Report from Dagstuhl Seminar 13101

Computational Geometry

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 13101 "Computational Geometry". The seminar was held from 3rd to 8th March 2013 and 47 senior and young researchers from various countries and continents attended it. Recent developments in the field were presented and new challenges in computational geometry were identified.

This report collects abstracts of the talks and a list of open problems.

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1 Executive Summary

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Computational Geometry and its Evolution

The field of computational geometry is concerned with the design, analysis, and implementation of algorithms for geometric and topological problems, which arise in a wide range of areas, including computer graphics, CAD, robotics computer vision, image processing, spatial databases, GIS, molecular biology, and sensor networks. Since the mid 1980s, computational geometry has arisen as an independent field, with its own international conferences and journals.

In the early years mostly theoretical foundations of geometric algorithms were laid and fundamental research remains an important issue in the field. Meanwhile, as the field matured, researchers have started paying close attention to applications and implementations of geometric and topological algorithms. Several software libraries for geometric computation (e.g. LEDA, CGAL, CORE) have been developed. Remarkably, this emphasis on applications and implementations has emerged from the originally theoretically oriented computational geometry community itself, so many researchers are concerned now with theoretical foundations as well as implementations.

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Seminar Topics

The seminar presented recent developments in the field and identified new challenges for computational geometry. Below we list some of the most interesting subareas of the field at this stage, covering both theoretical and practical issues in computational geometry.

- *Theoretical foundations* of computational geometry lie in combinatorial geometry and its algorithmic aspects. They are of an enduring relevance for the field, particularly the design and the analysis of efficient algorithms require deep theoretical insights.
- *Geometric Computing* has become an integral part of the research in computational geometry. Besides general software design questions, especially *robustness* of geometric algorithms is important. Several methods have been suggested and investigated to make geometric algorithms numerically robust while keeping them efficient, which lead to interaction with the field of computer algebra, numerical analysis, and topology.
- *Computational topology* concentrates on the properties of geometric objects that go beyond metric representation: modeling and reconstruction of surfaces, shape similarity and classification, and persistence are key concepts with applications in molecular biology, computer vision, and geometric databases.
- In its early years, computational geometry concentrated on low dimensions. *High-dimensional data* has become very important recently, in particular, in work related to machine learning and data analysis. Standard solutions suffer from the curse of dimensionality. This has led to extensive work on dimension-reduction and embedding techniques.
- Various *applications* such as robotics, GIS, or CAD lead to interesting variants of the *classical topics* originally investigated, including convex hulls, Voronoi diagrams and Delaunay triangulations, and geometric data structures. For example, Voronoi diagrams and nearest-neighbor data structures under various metrics have turned out to be useful for many applications and are being investigated intensively.
- *Massive geometric data* sets are being generated by networks of sensors at unprecedented spatial and temporal scale. How to store, analyze, query, and visualize them has raised several algorithmic challenges. New computational models have been proposed to meet these challenges, e.g., streaming model, communication-efficient algorithms, and maintaining geometric summaries.

Participants

47 researchers from various countries and continents attended the seminar, showing the strong interest of the community for this event. The feedback from participants was very positive.

Dagstuhl seminars on computational geometry have been organized in a two year rhythm since a start in 1990. They have been extremely successful both in disseminating the knowledge and identifying new research thrusts. Many major results in computational geometry were first presented in Dagstuhl seminars, and interactions among the participants at these seminars have led to numerous new results in the field. These seminars have also played an important role in bringing researchers together, fostering collaboration, and exposing young talent to the seniors of the field. They have arguably been the most influential meetings in the field of computational geometry.

No other meeting in our field allows young researchers to meet with, get to know, and work with well-known and senior scholars to the extent possible at Dagstuhl. To accommodate new, younger researchers, the organizers held a *lottery* for the first time this year. From an initial list of selected researchers, we randomly selected a certain number of senior, young, and female participants. Researchers on the initial list who were not selected by the lottery were notified by us separately per email, so that they knew that they were not forgotten, and to reassure them that—with better luck—they will have another chance in future seminars.

We believe that the lottery created space to invite younger researchers, rejuvenating the seminar, while keeping a large group of senior and well-known scholars involved. The seminar was much “younger” than in the past, and certainly more “family-friendly.” Five young children roaming the premises created an even cosier atmosphere than we are used in Dagstuhl. Without decreasing the quality of the seminar, we had a more balanced attendance than in the past. Feedback from both seminar participants and from researchers who were not selected was uniformly positive.

Dagstuhl itself is a great strength of the seminar. Dagstuhl allows people to really meet and socialize, providing them with a wonderful atmosphere of a unique closed and pleasant environment, which is highly beneficial to interactions. Therefore, we warmly thank the scientific, administrative and technical staff at Schloss Dagstuhl!

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3 Overview of Talks

3.1 Union of Random Minkowski Sums and Network Vulnerability Analysis

Pankaj Kumar Agarwal (Duke University – Durham, US)

Let $C = \{C_1, \ldots, C_n\}$ be a set of n pairwise-disjoint convex s-gons, for some constant s, and let π be a probability density function (pdf) over the non-negative reals. For each i, let K_i be the Minkowski sum of C_i with a disk of radius r_i, where each r_i is a random non-negative number drawn independently from the distribution determined by π. We show that the expected complexity of the union of $K_1, \ldots, K_n$ is $O(n \log n)$, for any pdf π; the constant of proportionality depends on s, but not on the pdf.

Next, we consider the following problem that arises in analyzing the vulnerability of a network under a physical attack. Let $G = (V, E)$ be a planar geometric graph where E is a set of n line segments with pairwise-disjoint relative interiors. Let $\phi : \Re_{\geq 0} \to [0, 1]$ be an *edge failure probability function*, where a physical attack at a location $x \in \Re^2$ causes an edge e of E at distance r from x to fail with probability $\phi(r)$; we assume that ϕ is of the form $1 - \Pi(x)$, where Π is a cumulative distribution function on the non-negative reals. The goal is to compute the most *vulnerable* location for G, i.e., the location of the attack that maximizes the expected number of failing edges of G. Using our bound on the complexity of the union of random Minkowski sums, we present a near-linear Monte-Carlo algorithm for computing a location that is an approximately most vulnerable location of attack for G.

