Coverability Synthesis in Parametric Petri Nets*

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Abstract

We study Parametric Petri Nets (PPNs), i.e., Petri nets for which some arc weights can be parameters. In that setting, we address a problem of parameter synthesis, which consists in computing the exact set of values for the parameters such that a given marking is coverable in the instantiated net.

Since the emptiness of that solution set is already undecidable for general PPNs, we address a special case where parameters are used only as input weights (preT-PPNs), and consequently for which the solution set is downward-closed. To this end, we invoke a result for the representation of upward closed set from Valk and Jantzen. To use this procedure, we show we need to decide universal coverability, that is decide if some marking is coverable for every possible values of the parameters. We therefore provide a proof of its ExpSpace-completeness, thus settling the previously open problem of its decidability.

We also propose an adaptation of this reasoning to the case of parameters used only as output weights (postT-PPNs). In this case, the condition to use this procedure can be reduced to the decidability of the existential coverability, that is decide if there exists values of the parameters making a given marking coverable. This problem is known decidable but we provide here a cleaner proof, providing its ExpSpace-completeness, by reduction to ω-Petri Nets.

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1 Introduction

The introduction of parameters in models aims to improve genericity. It also allows the designer to leave unspecified aspects, such as those related to the modeling of the environment. This increase in modeling power usually results in greater complexity in the analysis and verification of the model. Beyond verification of properties, the use of parameters opens the way to very relevant issues in design, such as the computation of the parameters values ensuring satisfaction of the expected properties. This is the synthesis problem: given a property, compute the exact set of all parameter values such that, instantiated with these values, the system satisfies this property. This notably permits an estimation of the *robustness*

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of a given instance of a model. Indeed, in full knowledge of "good values" for the parameters, we may be able to quantify the distance from a "bad value" providing an idea of how reliable is the system. Thus parameterised systems are of particular interest both in allowing the handling of more realistic classes of models and addressing more realistic verification issues. We therefore address here the case of parameterised concurrent systems modelled as parametric Petri nets.

Related work. The study of parameterised models and more specifically the synthesis has been studied in different parametrics settings. Parameters representing delays in timed systems modeled as timed automata have been particularly studied, but with very few decidability results [1]. Synthesis for such systems is only possible in very particular settings, such as bounded integer parameters computed symbolically in timed automata [15] or integer parameters in timed automata with parameters used only as upper bounds, or only as lower bounds, in timing constraints [4]. We focus here on a different type of parameters which represent discrete values. In Petri nets, this corresponds to parameterising the initial marking [5, 17] and the weights of the arcs in transitions [7]. The latter work deals with two decidability problems induced by the use of parameters: The *existential coverability*: does there exists an integer valuation v on the set of parameters such that m is coverable in the marked Petri net where parameters are replaced by the value given by v? And the *universal coverability*: is m is coverable in such a net for every possible valuation v? Those problems are both undecidable in the most general case, and syntactic subclasses restricting the use of parameters have been introduced, for which the different problems are decidable.

Contributions. We focus on computing the exact solution set to the synthesis problem for coverability in parametric Petri nets, i.e., the set of all parameter values such that in the net instantiated with these values, a given marking is coverable.

The emptiness and universality of the solution set being undecidable in general, computing this set can only be done in a restricted setting. We thus focus on the case when parameters are used only as input weights (preT-PPNs) or only as output weights (postT-PPNs). These assumptions give some structure to the solution set: we prove that it is then downward-closed wrt. the usual order on integer vectors for preT-PPNs, and upward-closed for postT-PPNs.

We show how a procedure by Valk and Jantzen from [20] can be used for computing a finite minimal basis of the solution set for postT-PPNs or its complement for preT-PPNs. This requires deciding universal coverability in preT-PPNs and existential coverability in postT-PPNs. The former is an open problem: our main result is it is ExpSpace-complete, which we prove by considering the more generic property of simultaneous unboundedness studied by Demri in [8]. The latter is known decidable but we provide a cleaner proof and additionally prove its ExpSpace-completeness. These results interestingly allow us to carry over a Rackoff upper bound into this parametric setting.

Finally, we prove that in what is called distinctT-PPNs in [7], i.e., when the set of parameters appearing as input weights, and the set of parameters appearing as output weights are disjoint, the solution set cannot be represented using any formalism for which the emptiness of the intersection with equality constraints is decidable.

Organization of the Paper. Section 2 gives basic notations and recalls useful mathematical results on orders and Petri nets. Section 3 presents Parametric Petri Nets and recalls some decidability problems. There, we also study the structure of the solution sets for preT- and postT-PPNs and show under which condition Valk and Jantzen's algorithm can be used to construct finite representation of those sets. In section 4 we give our construction for proving the ExpSpace-completeness of the universal coverability for preT-PPNs. Section 5 revisits the proof of the decidability of existential coverability in postT-PPNs and proves its ExpSpace-completeness. We also discuss the case of distinctT-PPNs. Finally, in Section 6, we conclude and present future work. Due to space contraints, proofs are omitted.

2 Background

2.1 Notations

We denote by $\mathbb{Z}$ the set of integers, and by $\mathbb{N}$ the set of natural numbers. As usual, $\mathbb{N}_\omega$ is the union $\mathbb{N} \cup \{\omega\}$ where for each $n \in \mathbb{N}$, $n + \omega = \omega$, $\omega - n = \omega$ and $\omega \leq \omega$. Moreover, if $n \in \mathbb{N}$, $n < \omega$. Let X be a finite set. We denote by 2^X the powerset of X and $|X|$ the size of X. If $X \subseteq \mathbb{N}^k$, $\neg X$ denotes its complement in $\mathbb{N}^k$. Given a finite set X, S_X denotes the symmetric group on X (*i.e.* the set of all permutations of elements of X). Given a set X, we define a *linear expression* on X by the following grammar: $\lambda ::= k \mid k * x \mid \lambda + \lambda$ where $k \in \mathbb{Z}$, $x \in X$. We denote by $\mathcal{L}(X)$ the set of linear expressions on X.

Let $V \subseteq \mathbb{N}$, a V-valuation for X is a function from X to V. We denote by V^X the set of V-valuations on X. Considering $v \in V^X$, we write $\mathtt{dom}(v)$ the domain of v (X in this case) and $\mathtt{im}(v)$ its image. We refer to $\mathbb{N}_\omega$-valuations as *extended* valuations and to $\mathbb{N}$-valuations simply as valuations. The set $V^\emptyset$ is reduced to a singleton $\{\emptyset_V\}$ where $\emptyset_V$ is the empty function. If X is finite, considering some arbitrary order on X, an (extended) valuation can be seen as a vector of size $|X|$. For any subset $X' \subseteq X$ and valuation $v \in V^X$, we define the restriction $v_{|X'}$ of v to X' as the unique V-valuation on X' such that $v_{|X'}(x) = v(x)$ for all $x \in X'$. We extend this notation to sets of valuations: given $Y \subseteq V^X$, $Y_{|X'}$ denotes its projection on X' that is to say $Y_{|X'} = \{v_{|X'} \mid v \in Y\}$. Given a value a of $\mathbb{N}_\omega$, we denote as $\mathbf{a}$ the uniform (extended) valuation that maps every element of X to a. Given an extended valuation v, we write $\omega(v)$ for the subset of X such that $x \in \omega(v)$ iff $v(x) = \omega$. We write $\mathbb{N}(v)$ for the subset of X such that $x \in \mathbb{N}(v)$ iff $v(x) \in \mathbb{N}$.

