σ. Suppose W_1 is an extended marking such that $W_1(p) \in \mathbb{N}$ for all $p \in P_{<\omega}$ and $W_1(p') = \omega$ for all $p' \in P_\times \cup P_{is}$. Also suppose that $W_1 \xrightarrow{\pi}_\rho W_r$. We want to measure the effect of π on places in $P_\times \cup P_{is}$ when we replace ω by large enough values in W_1. We define a vector $\Delta[\pi, W_1]$ of integers for this measurement. For a place $p \in P_\times$, all that a function $(\mathbf{A}[\rho], \mathbf{B}[\rho], \mathbf{C})$ can do to p is add 0 or 1 (this is due to the way $\mathbf{A}[\rho]$ and $\mathbf{B}[\rho]$ are defined in Definition 16). So we take $\Delta[\pi, W_1](p)$ to be the sum of all $\mathbf{B}[\rho](p)$ for all functions $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ occurring in π. For a place $p \in P_{is}$, we have to take into account the "interference" from other places. Since from Definition 17, each function in π isolates P_{is} from $P_\times \cup P_{is}$, the only places that can interfere with p are those in $P_{<\omega}$. Let $\pi = f_1 f_2 \cdots f_r$ and $W_1 \xrightarrow{f_1}_\rho W_2 \xrightarrow{f_2}_\rho \cdots \xrightarrow{f_{r-1}}_\rho W_r$. For each i between 1 and $r-1$, let $f_i = (\mathbf{A}_i, \mathbf{B}_i, \mathbf{C}_i)$. Following is the formal definition of $\Delta[\pi, W_1]$:

$$\Delta[\pi, W_1](p) = \sum_{i=1}^{r-1} \left(\sum_{p' \in P_{<\omega}} \mathbf{A}_i(p, p') W_i(p') + \mathbf{B}_i(p) \right) \text{ for all } p \in P_{is}$$

$$\Delta[\pi, W_1](p) = \sum_{i=1}^{r-1} \mathbf{B}_i[\rho](p) \text{ for all } p \in P_\times$$

Since all functions in π isolate $P_{<\omega}$ from $P_\times \cup P_{is}$, we infer that $\overline{\omega(W_r)} = \overline{\omega(W_1)} = P_{<\omega}$. It is routine to infer the following two facts from the definition of $\Delta[\pi, W_1]$.

- If M_1 is any marking such that $M_1 =_{P_{<\omega}} W_1$ and $M_1 \xrightarrow{\pi}_\rho M_2$, then $M_2(p) - M_1(p) = \Delta[\pi, W_1](p)$ for all $p \in P_\times \cup P_{is}$.
- Suppose $\pi = \pi_1\pi_2\pi_3$, $W_1 \xrightarrow{\pi_1}_\rho W' \xrightarrow{\pi_2}_\rho W' \xrightarrow{\pi_3}_\rho W_r$ and (π_2, W') is a $P_{<\omega}$-loop. Then $\Delta[\pi, W_1] = \Delta[\pi_1\pi_3, W_1] + \Delta[\pi_2, W']$.

The above two facts are sufficient to extend the technique used in [16] to ρ-transition systems. This technique was developed for Petri nets, where the effect of a sequence of functions can be determined from its Parikh image. This is not true in general for strongly increasing Affine nets, but the core idea can be lifted to ρ-transition systems.

Details of Steps 2–4 can be found in the full version. ◀

The following lemma establishes what value is "large enough" at the initial marking to ensure that crossed places are unbounded.

▶ **Lemma 21.** *Let $\rho = (P_{<\omega}, P_\times, P_{is})$ be a triple such that the sets $P_{<\omega}$, $P_\times$ and P_{is} partition the set of places P. Suppose σ is a ρ-pumping sequence enabled at M_0. If $M_0(p) \geq 3R|\sigma| + R + 1$ for all $p \in P_\times \cup P_{is}$, then $M_0 \xrightarrow{\sigma} M_1$ and $M_1 > M_0$.*

▶ **Definition 22.** Let $c = 2d$. The functions $\ell_2, t_2 : \mathbb{N} \to \mathbb{N}$ are as follows:

$$t_2(0) = 0 \qquad \ell_2(0) = (2R)^{cm^3}$$

$$t_2(i+1) = 4R\ell_2(i) + R + 1 \qquad \ell_2(i+1) = (2t_2(i+1)R)^{cm^3}$$

Due to the selection of the constant c above, we have $(2xR)^{cm^3} \geq x^i + (2Rx)^{dm^3}$ for all $x \in \mathbb{N}$ and all $i \in \{0, \ldots, m\}$.