3.2 Fast Point Location for Easy Points

Boris Aronov (Polytechnic Inst. of NYU, US)

Joint work of Aronov, Boris; de Berg, Mark; Roeloffzen, Marcel; Speckmann, Bettina
Main reference B. Aronov, M. de Berg, M. Roeloffzen, B. Speckmann, "Distance-Sensitive Planar Point Location," *WADS 2013*, to appear.
URL http://www.wads.org

Let $\mathcal{S}$ be a connected planar polygonal subdivision with n edges and of total area 1. We present a data structure for point location in $\mathcal{S}$ where queries with points far away from any region boundary are answered faster. More precisely, we show that point location queries can be answered in time $O(1 + \min(\log \frac{1}{\Delta_p}, \log n))$, where Δ_p is the distance of the query point p to the boundary of the region containing p. Our structure is based on the following result: any simple polygon P can be decomposed into a linear number of convex quadrilaterals with the following property: for any point $p \in P$, the quadrilateral containing p has area $\Omega(\Delta_p^2)$.

3.3 Geometry-driven collapses for simplifying Cech complexes

Dominique Attali (GIPSA Lab – Saint Martin d'Hères, FR)

Joint work of Attali, Dominique; Lieutier, André
Main reference D. Attali, A. Lieutier, "Geometry driven collapses for converting a cech complex into a triangulation of a nicely triangulable shape," arXiv:1304.3680v1 [cs.CG], 2013.
URL http://arxiv.org/abs/1304.3680v1

In many practical situations, the object of study is only known through a finite set of possibly noisy sample points. It is then desirable to try to recover the geometry and the topology of the object from this information.

In this talk, we will focus on an approach that approximates a shape from a set of sample points by returning the Rips complex of the points. Given a point set P and a scale parameter r, the Rips complex is the simplicial complex whose simplices are subsets of points in P with diameter at most 2r. Rips complexes have generally a size and dimension much too large to allow an explicit representation. Nonetheless, Rips complexes enjoy the property to be completely determined by the graph of theirs vertices and edges which thus provide a compressed form of storage (quadratic in the number of data points and linear in the ambient dimension). This suggests to reconstruct a shape by first building the Rips complex of the data points at some scale (encoded with its vertices and edges) and second by simplifying the result through a sequence of elementary operations. In previous work, we formulated conditions under which the Rips complex of the point set at some scale reflects the homotopy type of the shape [1, 2]. In this talk, we give conditions under which the complex can be transformed by a sequence of collapses into a triangulation of the shape [3].

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2 D. Attali, A. Lieutier, and D. Salinas. Vietoris-Rips complexes also provide topologically correct reconstructions of sampled shapes. *Computational Geometry: Theory and Applications (CGTA)*, 46:448–465, 2012.
3 D. Attali and A. Lieutier. Geometry driven collapses for converting a cech complex into a triangulation of a nicely triangulable shape. *arXiv preprint arXiv:1304.3680*, 2013.

3.4 Exact Symbolic-Numeric Computation of Planar Algebraic Curves

Eric Berberich (MPI für Informatik – Saarbrücken, DE)

Joint work of Berberich, Eric; Emeliyanenko, Pavel; Kobel, Alexander; Sagraloff, Michael
Main reference Eric Berberich, Pavel Emeliyanenko, Alexander Kobel, Michael Sagraloff, "Exact Symbolic-Numeric Computation of Planar Algebraic Curves," arXiv:1201.1548v1 [cs.CG], 2012.
URL http://arxiv.org/abs/1201.1548v1

We present a certified and complete algorithm to compute arrangements of real planar algebraic curves. It computes the decomposition of the plane induced by a finite number of algebraic curves in terms of a cylindrical algebraic decomposition. From a high-level perspective, the overall method splits into two main subroutines, namely an algorithm denoted Bisolve to isolate the real solutions of a zero-dimensional bivariate system, and an algorithm denoted GeoTop to compute the topology of a single algebraic curve. Compared to existing approaches based on elimination techniques, we considerably improve the corresponding lifting steps in both subroutines. As a result, generic position of the input system is never assumed, and thus our algorithm never demands for any change of coordinates. In addition, we significantly limit the types of symbolic operations involved, that is, we only use resultant and gcd computations as purely symbolic operations. The latter results are achieved by combining techniques from different fields such as (modular) symbolic computation, numerical analysis and algebraic geometry. We have implemented our algorithms as prototypical contributions to the C++-project Cgal. We exploit graphics hardware to expedite the remaining symbolic computations. We have also compared our implementation with the current reference implementations, that is, Lgp and Maple's Isolate for polynomial system solving, and Cgal's bivariate algebraic kernel for analyses and arrangement computations of algebraic curves. For various series of challenging instances, our exhaustive experiments show that the new implementations outperform the existing ones.

3.5 Four Soviets Walk the Dog – with an Application to Alt's Conjecture

Kevin Buchin (TU Eindhoven, NL)

Joint work of Buchin, Kevin; Buchin, Maike; Meulemans, Wouter; Mulzer, Wolfgang
Main reference K. Buchin, M. Buchin, W. Meulemans, W. Mulzer, "Four Soviets Walk the Dog-with an Application to Alt's Conjecture," arXiv:1209.4403v2 [cs.CG], 2012.
URL http://arxiv.org/abs/1209.4403v2

Given two polygonal curves in the plane, there are several ways to define a measure of similarity between them. One measure that has been extremely popular in the past is the Fréchet distance. Since it has been proposed by Alt and Godau in 1992, many variants and extensions have been described. However, even 20 years later, the original $O(n^2 \log n)$ algorithm by Alt and Godau for computing the Fréchet distance remains the state of the art (here n denotes the number of vertices on each curve). This has led Helmut Alt to conjecture that the associated decision problem is 3SUM-hard. In recent work, Agarwal et al. show how to break the quadratic barrier for the discrete version of the Fréchet distance, where we consider sequences of points instead of polygonal curves. Building on their work, we give an algorithm to compute the Fréchet distance between two polygonal curves in time $O(n^2(\log n)^{(1/2)}(\log\log n)^{(3/2)})$ on a pointer machine and in time $O(n^2(\log\log n)^2)$ on a word RAM. Furthermore, we show that there exists an algebraic decision tree for the Fréchet problem of depth $O(n^{(2-\epsilon)})$, for some $\epsilon > 0$. This provides evidence that computing the Fréchet distance may not be 3SUM-hard after all and reveals an intriguing new aspect of this well-studied problem.