Given a linear expression λ on X and an extended valuation v on $X' \subseteq X$, $v(\lambda)$ is the linear expression obtained when substituting each element x in X' from λ, by the corresponding value $v(x)$. If $X' = X$ we obtain an element of $\mathbb{N}_\omega$.

Given a set R, finite sets S, A, B such that $S = A \cup B$ and $A \cap B = \emptyset$, and functions $f \in R^A$ and $g \in R^B$, we write $f \cup g \in R^S$ the function defined by $(f \cup g)_{|A} = f$ and $(f \cup g)_{|B} = g$. We call $f \cup g$ the *union* of f and g. Note that given x in A, $y = f(x)$ is called the image of x and when there is no ambiguity (*i.e.* when f is injective), x is called the fiber of y by f.

Finally, let A be an alphabet and A^* be the free monoid over A. Let $w \in A^*$ be a word. We write $|w|$ the length of w. Given $a \in A$, $|w|_a$ is the number of occurrences of a in w. We define ϵ as the identity element of A^*. We write $t \sqsubseteq s$ when t is a prefix of s. We denote by $Pref(L)$ the prefix closure of a langage L, *i.e.* $Pref(L) = \bigcup_{s \in L}\{t \mid t \sqsubseteq s\}$.

2.2 Order

A *quasi order* (qo for short) $\lesssim$ on some set S is a reflexive and transitive binary relation on S. The pair $(S, \lesssim)$ is then called a quasiordered set. For $x, y \in S$ and given a qo $\lesssim$ on S, x and y are said *comparable* if either $x \lesssim y$ or $y \lesssim x$. A relation $<$ is a *strict order* on a set S if it is irreflexive and transitive (which implies asymmetry).

Given any quasi order $\lesssim$ on a set S we can define: (i) a strict order $<$ given by $x < y$ iff $x \lesssim y \wedge \neg(y \lesssim x)$, (ii) an equivalence relation $\sim$ given by $x \sim y$ iff $x \lesssim y \wedge y \lesssim x$, (iii) its dual quasi order $\gtrsim$ given by $y \gtrsim x$ iff $x \lesssim y$. A *well-quasi-ordering* (wqo for short) is a qo $\lesssim$ on a set S such that, for any infinite sequence $s = x_0, x_1, x_2, ...$ in S, there exists indexes $i < j$ with $x_i \lesssim x_j$. Now consider $\leq$, the qo on $\mathbb{N}^k$ component-wise. Formally, for every $x, y \in \mathbb{N}^k$, we write that $x \leq y$ iff for every component i of x and y, $x(i) \leq y(i)$. Dickson's Lemma [9] states that $(\mathbb{N}^k, \leq)$ is a well-quasi-ordered set. Let us also recall the following lemma[1]:

▶ **Lemma 1** ([9]). *Let $p_0, p_1, ..., p_n, ...$ be an infinite sequence of elements of $(\mathbb{N}_\omega)^k$. Then, there is an infinite sequence $p_{i_1}, p_{i_2}, ..., p_{i_n}, ...$ such that $p_{i_1} \leq p_{i_2} \leq ... \leq p_{i_n} \leq ...$. (with $i_1 < i_2 < \cdots < i_n < \ldots$).*

2.3 Downward and Upward closed sets

We reuse definitions and concepts from [10, 11] which are summed up in [12]. An upward closed set of the well quasi ordered set $(\mathbb{N}^k, \leq)$ is a subset U of $\mathbb{N}^k$ such that if $x \in U$, $y \in \mathbb{N}^k$ and $x \leq y$ then $y \in U$. The upward closure of a vector u, written $\uparrow u$ is the set $\{m \in \mathbb{N}^k \mid u \leq m\}$. Given a set U, we write $\uparrow U$ for the upward closure of U, defined as $\uparrow U = \bigcup_{u \in U} \uparrow u$. This implies that $\uparrow U$ is the least upward closed set in which U is included. Any upward closed set U can be represented by a finite set F, called basis, such that $U = \uparrow F$. The minimal elements of F still form a basis of U independently of F. This basis is minimal for inclusion among all bases and is thus called the minimal upward basis of F.

A downward closed set of the well quasi ordered set $(\mathbb{N}^k, \leq)$ is a subset D of $\mathbb{N}^k$ such that if $x \in D$, $y \in \mathbb{N}^k$ and $y \leq x$ then $y \in D$. The downward closure of a vector d, written $\downarrow d$ is the set $\{m \in \mathbb{N}^k \mid m \leq d\}$. Given a set D, we write $\downarrow D$ the downward closure of D, defined as $\downarrow D = \bigcup_{d \in D} \downarrow d$. This implies that $\downarrow D$ is the least downward closed set in which D is included. Moreover, the downward closure of a finite set is finite. To symbolically represent downward closed sets, we use the extension $\mathbb{N}^k_\omega$. The definitions remain otherwise the same. If D is a downward closed set, we can write $D = \mathbb{N}^k \cap \downarrow F$ where F is a finite set of $\mathbb{N}^k_\omega$. We call F a downward basis of D. The maximal elements of F still form a basis of D independently of F. This basis is minimal for the inclusion among all bases and is thus called the minimal downward basis of D.

We also recall results from [3]: the union and the intersection of two upward (resp. downward) closed sets is an upward (resp. downward) closed set. The complement of an upward closed set is a downward closed set and vice-versa. Given the basis of an upward closed set, it is possible to compute the basis of its complement using for instance the procedure suggested in Example 5 of [14], and vice versa by adapting this procedure.

Finally, Valk and Jantzen proposed in [20] a necessary and sufficient condition, recalled in Lemma 2, to ensure that a finite basis of an upward closed set is effectively computable.

▶ **Lemma 2** ([20]). *Given an upward closed set $U \subseteq \mathbb{N}^k$, a finite basis of U is effectively computable iff for each $v \in \mathbb{N}^k_\omega$, the emptiness of $\downarrow v \cap U$ is decidable, which is also equivalent to ask whether for all element $v \in \mathbb{N}^k_\omega$, it is decidable to answer whether $\downarrow v \cap \mathbb{N}^k \subseteq \neg U$.*

[1] The existence of such increasing subsequences can also be used as a definition for wqo, which leads to an equivalent notion. Note that a proof can also be found in [16].