▶ **Definition 23.** A *t_2-pumping pair enabled at W* is a pair (σ_1, σ_2) of sequence of functions satisfying the following conditions.
1. $W \xrightarrow{\sigma_1}_{t_2} W_1 \xrightarrow{\sigma_2}_{t_2} W_2$,
2. $\overline{\omega(W_2)} = \overline{\omega(W_1)}$ and
3. for some partition of $\overline{\omega(W_1)}$ into $P_\times$ and P_{is}, σ_2 is a ρ-pumping sequence enabled at $[W_1]_{\omega\to t_2}$, where $\rho = (\overline{\omega(W_1)}, P_\times, P_{is})$.

The following two lemmas prove that unboundedness in strongly increasing Affine nets is equivalent to the presence of a short t_2-pumping pair enabled at M_{init}. The ideas of these two lemmas are similar to those of Lemma 10 and Lemma 11 respectively. Lemma 20 is used in Lemma 25.

▶ **Lemma 24.** *If (σ_1, σ_2) is a t_2-pumping pair enabled at W, there is a self-covering sequence (σ_1', σ_2') enabled at $[W]_{\omega\to t_2}$.*

▶ **Lemma 25.** *If a self covering pair is enabled at M_{init}, there is a t_2-pumping pair (σ_1', σ_2') enabled at M_{init} such that $|\sigma_1'| + |\sigma_2'| \leq \ell_2(m)$.*

▶ **Lemma 26.** *Let $k = 8c$. Then $\ell_2(i) \leq (2R)^{k^{i+1} m^{3(i+1)}}$ for all $i \in \mathbb{N}$.*

▶ **Theorem 27.** *For some constant c_3, the Boundedness problem for strongly increasing Affine nets is in* NSPACE*($\mathcal{O}(2^{c_3 m \log m}(\log R + \log\|M_{init}\|_\infty))$).*

With the size of the input $n = (card(F)(m^2 + m) \log R + m \log\|M_{init}\|_\infty)$, we can infer from the above theorem an upper bound of SPACE($\mathcal{O}(2^{c_4 n \log n})$) for the boundedness problem.

6 Conclusions and Perspectives

We proved that coverability and boundedness are in SPACE($\mathcal{O}(2^{cn \log n})$) for strongly increasing Affine nets. The main difficulty in adapting Rackoff technique is that one cannot simply ignore places that have large enough values, as transitions may copy values from one place to another. From this result, we may immediately deduce the same result for the termination problem as one can add a new place p_{time} which is incremented by every transition. Then, the system terminates iff it is bounded. A natural question is to identify the properties that could be proved (with Rackoff techniques) to be EXPSPACE-complete for strongly increasing Affine nets. At least two (recent) classes of properties are candidates: the generalized unboundedness properties of Demri [5] and the CTL fragment of Blockelet and Schmitz [1]. As this last logic is proved undecidable for strongly increasing Affine nets, a natural restriction of this logic would be defined. We conjecture that replacing the predicates on the effect of a path by predicates on the Parikh image of the path would put the model checking problem for the logic in EXPSPACE.

As we have limited our study to Affine nets, another question would be to consider not only affine functions, but to find classes of recursive functions (that still forbids resets) for which the Rackoff techniques can still be applied. It is likely that the proof of coverability could be adapted by altering Prop. 6 to take into account the “maximum reduction” that functions can perform. For example, if we allow functions to halve the value in a place, it would suffice to say that the initial value is $2^{\ell_1(i)}$ times higher (of course, this would change the final upper bound obtained). However, the proof of boundedness relies heavily on the fact that a place is either fully copied, or not at all, so how to generalize it is unclear.