3.6 Trajectory Grouping Structures

Maike Buchin (TU Eindhoven, NL)

Joint work of Buchin, Kevin; Buchin, Maike; van Kreveld, Marc; Speckmann, Bettina; Staals, Frank
Main reference K. Buchin, M. Buchin, M. van Kreveld, B. Speckmann, F. Staals, "Trajectory Grouping Structures," arXiv:1303.6127v1 [cs.CG], 2013.
URL http://arxiv.org/abs/1303.6127v1

The collective motion of a set of moving entities like people, birds, or other animals, is characterized by groups arising, merging, splitting, and ending. Given the trajectories of these entities, we define and model a structure that captures all of such changes using the Reeb graph, a concept from topology. The trajectory grouping structure has three natural parameters that allow more global views of the data in group size, group duration, and entity inter-distance. We prove complexity bounds on the maximum number of maximal groups that can be present, and give algorithms to compute the grouping structure efficiently. We also study how the trajectory grouping structure can be made robust, that is, how brief interruptions of groups can be disregarded in the global structure, adding a notion of persistence to the structure. Furthermore, we showcase the results of experiments using data generated by the NetLogo flocking model and from the Starkey project. The Starkey data describe the movement of elk, deer, and cattle. Although there is no ground truth for the grouping structure in this data, the experiments show that the trajectory grouping structure is plausible and has the desired effects when changing the essential parameters. Our research provides the first complete study of trajectory group evolvement, including combinatorial, algorithmic, and experimental results.

3.7 Approximate Shortest Descending Paths

Siu-Wing Cheng (HKUST – Kowloon, HK)

Joint work of Cheng, Siu-Wing; Jin, Jiongxin
Main reference S.-W. Cheng, J. Jin, "Approximate Shortest Descending Paths," in Proc. of the 24th Annual ACM-SIAM Symp. on Discrete Algorithms (SODA'13), pp. 144–155, SIAM, 2013.
URL http://knowledgecenter.siam.org/0236-000023/

We present an approximate algorithm for the shortest descending path problem. Given a source s and a destination t on a terrain, a shortest descending path from s to t is a path of minimum Euclidean length on the terrain subject to the constraint that the height decreases monotonically as we traverse that path from s to t. Given any $\epsilon \in (0, 1)$, our algorithm returns in $O(n^4 \log(n/\epsilon))$ time a descending path of length at most $1 + \epsilon$ times the optimum. This is the first algorithm whose running time is polynomial in n and $\log(1/\epsilon)$ and independent of the terrain geometry.

3.8 Graph Induced Complex on Point Data

Tamal K. Dey (Ohio State University, US)

Joint work of Dey, Tamal K.; Fan, Fengtao; Wang, Yusu
Main reference T.K. Dey, F. Fan, Y. Wang, "Graph Induced Complex on Point Data," arXiv:1304.0662v1 [cs.CG]; to appear in Proc. of the 29th Annual Symp. on Computational Geometry 2013.
URL http://arxiv.org/abs/1304.0662v1

The efficiency of extracting topological information from point data depends largely on the complex that is built on top of the data points. From a computational viewpoint, the most favored complexes for this purpose have so far been Vietoris-Rips and witness complexes. While the Vietoris-Rips complex is simple to compute and is a good vehicle for extracting topology of sampled spaces, its size is huge–particularly in high dimensions. The witness complex on the other hand enjoys a smaller size because of a subsampling, but fails to capture the topology in high dimensions unless imposed with extra structures. We investigate a complex called the *graph induced complex* that, to some extent, enjoys the advantages of both. It works on a subsample but still retains the power of capturing the topology as the Vietoris-Rips complex. It only needs a graph connecting the original sample points from which it builds a complex on the subsample thus taming the size considerably. We show that, using the graph induced complex one can (i) infer the one dimensional homology of a manifold from a very lean subsample, (ii) reconstruct a surface in three dimension from a sparse subsample without computing Delaunay triangulations, (iii) infer the persistent homology groups of compact sets from a sufficiently dense sample. We provide experimental evidences in support of our theory.

3.9 Geometric Input Models

Anne Driemel (Utrecht University, NL)

Joint work of Driemel, Anne; Har-Peled, Sariel; Wenk, Carola; Raichel, Benjamin

The worst-case analysis of the running time and space complexities as a function of the input size is a fundamental method in algorithm design. However, it fails to describe the actual behavior when the worst case is a contrived geometric configuration which would never occur in practice. There are different approaches to reasoning about algorithms and data structures for real data that allow a theoretical analysis with provable bounds. I will outline some techniques we used and the results that we achieved. In particular, I will talk about two results (i) an approximation algorithm for the Fréchet distance [1] and (ii) bounding the complexity of Voronoi diagrams on terrains [2]. This research was carried out as a part of my PhD studies and is joint work with Carola Wenk (Tulane University), Sariel Har-Peled (UIUC) and Benjamin Raichel (UIUC).

References

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2. Anne Driemel, Sariel Har-Peled, and Benjamin Raichel. On the expected complexity of Voronoi diagrams on terrains. In *Proc. SoCG 2012*, pages 101–110, 2012.