2.4 Petri Nets

▶ **Definition 3** (Marked Petri Net). A Petri net $\mathcal{N}$ is a tuple $(P, T, Pre, Post)$ such that P is a finite set of places of $\mathcal{S}$, T is a finite set of transitions of $\mathcal{S}$, Pre and $Post$ are functions from $P \times T$ to $\mathbb{N}$. A *marking* of $\mathcal{N}$ is an $\mathbb{N}$-valuation on P. A marked Petri Net (PN) is a pair $\mathcal{S} = (\mathcal{N}, m_0)$ where $\mathcal{N}$ is a Petri net and m_0 the initial marking of $\mathcal{N}$.

Given a transition t of T, we define $Pre(t)$ as the univariate function on P at the point t which associates to each p of P the weight $Pre(p, t)$. We define $Post(t)$ in a similar way. A transition $t \in T$ is said *enabled* by a marking m when $m \geq Pre(t)$.

▶ **Definition 4** (PN Semantics). The semantics of a PN is a transition system $\mathcal{S}_\mathcal{T} = (Q, q_0, \rightarrow)$ where, $Q = \mathbb{N}^P$, $q_0 = m_0$, $\rightarrow \subseteq Q \times T \times Q$ such that for all $t \in T$, $m \xrightarrow{t} m' \Leftrightarrow m \geq Pre(t)$ and $m' = m - Pre(t) + Post(t)$

This relation can be extended to sequences of transitions as follows: (i) $m \xrightarrow{\epsilon} m'$ if $m = m'$ (ii) $m \xrightarrow{wt} m'$ if $\exists m'', m \xrightarrow{w} m'' \wedge m'' \xrightarrow{t} m'$ where $w \in T^*$ and $t \in T$. We write $\xrightarrow{*}$ the reflexive transitive closure of $\rightarrow$, i.e., $m \xrightarrow{*} m'$ when there exists $w \in T^*$ such that $m \xrightarrow{w} m'$.

▶ **Definition 5** (Reachability). Let $\mathcal{S} = (\mathcal{N}, m_0)$, where $\mathcal{N} = (P, T, Pre, Post)$, a marking m of $\mathbb{N}^P$ is reachable in $\mathcal{S}$ iff $m_0 \xrightarrow{*} m$.

The reachability set $\mathcal{RS}(\mathcal{S})$ of $\mathcal{S}$ is the set of all reachable markings of $\mathcal{S}$.

▶ **Definition 6** (Coverability). Let $\mathcal{S} = (\mathcal{N}, m_0)$, where $\mathcal{N} = (P, T, Pre, Post)$, and m a marking of $\mathbb{N}^P$, m is coverable in $\mathcal{S}$ iff $\exists m' \in \mathcal{RS}(\mathcal{S}), m' \geq m$.

The coverability set $\mathcal{CS}(\mathcal{S})$ of $\mathcal{S}$ is the set of markings coverable in $\mathcal{S}$. Coverability is decidable in marked Petri nets [16]. The coverability set is an over approximation of the reachability set in the sense that $CS(\mathcal{S}) = \downarrow\mathcal{RS}(\mathcal{S})$. Given a marked PN $\mathcal{S}$, and a marking m, we denote by $\mathtt{cov}(\mathcal{S}, m) \in \{True,\ False\}$ the coverability of m in $\mathcal{S}$. In Petri nets, coverability allows to verify safety properties. We recall that the coverability set of a marked Petri net is computable in the sense that its minimal downward basis is computable (see, *e.g.*, [12]).

3 Monotonicity in Parametric Petri Nets

3.1 Parametric Petri Nets and Parametric Problems

Following [7], we recall the definitions related to marked Parametric Petri Nets (PPNs). We omit the case of parametric initial markings which is a subcase of parametric output weights.

▶ **Definition 7** (Parametric Petri Net). A marked parametric Petri Net (PPN) is a pair $\mathcal{S} = (\mathcal{N}, m_0)$ where $\mathcal{N} = (P, T, Pre, Post, \mathbb{P})$ such that P is a finite set of places of $\mathcal{N}$, T is a finite set of transitions of $\mathcal{N}$, $\mathbb{P}$ is a finite set of parameters of $\mathcal{N}$, Pre and $Post$ are functions from $P \times T$ to $\mathbb{N} \cup \mathbb{P}$, m_0 is the initial marking of $\mathcal{N}$ belonging to $\mathbb{N}^P$.

We define the *parametric transitions* of $\mathcal{S}$, $\Theta \subseteq T$ as the set of transitions with at least one parameter on an input or output arc: $\Theta = \{t \in T \mid \exists p \in P\ s.t.\ Pre(p,t) \in \mathbb{P} \vee Post(p,t) \in \mathbb{P}\}$. We refer to $T \setminus \Theta$ as the set of plain transitions in echo to the notations of [13].

Considering an arbitrary ordering on places, parametric markings can be represented as vectors of linear combinations on the set of parameters *i.e.* from $\mathcal{L}(\mathbb{P})^{|P|}$. Similarly, Pre and $Post$ can be seen as matrices of $(\mathbb{N} \cup \mathbb{P})^{|P| \times |T|}$.

Given a marked PPN $\mathcal{S} = (\mathcal{N}, m_0)$, where $\mathcal{N} = (P, T, Pre, Post, \mathbb{P})$, for any $\mathbb{N}$-valuation v on a subset X of $\mathbb{P}$, we define the v-instance of $\mathcal{S}$ as the marked PPN $v(\mathcal{S}) = (v(\mathcal{N}), m_0)$ where $v(\mathcal{N}) = (P, T, v(Pre), v(Post), \mathbb{P} \backslash X)$. By $v(Pre)$ and $v(Post)$ we denote the functions/matrices obtained by replacing in their entries each parameter λ in $\mathtt{dom}(v)$ by $v(\lambda)$. If $X = \mathbb{P}$, $v(\mathcal{N})$ and $v(\mathcal{S})$ are respectively a Petri net and a marked Petri net. We also recall subclasses introduced in [7]. Given a PPN, $\mathcal{N} = (P, T, Pre, Post, \mathbb{P})$, if $Pre \in P \times T \to \mathbb{N}$, we call it a postT-PPN, whereas if $Post \in P \times T \to \mathbb{N}$, we call it a preT-PPN.

Given a PPN $\mathcal{S}$, and a marking m, we define two basic parametric decision problems: Does there exist a valuation v such that m is coverable in $v(\mathcal{S})$ (Existential coverability) ? Is m coverable in $v(\mathcal{S})$ for all valuations v (Universal coverability) ? Those problems were partly studied in [7]. In particular both *Existential coverability* and *Universal coverability* are proved to be undecidable for the generic class of PPNs. In this paper, we are interested in a more general question:

▶ **Definition 8** (*coverability*-Synthesis problem)**.** Compute all the valuations v, such that $\mathtt{cov}(v(\mathcal{SP}), m)$ is true.

We call this set of valuations the coverability synthesis set of a marked PPN $\mathcal{S}$ and a marking m, denoted by $\mathcal{CV}(\mathcal{S}, m)$. We also call it the solution set to the synthesis problem.

