Means and Limits of Decision

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Abstract

In this talk we survey recent work in the quest for expressive logics with good algorithmic properties, starting from the two-variable fragment of first-order logic and the guarded fragment. While tracing the boundary between decidable and undecidable fragments we describe their power, limitations, similarities and differences in order to stress out key properties responsible for their good or bad behaviour. We also highlight tools and techniques that have proven most effective for designing optimal algorithms, special attention giving to the more universal ones.

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1 Overview

In Computer Science, the use of logical formalisms to describe, query or manipulate structured data is now firmly embedded in both theory and practice. Having data described/specified within a logical formalism we often want such a specification to undergo static analysis – an automated procedure that optimizes the specification with respect to some correctness and efficiency criteria. Static analysis of specifications described in logical formalism often boils down to verifying one of the two basic logical properties, namely satisfiability and finite satisfiability.

Undecidability of the classical decision problem (=the satisfiability problem for first-order logic) results in two possible responses. The first one is to develop programs to test satisfiability of arbitrary collection of first-order formulas, accepting that, however well they generally work in practise, there will always be problem instances that defeat them. The second is to restrict attention to a *fragment* of first-order logic for which the satisfiability problem is decidable, exploiting the fact that in many real-life situations, the formulas we encounter fit comfortably into such fragments.

In this talk we overview recent work in the quest for expressive logics with good algorithmic properties. We concentrate mainly on fragments of first-order logic defined by restricting the number of variables (to gain decidability – to *two* [9, 7]) and usage of quantifiers to *guarded* quantification [1], and their variants or extensions motivated by real-life applications. We are equally interested in satisfiability and finite satisfiability, as in many application areas we want to model systems and computation to be essentially finite.

While tracing the boundary between decidable and undecidable fragments we study their similarities and differences to understand their power and limitations and to stress out key properties responsible for (un)decidability or (in)tractability. We give examples of fragments enjoying the *finite model property*: any satisfiable formula is true in some finite structure, and the *tree model property*: any satisfiable formula is true is some tree-like structure. We present fragments for which these two key properties led to optimal decision procedures (e.g. [4], [3], [11]) and contrast these fragments with their extensions where more sophisticated reasoning is required (e.g. [6, 5, 10]). We give special attention to linear and integer programming techniques that have recently proved useful to design optimal algorithms to decide uniformly both, the finite and the unrestricted satisfiability problems for certain expressive fragments.

The talk involves recent and ongoing work with Emanuel Kieroński, Jakub Michaliszyn, Ian Pratt-Hartmann, Wiesław Szwast, Georg Gottlob and Andreas Pieris.

The title of the talk has been inspired by Quine [8].

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