3.10 Efficiently hex-meshing things with topology

Jeff Erickson (University of Illinois – Urbana, US)

Main reference J. Erickson, "Efficiently hex-meshing things with topology," to appear in Proc. of the 28th Annual Symp. on Computational Geometry 2013.
URL http://www.cs.uiuc.edu/~jeffe/pubs/hexmesh.html

A topological quadrilateral mesh Q of a connected surface in $\mathbb{R}^3$ can be extended to a topological hexahedral mesh of the interior domain Ω if and only if Q has an even number of quadrilaterals and no odd cycle in Q bounds a surface inside Ω. Moreover, if such a mesh exists, the required number of hexahedra is within a constant factor of the minimum number of tetrahedra in a triangulation of Ω that respects Q. Finally, if Q is given as a polyhedron in $\mathbb{R}^3$ with quadrilateral facets, a topological hexahedral mesh of the polyhedron can be constructed in polynomial time if such a mesh exists. All our results extend to domains with disconnected boundaries. Our results naturally generalize results of Thurston, Mitchell, and Eppstein for genus-zero and bipartite meshes, for which the odd-cycle criterion is trivial.

3.11 Theory Meets Practice: Two Videos

Sándor Fekete (TU Braunschweig, DE)

Joint work of Fekete, Sándor; Friedrichs, Stephan; Kröller, Alexander; Schmidt, Christiane; Borrmann, Dorit; de Rezende, Pedro J.; de Souza, Cid C.; Tozoni, Davi C.; Becker, Aaron; Lee, SengKyou; McLurkin, James

One of the driving engines of Computational Geometry is the interaction with practical problems; one of the application areas with strong ties to geometry is the filed of robotics. In this talk, I present two videos that document ongoing collaborations with colleagues from robotics.

The first [1] considers exploration and triangulation with a swarm of small robots with relatively few individual capabilities; we develop ideas, provide theory and present a practical demonstration of how such a swarm can be used to explore an unknown territory, and guard it. This is joint work with colleagues from Rice University (USA).

The second [2] shows how building detailed three-dimensional maps with a robot platform that carries a powerful laserscanner is related to the classical Art Gallery Problem (AGP). We develop different methods for solving such problems to optimality, and demonstrate the resulting application. This is joint work with colleagues from the University of Campinas (Brazil) and Jacobs University Bremen (Germany).

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1 A. Becker, S.P. Fekete, A. Kröller, L.S. Kyou, J. McLurkin and C. Schmidt. Triangulating Unknown Environments Using Robot Swarms, Video and abstract. To appear in: *Proc. SoCG 2013*.

2 D. Borrmann, P.J. de Rezende, C.C. de Souza, S.P. Fekete, S. Friedrichs, A. Kröller, A. Nüchter, C. Schmidt and D.C. Tozoni. Point Guards and Point Clouds: Solving General Art Gallery Problems, Video and abstract. To appear in: *Proc. SoCG 2013*

3.12 Random hypergraphs and small silhouettes

Marc Glisse (INRIA Saclay – Île-de-France – Orsay, FR)

Joint work of Devillers, Olivier; Glisse, Marc; Goaoc, Xavier; Lazard, Sylvain; Michel, Julien; Pouget, Marc

We present a new simple scheme for the analysis of random geometric structures, which we illustrate on convex hulls and Delaunay triangulations. We then introduce some refinements of the analysis which tighten the bounds and give large-deviation-related results. Those refinements are finally used to deduce a worst-case bound on the size of the silhouettes of a random polytope.

3.13 Simplifying inclusion-exclusion formulas

Xavier Goaoc (INRIA Lorraine, FR)

Joint work of Goaoc, Xavier; Matoušek, Jiří; Paták, Pavel; Safernová, Zuzana; Tancer, Martin
Main reference X. Goaoc, J. Matoušek, P. Paták, Z. Safernová, M. Tancer, "Simplifying inclusion-exclusion formulas," arXiv:1207.2591v1 [math.CO] .
URL http://arxiv.org/abs/1207.2591v1

Let $F = \{F_1, F_2, \ldots, F_n\}$ be a family of n sets on a ground set X, such as a family of balls in $\mathbb{R}^d$. For every finite measure μ on X, such that the sets of F are measurable, the classical *inclusion-exclusion formula* asserts that $\mu(F_1 \cup F_2 \cup \cdots \cup F_n) = \sum_{I:\emptyset \neq I \subseteq [n]} (-1)^{|I|+1} \mu\left(\bigcap_{i \in I} F_i\right)$; that is, the measure of the union is expressed using measures of various intersections. The number of terms in this formula is exponential in n, and a significant amount of research, originating in applied areas, has been devoted to constructing simpler formulas for particular families F. We provide an upper bound valid for an arbitrary F: we show that every system F of n sets with m nonempty fields in the Venn diagram admits an inclusion-exclusion formula with $m^{O(\log^2 n)}$ terms and with ± 1 coefficients, and that such a formula can be computed in $m^{O(\log^2 n)}$ expected time. We also construct systems of n sets on n points for which every valid inclusion-exclusion formula has the sum of absolute values of the coefficients at least $\Omega(n^{3/2})$.

3.14 Fréchet Queries in Geometric Trees

Joachim Gudmundsson (University of Sydney, AU)

Joint work of Gudmundsson, Joachim; Smid, Michiel

Let T be a tree that is embedded in the plane and let $\Delta > 0$ be a real number. The aim is to preprocess T into a data structure, such that, for any query polygonal path Q, we can decide if T contains a path P whose Fréchet distance $\delta_F(P, Q)$ to Q is less than Δ. We present an efficient data structure that solves an approximate version of this problem, for the case when T is c-packed and each of the edges of T and Q has length $\Omega(\Delta)$ (not required if T is a path): If the data structure returns NO, then there is no such path P. If it returns YES, then $\delta_F(P, Q) \leq \sqrt{2}(1 + \epsilon)\Delta$ if Q is a line segment, and $\delta_F(P, Q) \leq 3(1 + \epsilon)\Delta$ otherwise.

3.15 Geometric properties of space-filling curves: some results and open problems

Herman J. Haverkort (TU Eindhoven, NL)

Main reference H. Haverkort, "Recursive tilings and space-filling curves with little fragmentation," Journal of Computational Geometry, 2(1):92–127, 2011.
URL http://www.jocg.org/index.php/jocg/article/view/68

A space-filling curve is a continuous surjective function f that maps the unit interval [0,1] to a higher-dimensional region, such as the unit square. Such a curve is usually defined on the basis of a recursive tiling, such that the curve traverses the tiles of each level of the tiling one by one, and the curve can be parameterized such that the union of all points f(t) over all t in [a,b] is a region of measure exactly b-a. In this presentation I focus on two open problems about space-filling curves.