▶ Remark. From any PN $\mathcal{S}$, we can build a PPN $\mathcal{S}'$ by adding an unused parameter. Then checking existential or universal coverability on $\mathcal{S}'$ is equivalent to checking coverability on $\mathcal{S}$. Those parametric problems are therefore EXPSPACE-hard. The same reasoning applies for other properties such as (simultaneous) unboundedness.

3.2 Special Structure of the Coverability Synthesis Set for PreT-PPNs and PostT-PPNs

When we restrict the use of parameters to input arcs, we ensure that any marking coverable in a v-instance remains coverable for any v'-instance such that $v' \leq v$. Intuitively, decreasing the valuation leads to a more permissive firing condition. Symmetrically, when we restrict the use of parameters to output arcs, we ensure that any marking coverable in a v-instance, remains coverable for any v'-instance such that $v \leq v'$. Intuitively, firing the same parametric transition while increasing the valuation leads to greater (and thus more permissive) markings. Those results are formalized in Lemma 9.

▶ **Lemma 9.** *Let $\mathcal{S}_{pre}$ and $\mathcal{S}_{post}$ be respectively a marked preT-PPN and postT-PPN of initial marking m_0 and s_0.*

- *For every transitions sequence w of $\mathcal{S}_{pre}$ and for every valuation v, if $m_0 \xrightarrow{w} m$ in $v(\mathcal{S}_{pre})$, then for every valuation $v' \leq v$, there exists $m' \geq m$ such that $m_0 \xrightarrow{w} m'$ in $v'(\mathcal{S}_{pre})$.*
- *For every transitions sequence w of $\mathcal{S}_{post}$ and for every valuation v, if $s_0 \xrightarrow{w} s$ in $v(\mathcal{S}_{post})$, then for every valuation $v' \geq v$, there exists $s' \geq s$ such that $s_0 \xrightarrow{w} s'$ in $v'(\mathcal{S}_{post})$.*

Note that those properties of monotonicity directly provide a notable structure for the solution set of the synthesis for those two subclasses presented in Corollary 10.

▶ **Corollary 10.** *Given $\mathcal{S}_{pre}$, $\mathcal{S}_{post}$ a marked preT-PPN and a marked postT-PPN respectively and goal markings m and s for each of those nets,*

- *$\mathcal{CV}(\mathcal{S}_{pre}, m)$ is downward closed.*
- *$\mathcal{CV}(\mathcal{S}_{post}, s)$ is upward closed.*

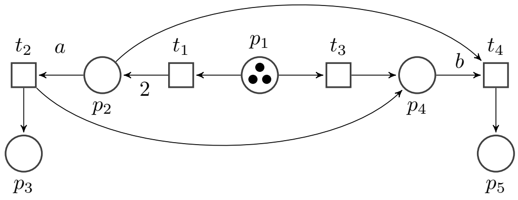

Figure 1 $\mathcal{S}_1$ where $\mathbb{P} = \{a, b\}$.

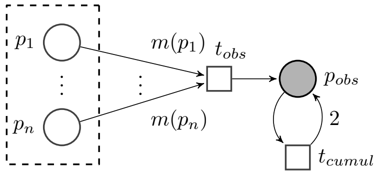

Figure 2 Reduction of coverability to the place boundedness.

3.3 Reduction of Valk and Jantzen Condition for PreT-PPNs and PostT-PPNs

Given a preT-PPN $\mathcal{S}$ and a marking m, one way to compute $\mathcal{CV}(\mathcal{S}, m)$ is thus to find its finite minimal basis. A naive enumeration is not possible however since this set may be infinite. In particular, the strategy that consists in testing for universality and, in the negative case, enumerating until a witness of non coverability is found would in general provide only a subset of $\mathcal{CV}(\mathcal{S}, m)$. In fact, the main difficulty here resides in the fact that the elements of the minimal basis have to be found among the complete lattice induced by $\leq$ on $\mathbb{N}_\omega^{\mathbb{P}}$.

In order to represent a finite basis of a downward closed set of valuations, we need to extend valuations to $\mathbb{N}_\omega$. Given a preT-PPN and an extended valuation v, we extend the predicate $\mathtt{cov}(v(\mathcal{S}), m)$ to extended valuations as follows: $\mathtt{cov}(v(\mathcal{S}), m) \overset{def}{\Leftrightarrow} \forall v' \in \mathbb{N}^{\omega(v)}, \mathtt{cov}(v'(v_{|\mathbb{N}(v)}(\mathcal{S})), m)$.

Figure 1 presents a preT-PPN with two parameters a and b. If we consider the valuation v defined by $v(a) = 1$ and $v(b) = \omega$, $\mathtt{cov}(v(\mathcal{S}), m)$ is therefore equivalent to the universal coverability of m in $v_{|\{a\}}(\mathcal{S})$ where $v_{|\{a\}}$ is a valuation defined by $v(a) = 1$, that is to say "can we cover m in $\mathbf{1}_{|\{a\}}(\mathcal{S})$ for any value of b ?". Note that this extension of $\mathtt{cov}(v(\mathcal{S}), m)$ is consistent with the classic behavior: if $\mathbb{N}(v) = \mathbb{P}$, then $\mathtt{cov}(v(\mathcal{S}), m)$ asks the coverability of m in the marked Petri Net $v(\mathcal{S})$.

We recall that in postT-PPNs, universal coverability is true iff $\mathtt{cov}(\mathbf{0}(\mathcal{S}), m)$. In a similar manner to preT-PPN, we extend the notation of $\mathtt{cov}$, as follows: given a postT-PPN and an extended valuation v, we extend the predicate $\mathtt{cov}(v(\mathcal{S}), m)$ to extended valuations as follows: $\neg\mathtt{cov}(v(\mathcal{S}), m) \overset{def}{\Leftrightarrow} \forall v' \in \mathbb{N}^{\omega(v)}, \neg\mathtt{cov}(v'(v_{|\mathbb{N}(v)}(\mathcal{S})), m)$. This definition is similar to the definition extended for preT-PPNs where coverability has been replaced by non-coverability.

With those extended notations, we now wonder if it is possible to compute a finite basis of $\mathcal{CV}(\mathcal{S}, m)$ where $\mathcal{S}$ is a preT-PPN or a postT-PPN and m a goal marking. To this end, we suggest to use an algorithm by Valk and Jantzen [20] to compute a finite representation of those sets. Nevertheless, to ensure that this algorithm is suitable to our context and that those basis are effectively computable, we need to clarify two points:

- First, this algorithm is used to compute bases of upward closed sets.
- Second, the necessary and sufficient condition recalled in Lemma 2 must be satisfied.