Coverability and boundedness for Petri nets with an unique Reset/Transfer/Zero-test extended arc have been recently proved to be decidable [2]. For the Zero-test case, the complexity of coverability is at least as hard as reachability for Petri Nets, so there is not much hope of applying this technique. We conjecture that it could be applied to the one Reset or Transfer case, even if it would yield an upper bound greater than EXPSPACE.

References

1 M. Blockelet and S. Schmitz. Model checking coverability graphs of vector addition systems. In *Proceedings of MFCS*, volume 6907 of *LNCS*, pages 108–119, 2011.

2 R. Bonnet. The reachability problem for vector addition systems with one zero-test. In *Proceedings of MFCS*, volume 6907 of *LNCS*, pages 145–157, 2011.

3 R. Bonnet, A. Finkel, S. Haddad, and F. Rosa-Velardo. Comparing Petri data nets and timed Petri nets. Research Report LSV-10-23, CNRS, ENS Cachan, France, 2010.

4 R. Bonnet, A. Finkel, and M. Praveen. Extending the Rackoff technique to affine nets. Technical report, LSV, ENS Cachan, 2012. http://www.lsv.ens-cachan.fr/Publis/.

5 S. Demri. On selective unboundedness of VASS. In *Proceedings of INFINITY*, volume 39 of *EPTCS*, pages 1–15, 2010.

6 S. Demri, M. Jurdziński, O. Lachish, and R. Lazić. The covering and boundedness problems for branching vector addition systems. In *Proceedings of FSTTCS*, volume 4 of *LIPIcs*, pages 181–192, 2009. Journal version to appear in Journal of Computer and System Sciences, 2012.

7 C. Dufourd and A. Finkel. Polynomial-time many-one reductions for Petri nets. In *Proceedings of FSTTCS*, volume 1346 of *LNCS*, pages 312–326, 1997.

8 C. Dufourd, A. Finkel, and Ph. Schnoebelen. Reset nets between decidability and undecidability. In *Proceedings of ICALP*, volume 1443 of *LNCS*, pages 103–115, 1998.

9 A. Finkel, P. McKenzie, and C. Picaronny. A well-structured framework for analysing Petri net extensions. *Information & Computation*, 195(1-2):1–29, 2004.

10 A. Finkel and Ph. Schnoebelen. Well-structured transition systems everywhere! *Theoretical Computer Science*, 256(1-2):63–92, 2001.

11 S. R. Kosaraju. Decidability of reachability in vector addition systems. In *Proceedings of STOC*, pages 267–281, 1982.

12 R. Lazic, T. Newcomb, J. Ouaknine, A. W. Roscoe, and J. Worrell. Nets with tokens which carry data. In *Proceedings of ICATPN*, volume 4546 of *LNCS*, pages 301–320, 2007.

13 J. Leroux. Vector addition system reachability problem: a short self-contained proof. In *Proceedings of POPL*, pages 307–316, 2011.

14 R. J. Lipton. The reachability problem requires exponential space. Technical Report 62, Yale University, 1976.

15 E. W. Mayr. An algorithm for the general Petri net reachability problem. In *Proceedings of STOC*, pages 238–246, 1981.

16 C. Rackoff. The covering and boundedness problems for vector addition systems. *Theoretical Computer Science*, 6(2):223–231, 1978.

17 K. Reinhardt. Reachability in Petri nets with inhibitor arcs. In *Proceedings of RP*, volume 223 of *ENTCS*, pages 239–264, 2008.

18 F. Rosa-Velardo and D. de Frutos-Escrig. Name creation vs. replication in Petri net systems. *Fundamenta Informaticae*, 88(3):329–356, 2008.

19 Ph. Schnoebelen. Lossy counter machines decidability cheat sheet. In *Proceedings of RP*, volume 6227 of *LNCS*, pages 51–75, 2010.

20 Ph. Schnoebelen. Revisiting Ackermann-hardness for lossy counter machines and reset Petri nets. In *Proceedings of MFCS*, volume 6281 of *LNCS*, pages 616–628, 2010.

21 R. Valk. Self-modifying nets, a natural extension of Petri nets. In *Proceedings of ICALP*, volume 62 of *LNCS*, pages 464–476, 1978.