1. The Arrwwid number of a three-dimensional space-filling curve is the smallest number A, such that any ball with volume B can be covered by A pieces of the curve of total size O(B). The three-dimensional curve with the lowest known Arrwwid number has Arrwwid number 4. We can prove that this is a lower bound for any three-dimensional curve that traverses the tiles of a recursive tiling with convex tiles one by one. Can we also prove this lower bound for curves that are not based on convex tiles? We would be able to prove this if we could prove a certain relation between the number of vertices, the number of tiles and the number of vertex-tile incidences that holds for any "reasonable" tiling in three dimensions.

2. The dilation of a two-dimensional space-filling curve is the maximum, over all a,b in [0,1], of the squared distance between f(a) and f(b), divided by (b-a). We can prove that each two-dimensional space-filling curve must have dilation at least 4/pi. The curve with the lowest known dilation has dilation 4. Can we improve the lower bound—or can we find a space-filling curve with dilation less than 4?

3.16 Qualitative symbolic perturbations

Menelaos Karavelas (University of Crete – Heraklion, GR)

Joint work of Devillers, Olivier; Karavelas, Menelaos; Teillaud, Monique
Main reference O. Devillers, M. Karavelas, M. Teillaud, "Qualitative Symbolic Perturbation: a new geometry-based perturbation framework," HAL, RR-8153, 2012.
URL http://hal.inria.fr/hal-00758631/

In classical *Symbolic Perturbations*, degeneracies are resolved by using a sequence of predicates obtained by algebraic substitution of polynomials in ε to the input. Instead of a single perturbation, we propose to use a sequence of (simpler) perturbations and to look at their effect geometrically instead of algebraically. We obtain solutions for Apollonius predicates which were not solvable using the algebraic approach.

3.17 Bottleneck Non-Crossing Matching in the Plane

Matthew J. Katz (Ben Gurion University – Beer Sheva, IL)

Joint work of Abu-Affash, A. Karim; Carmi, Paz; Katz, Matthew J.; Trabelsi Yohai
Main reference A. Karim Abu-Affash, P. Carmi, M.J. Katz, Y. Trabelsi, "Bottleneck Non-Crossing Matching in the Plane," in Proc. of the 20th Annual European Symp. on Algorithms (ESA'12), LNCS, Vol. 7501, pp. 36–47, Springer, 2012.
URL http://dx.doi.org/10.1007/978-3-642-33090-2_5

Let P be a set of $2n$ points in the plane, and let M_{C} (resp., M_{NC}) denote a bottleneck matching (resp., a bottleneck non-crossing matching) of P. We study the problem of computing M_{NC}. We first prove that the problem is NP-hard and does not admit a PTAS. Then, we present an $O(n^{1.5}\log^{0.5} n)$-time algorithm that computes a non- crossing matching M of P, such that $bn(M) \leq 2\sqrt{10} \cdot bn(M_{\mathrm{NC}})$, where $bn(M)$ is the length of a longest edge in M. An interesting implication of our construction is that $bn(M_{\mathrm{NC}})/bn(M_{\mathrm{C}}) \leq 2\sqrt{10}$.

3.18 On the Complexity of Higher Order Abstract Voronoi Diagrams

Rolf Klein (Universität Bonn, DE)

Joint work of Bohler, Cecilia; Cheilaris, Panagiotis; Klein, Rolf; Liu, Chih-Hung; Papadopoulou, Evanthia; Zavershynskyi, Maksymolf
Main reference C. Bohler, P. Cheilaris, R. Klein, C. Liu, E. Papadopoulou, M. Zavershynskyi, "On the Complexity of Higher Order Abstract Voronoi Diagrams," to appear in the Proc. of the 40th Int'l Colloquium on Automata, Languages and Programming (ICALP '13), Riga, 2013.

Abstract Voronoi diagrams are based on bisecting curves enjoying simple combinatorial properties, rather than on the geometric notions of sites and circles. They serve as a unifying concept. Once the bisector system of any concrete type of Voronoi diagram is shown to fulfill the AVD properties, structural results and efficient algorithms become available without further effort. For example, the first optimal algorithms for constructing nearest Voronoi diagrams of disjoint convex objects, or of line segments under the Hausdorff metric, have been obtained this way.

In a concrete order-k Voronoi diagram, all points are placed into the same region that have the same k nearest neighbors among the given sites. This paper is the first to study abstract Voronoi diagrams of arbitrary order k. We prove that their complexity is upper bounded by $2k(n-k)$. So far, an $O(k(n-k))$ bound has been shown only for point sites in the Euclidean and L_p plane, and, very recently, for line segments. These proofs made extensive use of the geometry of the sites.

Our result on AVDs implies a $2k(n-k)$ upper bound for a wide range of cases for which only trivial upper complexity bounds were previously known, and a slightly sharper bound for the known cases.

3.19 On numerical algorithms for the topology of curves with simple singularities

Guillaume Moroz (INRIA Grand Est – Nancy, FR)

Joint work of Moroz, Guillaume; Pouget, Marc

Let C be the planar curve defined by a polynomial equation f (x, y) = 0. If C is smooth, its topology can be computed with adaptive numerical algorithms. Otherwise, computing the topology requires a different set of tools that induce a significant gap between the analysis of smooth curves and singular curves, even with simple multiplicity structure. Such tools include the computation of a resultant, the subdivision until a global separation. We present works in progress to fill this gap for singular curves with simple self-intersections.