To address the first point, by Corollary 10 notice that in the case of postT-PPNs, the set $\mathcal{CV}(\mathcal{S}, m)$ is upward closed, and the procedure could be applied directly on it. In the case of preT-PPNs, since $\mathcal{CV}(\mathcal{S}, m)$ is downward closed, we need to consider $\neg\mathcal{CV}(\mathcal{S}, m)$, which is upward closed as recalled in Section 2.3. We also recalled in that section that given a finite basis of an upward closed set, it is possible to compute a finite basis of its complement. It is therefore equivalent to being able to compute a finite basis of the set of valuations for which it is not possible to cover m or to be able to compute a finite basis of the set of valuations for which it is possible to cover m. Thus, a finite basis of $\neg\mathcal{CV}(\mathcal{S}, m)$ is effectively computable iff a finite basis of $\mathcal{CV}(\mathcal{S}, m)$ is computable. Considering this reasoning, we address the second point through the following Lemma:

▶ **Lemma 11.** *The Valk and Jantzen condition can be reduced to the following criteria:*
1. *we can compute a finite representation of the coverability synthesis set in preT-PPNs iff universal coverability is decidable in preT-PPNs*
2. *we can compute a finite representation of the coverability synthesis set in postT-PPNs iff existential coverability is decidable in postT-PPNs*

We now start by focusing on universal coverability in preT-PPNs.

4 Universal Coverability in preT-PPNs

We address the problem of universal coverability through that of the more general universal simultaneous unboundedness. We will prove that both are ExpSpace-complete.

4.1 Simultaneous Unboundedness

▶ **Definition 12** (Simultaneous Unboundedness [8]). Given $\mathcal{N} = (P, T, Pre, Post)$, and $\mathcal{S} = (\mathcal{N}, m_0)$, considering a subset $X \subseteq P$, $\mathcal{S}$ is simultaneous X unbounded if for any $B \geq 0$, there is a run w such that $m_0 \xrightarrow{w} m$ and for all $i \in X$, we have $m(i) \geq B$. If X is reduced to a singleton $\{p\}$, $\mathcal{S}$ is said p-unbounded.

▶ Remark. Notice that coverability can be easily reduced to simultaneous unboundedness by the use of an observer as depicted in Figure 2. The transition t_{obs} has an input condition equal to the marking we want to cover m. Its output effect provides a token in a place p_{obs}, that, once is marked, can become unbounded through the firing of t_{cumul}. With this construction, m is coverable in the net iff it is simultaneous p_{obs}-unbounded.

Since there exist polynomial translations from VASS to VAS and PN and from PN to VAS (and VASS) [18, 2], we have the following Theorem, initially stated with VASS in [8].

▶ **Theorem 13** ([8]). *Simultaneous unboundedness for PNs is ExpSpace-complete.*

4.2 Notion of Incremental Model

To prove the decidability of universal coverability in preT-PPNs, we will prove the decidability of universal simultaneous unboundedness. We will also prove that the latter belong to ExpSpace. Together with Remark of Section 3.1, we can then conclude that both problems are ExpSpace-complete.

Formally, given a parametric Petri net, and a set of places X, the parametric net is *universally simulatenous X unbounded* iff for every possible valuation v of its parameters, the v-instance of this net is simultaneous X unbounded.

We first show that it is sufficient for a net to be simultaneous unbounded on a set of places in infinitely many instances (under uniform valuations) of the parametric Petri net to be universally simultaneous unbounded on this set of places. Indeed, for any valuation v, we can find a uniform valuation $\mathbf{k}$ such that $v \leq \mathbf{k}$ and apply Lemma 9.

▶ **Lemma 14.** *Given a marking m and a marked preT-PPN $\mathcal{S}$ and X a subset of places of $\mathcal{S}$, the two following propositions are equivalent :*

1. $(\mathcal{S}, m_0)$ *is universally simultaneous X unbounded*
2. $\{k \in \mathbb{N} \mid (\mathbf{k}(\mathcal{S}), m_0)$ *is simultaneous X unbounded$\}$ is infinite*

This Lemma is used for the proof of the upcoming Lemma 15. It is indeed an important result since it reduces the infinite set of valuations over which we should investigate to the infinite set of uniform valuation that is totally ordered *i.e.* two elements of this set are always comparable.

We can now address the problem of universal simultaneous unboundedness. To solve this problem, we reduce it to the existence of a classic Petri net built upon our parametric model satisfying an adequately chosen simultaneous unboundedness property. The classic Petri net is in fact obtained by evaluating a preT-PPN, called incremental net, under the uniform valuation $\mathbf{0}$. The incremental net has a polynomial size in the original preT-PPN and it directly depends on the original preT-PPN and a sequence of distinct parametric transitions. This Section is thus driven by the idea that universal simultaneous X unboundedness on a preT-PPN $\mathcal{S}$ is equivalent to the existence of a sequence σ of distinct parametric transitions of $\mathcal{S}$, such that the incremental model build on $\mathcal{S}$ and σ evaluated under $\mathbf{0}$ satisfies a simultaneous unboundedness property depending on X and σ.

Before providing the theoretical definition, let us consider the main intuition of our construction. If a net is universally simultaneous unbounded on a set of places X, two main cases are possible: we can either find a path such that the places of X are unbounded without using any parametric transition, and then the corresponding run works for any valuation, or we need at least one parametric transition.

In the latter case, since there is an infinite number of valuations and a finite number of parametric transitions, using the pigeonhole principle, there is at least one such transition that must be used as the first parametric transition in the run for an infinite number of valuations. The input places of its parametric arcs are therefore not bounded. Thus, the valuation of the input parametric arcs of this transition is not limiting anymore since we can generate an arbitrary large amount of tokens in the corresponding places. Therefore, we will later evaluate[2] those parameters to 0 in order to perform the verification on a classic Petri net.

Nevertheless, we need to ensure that the set of input places of the parametric arcs are not bounded (without using that transition). This is exactly the goal of this incremental model. Indeed, once fired, we could then consider a new net where the first parametric transition can be involved as well as non parametric transitions and investigate for the newly unbounded places. Either we can unbound the places of the goal set X or we can reuse previous reasoning and choose a new parametric transition that has to be involved in infinitely many instances. What is important to note here is that at each firing of a new parametric transition, that never occurred in the run, we need to ensure that its input places of parametric input arcs were unbounded using only previous transitions of the run and to remember what are the new places that can be unbounded through the use of this new transition. We will now formalize how it is possible to remember the boundedness of the input places of parametric arcs by presenting exhaustively the model of incremental nets.

Given a preT-PPN $\mathcal{N} = (P, T', Pre, Post, \mathbb{P})$ and a partition of its transitions $T' = T \cup \Theta$ between its plain and parametric transition, we denote by $\mathcal{N}_p$ the Petri Net obtained from

[2] Note that any other finite valuation would be suitable since the input places of the parametric arcs are unbounded.