3.20 Output-Sensitive Well-Separated Pair Decompositions for Dynamic Point Sets

David M. Mount (University of Maryland – College Park, US)

Joint work of Park Eunhui; Mount, David M.
Main reference (Unpublished manuscript, submitted for publication)

The well-separated pair decomposition (WSPD) is a fundamental structure in computational geometry. Given a set of n points in d-dimensional space and a positive parameter s, it is known that there exists an s-WSPD of size $O(s^d n)$. While this is linear in n, the factor of s^d is a significant consideration when the dimension d is even a moderately large constant. The actual number of pairs may be much smaller than this worst-case bound, for example, if the points are clustered near a lower dimensional subspace. Batch WSPD constructions are output sensitive, but existing algorithms for maintaining the WSPD of a dynamic point set are not. In this paper we present output-sensitive algorithms for maintaining the WSPD of a dynamic point set under insertion and deletion.

3.21 Improved Approximation for Geometric Unique Coverage Problems

Yoshio Okamoto (University of Electro-Communications, Tokyo, JP)

Joint work of Ito, Takehiro; Nakano, Shin-ichi; Okamoto, Yoshio; Otachi, Yota; Uehara, Ryuhei; Uno, Takeaki; Uno, Yushi
Main reference T. Ito, S.-I. Nakano, Y. Okamoto, Y. Otachi, R. Uehara, T. Uno, Y. Uno, "A 4.31-Approximation for the Geometric Unique Coverage Problem on Unit Disks," in Proc. of the 23rd Int'l Symp. on Algorithms and Computation (ISAAC'12), LNCS, Vol. 7676, pp. 372–381, Springer, 2012.
URL http://dx.doi.org/10.1007/978-3-642-35261-4_40
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URL http://dx.doi.org/10.1007/978-3-642-31155-0_3

Given a set of points and a set of objects, both in the plane, we wish to find a subset of the objects that maximizes the number of points contained in exactly one object in the subset. Erlebach and van Leeuwen [1] introduced this problem as the geometric version of the unique coverage problem, and gave polynomial-time approximation algorithms. Their approximation ratios were 18 when the objects were unit disks, and 4 when the objects were axis-parallel unit squares (which was later improved to 2 by van Leeuwen [2]). We improve the approximation ratios to 4.31 for unit disks and $1 + \varepsilon$ for axis-parallel unit squares.

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3.22 Kinetic data structures in the black-box model

Marcel J. M. Roeloffzen (TU Eindhoven, NL)

Joint work of Berg, Mark de; Roeloffzen, Marcel; Speckmann, Bettina

Over the past decade, the kinetic-data-structures framework has become the standard in computational geometry for dealing with moving objects. A fundamental assumption underlying the framework is that the motions of the objects are known in advance. This assumption severely limits the applicability of KDSs. We study KDSs in the black-box model, which is a hybrid of the KDS model and the traditional time-slicing approach. In this more practical model we receive the position of each object at regular time steps and we have an upper bound on $d_{\max}$, the maximum displacement of any point in one time step.

In this talk we describe the black-box model and give an overview of the results obtained for maintaining the convex hull, Delaunay triangulation and compressed quadtree of a set of points in the black-box model. We also go into some more detail on the latest result on maintaining the Euclidean 2-center.

3.23 α-Visibility

Jörg-Rüdiger Sack (Carleton University – Ottawa, CA)

Joint work of Ghodsi, Mohammad; Maheshwari, Anil; Nouri, Mostafa; Sack, Jörg-Rüdiger; Zarrabi-Zadeh, Hamid
Main reference M. Ghodsi, A. Maheshwari, M. Nouri, J.-R. Sack, H. Zarrabi-Zadeh, "α-Visibility," in Proc. of the 13th Scandinavian Symposium and Workshops on Algorithm Theory (SWAT'2012), LNCS, Vol. 7357, pp. 1–12, Springer, 2012.
URL http://dx.doi.org/10.1007/978-3-642-31155-0_1

We study a new class of visibility problems based on the notion of α-visibility. Given an angle α and a collection of line segments S in the plane, a segment t is said to be α-visible from a point p, if there exists an empty triangle with one vertex at p and the side opposite to p on t such that the angle at p is α. In this model of visibility, we study the classical variants of point visibility, weak and complete segment visibility, and the construction of the visibility graph. We also investigate the natural query versions of these problems, when α is either fixed or specified at query time.

3.24 Parallel computation of the Hausdorff distance between shapes

Ludmila Scharf (FU Berlin, DE)

Joint work of Alt, Helmut; Scharf, Ludmila

We show that the Hausdorff distance for two sets of n non-intersecting line segments can be computed in parallel in $O(\log^2 n)$ time using $O(n)$ processors in a CREW-PRAM computation model. We discuss how some parts of the sequential algorithm can be performed in parallel using previously known parallel algorithms; and identify the so-far least efficiently solved part of the problem for the parallel computation, which is the following: Given two sets of x-monotone curve segments, red and blue, for each red segment find its extremal intersection points with the blue set, i.e. points with the minimal and maximal x-coordinate. Each segment set is assumed to be intersection free. The best known parallel algorithm for this problem has total work of $O(n \log^3 n)$ and uses $O(n \log^2 n)$ space. The algorithm presented here improves the theoretical time and space performance while still being practically feasible.

3.25 Convex Transversals

Lena Schlipf (FU Berlin, DE)

The talk gives an overview of the work on convex transversals. The question about convex transversals was initially posed by Arik Tamir at the Fourth NYU Computational Geometry Day (1987): "Given a collection of compact sets, can one decide in polynomial time whether there exists a convex body whose boundary intersects every set in the collection?" So far, there have been very few results. One of these rare results is an $O(n \log n)$ algorithm by Goodrich and Snoeyink [1] that solves this problem when the sets are n parallel line segments. We show that when the sets are segments in the plane, deciding existence of the convex stabber is NP-hard (this is joint work with Arkin, Dieckmann, Knauer, Mitchell, Polishchuk, Yang [2]). The problem remains NP-hard when the sets are simple regular polygons. Additionally, we prove the problem to be NP-hard when the sets are disjoint bends in the plane.

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3.26 Delaunay and other triangulations of moving point sets: What's going on?

Micha Sharir (Tel Aviv University, IL)

Joint work of Sharir, Micha; Agarwal, Pankaj; Kaplan, Haim; Rubin, Natan; and several others

In this talk we review several recent works addressing the following problem: Given a set P of n moving points in the plane, where the motion of each point is semialgebraic of constant description complexity, we want to maintain the Delaunay (or some other) triangulation of P kinetically, updating it after each discrete change that it experiences.

The Delaunay triangulation, DT(P), besides its many useful properties, is ideal for such a maintenance, because it admits local certification, requiring the circumdisk of each triangle to be empty. The main problem is to show that the number of discrete changes in DT(P) is "small", meaning nearly quadratic in n. (A quadratic lower bound is known.) This is still open, and is considered one of the hardest open problems in combinatorial and computational geometry.

We review several recent attempts to address this issue:

(1) Developing other triangulation schemes, with a provably near-quadratic number of changes. This has been done by Agarwal, Wang and Yu, and later by Kaplan, Rubin and Sharir.

(2) Maintaining the Delaunay triangulation of P under a polygonal, non-Euclidean norm. Here too one can show that the diagram experiences only a near-quadratic number of changes, and can be maintained efficiently, but it has several drawbacks. This goes back to Chew, and has been treated in a more general and complete manner by Agarwal, Kaplan, Rubin, and Sharir (work in progress).

(3) Maintaining only a "stable" portion of the Delaunay diagram, roughly corresponding to edges whose dual Voronoi edges are seen from their sites at a sufficiently large angle. Again, a near-quadratic bound on the number of changes can be established, and the stable portion has several drawbacks. This is work in progress by Agarwal and many other authors, originally presented at SoCG many years ago.

(4) Most importantly, we review a recent work by Rubin, where he manages to establish a near-quadratic bound for the Euclidean Delaunay triangulation for points moving at unit speeds. The analysis is quite involved, and we review some of its main technical ingredients.

3.27 Towards Elastic Shape Matching

Fabian Stehn (Universität Bayreuth, DE)

Joint work of Knauer, Christian; Stehn, Fabian

Geometric shape matching problems are one of the core research topics in the field of computational geometry. The general question of a geometric matching problem is as follows: given are two geometric objects – a pattern and a model – and a transformation class as well as a similarity measure. One seeks a transformation t of the given class such that the similarity measure of the pattern transformed by t to the model is maximized.

We introduced the concept of *elastic* (non-uniform) geometric shape matching problems. In an elastic geometric shape matching problem the pattern is not transformed by a single transformation, but by a so-called *transformation ensemble*. Transformation ensembles allow non-uniform deformations of the pattern – different parts of the pattern can be transformed by different mappings. Another benefit of transformation ensembles is the possibility to incorporate temporal dependencies of the pattern and changes of its shape over time in this modeling. This allows to compute registrations that are valid within a certain time frame even if the reference objects change during this time period. This is achieved by linking together transformation ensembles at different points in time and by applying suitable temporal and spacial interpolation methods.

The modeling as an elastic geometric shape matching problem has various benefits from a theoretical as well as practical point of view. *Classical* geometric shape matching problems form a special case in this modeling and hence allow a direct comparison to results of elastic geometric shape matching problems. On the other hand, many practical applications (such as navigated surgeries for example) will benefit from algorithms and data structures that compute transformation ensembles: in these applications one often has to deal with local deformations of the input (e.g. due to distortion caused by magnetic fields), as well as entities that vary of time (for example soft tissue deformation in the aforementioned context of navigated surgeries).In this talk we introduced first results on elastic shape matching problems for point sequences under translations.

3.28 Extended Formulations for polytopes

Hans Raj Tiwary (University of Brussels, BE)

Joint work of Tiwary, Hans Raj; Avis, David
Main reference D. Avis, H. Raj Tiwary, "On the extension complexity of combinatorial polytopes," arXiv:1302.2340v2 [math.CO]; accepted in ICALP 2013.
URL http://arxiv.org/abs/1302.2340v2

A polytope Q is said to be an extended formulation (EF) for a polytope P, iff P is the projection of Q. The notion of extended formulations are not only important in many areas of applied sciences but also interesting from a theoretical perspective. In a certain sense a compact EF encodes "faithful" linear programs for solving optimization problems. In this talk, I will discuss some basics, some recent new results, and a purposely vague open problem related to the existence of compact EFs.

3.29 Reverse stabbing queries in galleries

Suresh Venkatasubramanian (University of Utah, US)

Joint work of Daruki, Samira; Hillyard, Peter; Patwari, Neal; Venkatasubramanian, Suresh

Radio Tomographic Imaging (RTI) is an emerging technology that locates moving objects in areas surrounded by simple and inexpensive radios. RTI is useful in emergencies, rescue operations, and security breaches, since the objects being tracked need not carry an electronic device. Tracking humans moving through a building, for example, could help firefighters save lives by locating victims quickly. RTI works by placing small inexpensive radios in a region of interest. The radios can send and receive wireless signals, and form a network of links that cover the region of interest. When a person walks through the region, they interfere with the links, creating a "shadow" of broken links that can be used to infer presence and track individuals.

This yields the following problem: given a collection of radios and a set of "visible" links, infer the trajectory of a person moving through the region. This inference must be robust under link errors and occlusion, as well as be performed in real time. In addition, there is an associated planning problem of where to place the radios in order to make the tracking algorithm as effective as possible.

In this talk, I present geometric algorithms for these questions. The key technical developments include a generalization of stabbing line and transversal problems, as well as a novel generalization of traditional art gallery problems.

3.30 A Faster Algorithm for Computing Motorcycle Graphs

Antoine Vigneron (KAUST – Thuwal, SA)

Joint work of Vigneron, Antoine; Lie, Yan
Main reference To appear in the Proceedings of the Symp. on Computational Geometry 2013.

We present a new algorithm for computing motorcycle graphs. Its running time is $O(n^{4/3})$, where n is the size of the input. When the motorcycles start from the side of a simple polygon, and input coordinates are $O(\log n)$-bit rational numbers, the time bound improves to $O(n \log^3 n)$. It yields an $O(n \log^3 n)$ expected time algorithm for computing the straight skeleton of a simple polygon.