$\mathcal{N}$ by removing all transitions of Θ from $\mathcal{N}$. An example is given in Figure 3. Let us now consider a finite sequence $\sigma \in Pref(S_\Theta)$ where S_Θ is the symmetric group over Θ seen as a language. Let $|T| = m$, $|P| = n$ and $|\sigma| = k$. We define the incremental model $\mathcal{I}$ of $\mathcal{N}$ along σ. We write $\mathcal{I} = \texttt{incr}(\mathcal{N}, \sigma)$ to denote this preT-PPN. This model is illustrated by the example[3] at the right hand side of Figure 3. Its construction consists of the following main steps:

(i) Consider $\mathcal{N}_p$ and k copies of $\mathcal{N}_p$, where to each of those $k+1$ subnets is associated a global lock place, ensuring that exactly one copy is active at any given instant. The copies correspond to the black subnet of this example, whereas the global locks p_0^i's are depicted in blue and dotted arcs.

(ii) Add a copy of the first i transitions of σ to each i^{th} copy of $\mathcal{N}_p$, for $1 \leq i \leq k$.

(iii) Between the $i-1^{th}$ and i^{th} subnets, add a copy of the $i+1^{th}$ transition of σ, depicted in plain green arcs in Figure 3, for $1 \leq i \leq k$. Notice that this copy presents a special behaviour: its input effect impacts the $i-1^{th}$ subnet and its lock p_0^{i-1} whereas its output effect impacts the i^{th} subnet and its lock p_0^i. This ensures that we change of active subnet only after the firing of a the first occurrence of a precise parametric transition.

(iv) Finally, we ensure that given every copy of a transition, including the intermediate copies that allow to change the active copy, it modifies simultaneously the places in the associated copy as explained above, but also all copies of greater index (*i.e.* those that have not been activated yet). Those arcs ensure that every later subnet always has the "same" marking as the active copy. They are depicted by dashed red arcs in Figure 3. Note that we synchronise the different copies and do not merge them because we use them to remember the order in which the different input places to parametric transitions become unbounded.

Let us suppose we evaluated this incremental model in order to perform an execution. At the beginning of any execution, given a precise subnet, let us say the i^{th} subnet, it follows the behavior of the subnets with lower index because of synchronisations introduced in item (iv). Then, once this copy become active, after the firing of a given parametric transition introduced in item (iii), it will dictate the behavior of the global net (and thus of the subnets with greater index through synchronisations). Once the next copy becomes active, our original i^{th} subnet cannot change its state anymore. It is now literally an historic state of the global run of the incremental net.

More formally, the incremental net $\texttt{incr}(\mathcal{N}, \sigma)$ is the preT-PPN $(\mathcal{P}, \mathcal{T}, \textsc{Pre}, \textsc{Post}, \mathbb{P})$ such that $\mathcal{P} = \{p_j^i \mid 0 \leq i \leq k \wedge 0 \leq j \leq n\}$ where p_0^i represents the lock related to the i^{th} copy of $\mathcal{N}$ whereas p_j^i with $j > 0$ represents the copy of the place p_j of P in the i^{th} subnet and $\mathcal{T} = \{t_j^i \mid 0 \leq i \leq k \wedge 1 \leq j \leq m\} \cup \{\theta_j^i \mid 1 \leq j \leq i \leq k\} \cup \left(\cup_{1 \leq i \leq k} \{\theta_i^0\}\right)$ where t_j^i represents the copy of the transition t_j of T in the i^{th} subnet, θ_j^i represents the copy of the transition θ_j of Θ in the i^{th} subnet, θ_i^0 represents a copy of the transition θ_i from σ which is used to change the active copy (from the $i-1^{th}$ to the i^{th}).

We define with this construction a net mapping to relate places and transitions from both models. Given the two nets $\mathcal{N}$ and $\mathcal{I}$ defined as above, considering previous notations, for each $0 \leq i \leq |\sigma|$ we define the application $f^i_{\mathcal{N}\to\mathcal{I}}$ that links the original net $\mathcal{N}$ to its i^{th} copy in $\mathcal{I}$ (except the corresponding lock). We define $f^i_{\mathcal{N}\to\mathcal{I}} : T \cup \{\theta_j \in \sigma \mid j \leq i\} \cup P \to \mathcal{T} \cup \mathcal{P}$ such that for $t_j \in T$ (resp. $\theta_j \in \sigma$ and $p_j \in P$), $f^i_{\mathcal{N}\to\mathcal{I}}(t_j) = t_j^i$ (resp. $f^i_{\mathcal{N}\to\mathcal{I}}(\theta_j) = \theta_j^i$ and $f^i_{\mathcal{N}\to\mathcal{I}}(p_j) = p_j^i$). We can then define f^{-1} the function that maps components of the copies

[3] The exact meaning of the notations used to refer to the different components of this example will be provided after this informal intuition on the construction.

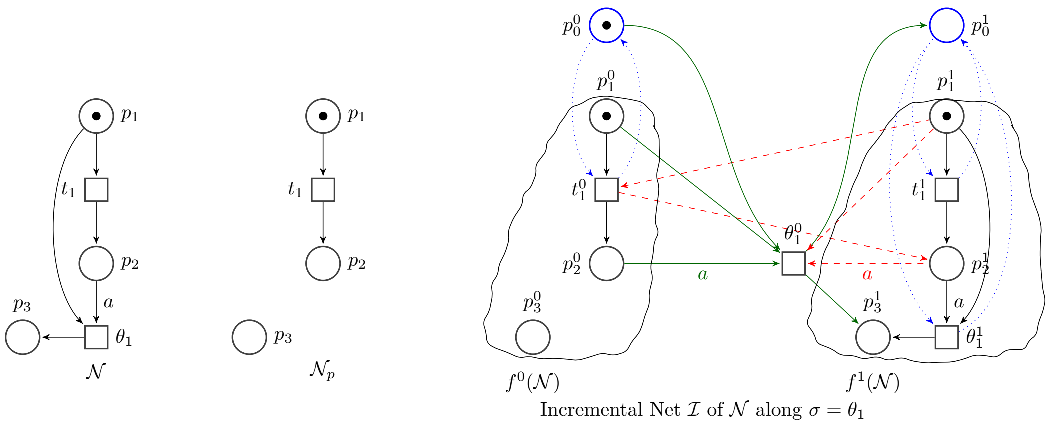

Figure 3 Construction of an Incremental Net3.

of $\mathcal{N}$ in $\mathcal{I}$ to their original fiber by the previous application. Formally, f^{-1} is defined by: $f^{-1} : \cup_{0 \leq i \leq k} \mathtt{im}(f^i_{\mathcal{N} \to \mathcal{I}}) \to T \cup P \cup \Theta$ and associates to $t^i_j \in \mathcal{T}$ (resp. $\theta^i_j \in \mathcal{T}$ and $p^i_j \in \mathcal{P}$) $f^{-1}(t^i_j) = t_j$ (resp. $f^{-1}(\theta^i_j) = \theta_j$ and $f^{-1}(p^i_j) = p_j$). Finally, we define the application $h_{\mathcal{I} \to \mathcal{N}} : \cup_{1 \leq i \leq k}\{\theta^0_i\} \subseteq \mathcal{T} \to \Theta$ that maps the intermediate parametric transitions between each copies of $\mathcal{N}$ in $\mathcal{I}$, θ^0_i to their original fiber from $\mathcal{N}$ and occuring in σ, that is to say the i^{th} transition of σ.