3.31 Towards Understanding Gaussian Weighted Graph Laplacian

Yusu Wang (Ohio State University, US)

The Gaussian-weighted graph Laplacian, as a special form of graph Laplacians with general weights, has been a popular empirical operator for data analysis applications, including semi-supervised learning, clustering, and denoising. There have been various studies of the properties and behaviors of this empirical operator; most notably, its convergence behavior as the number of points sampled from a hidden manifold goes to infinity.

In this talk we present two new results on the theoretical properties of the Gaussian-weighted Graph Laplacian. The first one [1] is about its behavior as the input points where we construct the graph are sampled from a, what we call, *singular manifold*; while previous theoretical study of the Gaussian-weighted Graph Laplacian typically assumes that the hidden domain is a compact smooth manifold. A singular manifold can consist of a collection of potentially intersecting manifolds with boundaries, and represents one step towards modeling more complex hidden domains.

The second result we present is about the stability of the Gaussian-weighted Graph Laplacian as the hidden manifold where input points are sampled from have certain small perturbation [2]. The goal is to understand how the spectrum of Gaussian-weighted Graph Laplacian changes with respect to perturbations of the domain.

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4 Open Problems

▶ Problem 1 (Otfried Cheong). 3-*dimensional Kakeya problem:* Find a smallest-volume three-dimensional convex body K such that, for any direction u, $\text{width}(K, u) \geq 1$.

In two dimensions, the optimal body is the equilateral triangle of height 1. In three dimensions, a regular tetrahedron where the distance between opposite edges is 1 is not optimal. One can "shave off" the corners in order to decrease the area while maintaining the width-condition.

▶ Problem 2 (Jeff Erickson). *Almost simple polygons:* A polygon P with vertices $p_1, p_2, \ldots, p_n \in \mathbb{R}^2$ is *almost simple* if, for any $\varepsilon > 0$, there is a simple polygon Q with vertices $q_1, q_2, \ldots, q_n$ such that $\|p_i - q_i\| < \varepsilon$ for each index i. Equivalently, an n-gon P is weakly simple if there are simple n-gons with arbitrarily small Fréchet distance to P.

Is there a polynomial-time algorithm to determine whether a given sequence of points is the vertex sequence of an almost-simple polygon?

There is an algorithm to decide whether a *spur-free* polygon is almost simple in $O(n \log n)$ time, where a *spur* is a vertex with a zero-degree angle, or equivalently, a pair of consecutive edges that overlap. A spur-free polygon is weakly simple if and only if it contains no crossing subwalks and its winding number is ± 1. (Two spur-free walks $a_0 b_1 \cdots b_k a_{k+1}$ and $c_0 b_1 \cdots b_k c_{k+1}$ *cross* if either the triples a_0, b_1, c_0 and a_{k+1}, b_k, c_{k+1} have the same orientation, or $k \leq 1$ and the walks intersect transversely.) However, this characterization does not extend to polygons with spurs, in part because the winding number is not well-defined.

▶ Problem 3 (Fabian Stehn). Let $S = \{s_1, \ldots, s_n\}$ be a set of segments in the plane. Compute n translations $t_1, \ldots, t_n$ such that the set $S' = \{s'_i \mid s'_i = s_i + t_i, i = 1, \ldots, n\}$ is disjoint and the convex hull of S' has minimum area.
Remark: Two segments of S' are allowed to have a common endpoint and the endpoint of a segment in S' is allowed to lie on another segment.

▶ Problem 4 (Suresh Venkatasubramanian). *MDS:* Given a distance matrix $[d_{ij}]$, where d_{ij} which is the distance between the ith and the jth object, find an embedding of points $x_1, \ldots, x_n \in \mathbb{R}^k$ such that $\sum(d_{ij} - ||x_i - x_j||)^2$ is minimal.

Other versions of the problem are, e.g., minimize $\sum(d_{ij}^2 - ||x_i - x_j||^2)$. Even the case where $k = 1$ is open.

▶ Problem 5 (Maarten Löffler). Given a unit square, find a set of lines such that there is a disk of radius ε centered on each line inside the unit square and such that no disk intersects another disk or another line. What is the maximum number of lines that can be placed?
Known bounds: $\Omega(1/\varepsilon)$ and $O(1/\varepsilon^2)$.

▶ Problem 6 (Günter Rote). This problem is due to Sergio Cabello and Maria Saumell.

Let P be a polygon of area 1. Let $C_{\max}$ be the area of the largest convex polygon contained in P. It is easy to see that

$$C_{\max}^2 \leq \text{probability}(x \text{ sees } y | x, y \in P).$$

The question is whether the following reverse bound holds

$$\text{probability}(x \text{ sees } y | x, y \in P) \leq O(C_{\max}).$$

Other variants:
a) P is star-shaped.
b) P is any region.
c) P is a polygonal region with holes. *Remark:* Sándor Fekete has pointed out that a convex region with many small holes (punctures) will be a counterexample.

▶ Problem 7 (Yoshio Okamoto). Given a polygonal domain P with a total number of n vertices, what is the maximum number of local maxima of the geodesic distance function $d(p, q)$, $p, q \in P$, on P? Known results: $O(n^7)$ and $\Omega(n^2)$ (the lower bound is tight for polygons without holes) [1].

▶ Problem 8 (Rolf Klein). *Lion problem*: We are given a $n \times n$ grid, each cell is contaminated or clean. Additionally, we are given a fixed number of lions. We consider discrete time steps; in each step the contamination of a cell spreads to its four adjacent cells. A lion can clean one adjacent cell per step.

How many lions are needed to clean the grid?

It is obvious that n lions are enough but it is an open question whether $n - 1$ lions are enough. Dumitrescu et al. [4] proved that $\sqrt{n}$ lions are not enough. Later on, Brass et al. [3], and independently Berger et al. [2], showed that $\lfloor n/2 \rfloor$ lions are not enough.
In general, it is assumed that in the beginning all cells are contaminated but there are many variants for the problem:
– different number of contaminated cells at the beginning
– remove boundaries
– etc.

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