Those applications allow us to define formally the functions Pre and Post. Given i' and j', let $x^{i'}_{j'}$ denote either t^i_j or θ^i_j from $\{t^i_j \mid 0 \leq i \leq k \wedge 1 \leq j \leq m\} \cup \{\theta^i_j \mid 1 \leq j \leq i \leq k\}$ in the following expressions:

- $\textsc{Pre}(p^i_j, x^{i'}_{j'})(resp.\ \textsc{Post}(p^i_j, x^{i'}_{j'})) = \begin{cases} 0 \text{ if } (i < i') \text{ or } (i > i' \text{ and } j = 0) \\ 1 \text{ if } i = i' \text{ and } j = 0 \\ Pre(f^{-1}(p^i_j), f^{-1}(x^{i'}_{j'})) \text{ otherwise} \\ (resp.\ Post(f^{-1}(p^i_j), f^{-1}(x^{i'}_{j'}))) \text{ otherwise} \end{cases}$
- $\textsc{Pre}(p^i_j, \theta^0_{i'}) = \begin{cases} 0 & \text{if } (i+1 < i') \text{ or } (i+1 > i' \text{ and } j = 0) \\ 1 & \text{if } i+1 = i' \text{ and } j = 0 \\ Pre(f^{-1}(p^i_j), h^{-1}(\theta^0_{i'})) & \text{otherwise} \end{cases}$
- $\textsc{Post}(p^i_j, \theta^0_{i'}) = \begin{cases} 0 & \text{if } (i < i') \text{ or } (i > i' \text{ and } j = 0) \\ 1 & \text{if } i = i' \text{ and } j = 0 \\ Post(f^{-1}(p^i_j), h^{-1}(\theta^0_{i'})) & \text{otherwise} \end{cases}$

Given a net $\mathcal{N}$ and the function $f^i_{\mathcal{N} \to \mathcal{I}}$ we extend the definition of $f^i_{\mathcal{N} \to \mathcal{I}}$ to sets by $f^i_{\mathcal{N} \to \mathcal{I}}(X) = \{f^i(x) \mid x \in X\}$ and nets by defining $f^i_{\mathcal{N} \to \mathcal{I}}(\mathcal{N})$ as $(f^i(P), f^i(T), \textsc{Pre}_{f^i(P) \times f^i(T)}, \textsc{Post}_{f^i(P) \times f^i(T)}, \mathbb{P})$. When the context is clear, we omit the subscript $\mathcal{N} \to \mathcal{I}$. As examples, $f^0(\mathcal{N})$ and $f^1(\mathcal{N})$ are provided in Figure 3. Finally, we associate to a marking m of $\mathcal{N}$ the marking $f(m)$ defined by for all p of $\cup_{0 \leq i \leq k}(f^i(P))$, $f(m)(p) = m(f^{-1}(p))$. Notice that this ignores the locks introduced in the net. Given the initial marking m_0, we thus define the initial marking of the incremental net μ_0 as $f(m_0)$ for the copies of the places, and 0 in all locks except the first one which receives 1. Formally, $\mu_0(p^i_j) = m_0(p_j)$ if $j \neq 0$, 1 if $i = j = 0$ and 0 otherwise.

The idea behind this construction is double. First, we can enforce the order of the first occurrence of a parametric transition which is dictated by the sequence σ. Second, we can access the exact amount of tokens stored in a place before the firing of the first occurrence of a parametric transition and thus keep an historic of the state of a run, just before the firing of this parametric transition, through the copies of the original net. Based on those two observations, we will be able to observe if the input places of the parametric arcs of the first occurrence of a parametric transition in a run are bounded or not.

4.3 Complexity of Universal Simultaneous Unboundedness

We will now see that universal simultaneous unboundedness can be reduced to the existence of a sequence σ of distinct parametric transitions such that the incremental net built upon this sequence is simultaneous unbounded on an adequately defined set of places. We first provide the intuition behind this statement before providing its formal version. We must ensure that each input place of a parametric arc of the first occurrence of a parametric transition is unbounded. Based on the previous construction, one can notice that given a parametric transition θ_i occurring in σ, its input places are only impacted by the transitions occurring in the first i copies of the net. Thus, once θ_i is fired in the incremental net, the new feasible transitions will not impact the amount of tokens stored in the i first subnets. We will thus be able to verify if the input places were bounded or not before its firing, by observing the places of the copy that occurs just before the first firing of this transition. For each parametric transition of σ, we should thus verify that the input places in the corresponding copies are unbounded, and finally verify that the places that should be unbounded as part of the original property are indeed unbounded in the last copy of the net and that for each instance of the incremental net under a uniform valuation. Nevertheless, since the corresponding input places of parametric transitions are unbounded, it is sufficient to verify this property for only one instance of the incremental net, and in particular we will later choose the **0**-instance. Indeed, if such a property is verified for any **k**-instance, then, it could be verified for any **k′**-instance (with $k' > k$) by exhibiting the witness run and performing more loops.

▶ **Lemma 15.** *Let $\mathcal{N} = (P, T', Pre, Post, \mathbb{P})$ be a preT-PPN, such that $T' = T \cup \Theta$ where Θ represents the parametric transitions of $\mathcal{N}$ and T its plain transitions. For every a set of places of $X \subseteq P$, the following propositions are equivalent:*

1. *$(\mathcal{N}, m_0)$ is universally simultaneous X unbounded*
2. *$\exists\sigma = t_1, ..., t_l \in Pref(S_\Theta)$, considering the incremental model $\mathcal{I}$ of $\mathcal{N}$ along σ, $\mathcal{I} = \boldsymbol{incr}(\mathcal{N}, \sigma)$, $\exists k \in \mathbb{N}$, $(\mathbf{k}(\mathcal{I}), \mu_0)$ is simultaneous Y unbounded where $Y = f^l_{\mathcal{N}\to\mathcal{I}}(X) \cup \left(\bigcup_{t_i\in\sigma} f^{i-1}_{\mathcal{N}\to\mathcal{I}}(\Pi(t_i))\right)$.*
3. *$\exists\sigma = t_1, ..., t_l \in Pref(S_\Theta)$, considering the incremental model $\mathcal{I}$ of $\mathcal{N}$ along σ, $\mathcal{I} = \boldsymbol{incr}(\mathcal{N}, \sigma)$, $\forall k \in \mathbb{N}$, $(\mathbf{k}(\mathcal{I}), \mu_0)$ is simultaneous Y unbounded where $Y = f^l_{\mathcal{N}\to\mathcal{I}}(X) \cup \left(\bigcup_{t_i\in\sigma} f^{i-1}_{\mathcal{N}\to\mathcal{I}}(\Pi(t_i))\right)$.*

Following the notations, $Pref(S_\Theta)$ corresponds to the finite set of sequences of distinct parametric transitions. Remark that $f^l_{\mathcal{N}\to\mathcal{I}}(X)$ represents the copy of the places of X in the last subnet of $\mathcal{I}$. The set $\bigcup_{t_i\in\sigma} f^{i-1}_{\mathcal{N}\to\mathcal{I}}(\Pi(t_i))$ is a bit more complex: for each transition $t_i \in \sigma$, $\Pi(t_i)$ represents the input places of the parametric arcs. We therefore address here the unboundedness of the copies of those places in the corresponding subnet of the $\mathcal{I}$

Proof. We provide here the sketch of the implication (1) ⇒ (3). The goal is to find the sequence of parametric transitions along which we construct the incremental model seen in Section 4.2. This proof is done by induction on the number of parametric transitions in $\mathcal{N}$.

- In the base case, $\mathcal{N}$ is a PN. Therefore the incremental model considered is isomorphic to $\mathcal{N}$ and the property is immediate.

- In the inductive step, the case where it is possible that the places from X are unbounded without using parametric transitions is straightforward. In the other case, we show that there is a parametric transition θ that can be used as the first parametric transition occurring in a run leading to some simultaneous X unbounded markings in the coverability tree of $(\mathbf{k}(\mathcal{N}), m_0)$ for an infinite number of uniform valuation $\mathbf{k}$.
  1. We then prove that the input places of the parametric input arcs of θ must be unbounded in $(\mathcal{N}_p, m_0)$, that is to say, there is a marking z with some ω's on those components in the basis of the coverability set of this net.
  2. We now consider the net where those places have been removed and the projection of the marking obtained by firing θ from z. In this net θ is then a plain transition. Using the induction assumption, we can build an incremental model $\mathcal{J}$ along a sequence σ' for this reduced net.
  3. The end of this proof consists in building the incremental model $\mathcal{I}$ of $\mathcal{N}$ along the sequence $\theta\sigma'$ and to construct the set Y. There is no particular difficulty in this last point but the construction is a bit involved.

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From Lemma 15 we can observe that answering the universal simultaneous X unboundedness on a preT-PPN can be reduced to guessing an element σ of the finite set $Pref(S_\Theta)$ such that $(\mathbf{0}(\mathcal{I}), \mu_0)$ is simultaneous Y unbounded. Since checking simultaneous X unboundedness can be done in ExpSpace as recalled in Theorem 13, we obtain a NExpSpace procedure. Then, a well-known theorem by Savitch [19] stating that there is therefore a ExpSpace deterministic procedure answering this problem and the Remark from Section 3.1 allow us to deduce Theorem 16. Note that the following Corollary directly comes from Theorem 16 and Lemma 11.

▶ **Theorem 16.** *The Universal Simultaneous Unboundedness problem for preT-PPNs is ExpSpace-complete.*

▶ **Corollary 17.** *Given a marked preT-PPN $\mathcal{S}$ and a marking m, we can compute a finite representation of $\mathcal{CV}(\mathcal{S}, m)$.*

5 Synthesis in postT-PPNs and distinctT-PPNs

5.1 Complexity of Existential Coverability in postT-PPNs

We propose here a cleaner proof for the decidability of the existential coverability in postT-PPNs, and provide its complexity. We use a polynomial time transformation[4] from postT-PPN to ωPN (see [13]) which preserves existential coverability and invoke a transformation from ωPN to PN underlined in [13]. First, we recall definitions and results from [13].

▶ **Definition 18** (ω-Petri Net [13])**.** An ω-Petri Net (ωPN) is a tuple $(P, T, Pre, Post)$ where P and T are respectively a finite set of places and transitions. Pre (resp. $Post$) is a function of $P \times T$ to $\mathbb{N}_\omega$ that gives the input (resp. output) effect of a transition t on a place p.

▶ **Definition 19** (ωPN Semantics)**.** Given a marking m, and a transition t such that $m \geq Pre(t)$, firing t from m gives a new marking m' s.t. $\forall p \in P, m'(p) = m(p) - Pre(p,t) + o$

[4] More specifically we obtain an ω-output-PN or ωOPN for short, which corresponds to the natural subclass of ωPNs where $Pre \in P \times T \mapsto \mathbb{N}$.

where $o = Post(t,p)$ if $Post(p,t) \in \mathbb{N}$ and $o \geq 0$ if $Post(p,t) = \omega$. We denote this by $m \xrightarrow{t} m'$. Thus $Post(p,t) = \omega$ means that an arbitrary number of tokens are generated in p.

From this semantics, we can notice that ω's play a role not unlike output parameters, with the crucial difference that their "value" can change along the execution of the net. Let consider a postT-PPN $\mathcal{N}$ and let us associate to this model the ωOPN $\mathcal{N}'$ such that we replace each parametric arc of $\mathcal{N}$ by an ω arc.

▶ **Lemma 20** (postT-PPNs to ωOPNs). *Let $\mathcal{N}$ be a postT-PPN (which involves parameters of a set $\mathbb{P}$) and let $\mathcal{N}'$ be its corresponding ωOPN (with the same set of places and transitions) and let m_0 be their common initial marking. Given a marking $m \in \mathcal{RS}(\mathcal{N}', m_0)$, there exists a valuation v such that there exists a marking $m' \geq m$ with $m' \in \mathcal{RS}(v(\mathcal{N}), m_0)$. Moreover*[5]*, $\cup_{v \in \mathbb{N}^{\mathbb{P}}} \mathcal{RS}(v(\mathcal{N}), m_0) \subseteq \mathcal{RS}(\mathcal{N}', m_0)$.*

We can thus directly deduce Theorem 21 by reducing existential coverability in postT-PPNs to coverability in ωPN which belongs to ExpSpace by [13]. Note that the following Corollary comes from Theorem 21 and Lemma 11.

▶ **Theorem 21** (Complexity of Existential Coverability). *The existential coverability problem on postT-PPNs is ExpSpace-complete.*

▶ **Corollary 22.** *Given a marked postT-PPN $\mathcal{S}$ and a marking m, we can compute a finite representation of $\mathcal{CV}(\mathcal{S}, m)$.*

5.2 Representing the Coverability Synthesis Set for DistinctT-PPNs

Let us finally consider the case of PPNs in which the set of parameters used as input weights, and the set of parameters used as output weights, are disjoint. For this class, called distinctT-PPNs, the emptiness of the solution set to the synthesis problem for coverability is decidable [7]. Interestingly, we can adapt an idea originally used for L/U-automata in [15] to prove that the structure of this set is however much more complex than for preT-PPNs or postT-PPNs. In particular, one cannot represent this set with a finite set, a finite union of downward and/or upward closed sets or a finite union of convex polyhedra.

▶ **Lemma 23.** *If it can be computed, the solution of the synthesis of coverability in distinctT-PPN cannot, in general, be represented using any formalism for which emptiness of the intersection with equality constraints is decidable.*

6 Conclusion

It can be challenging to find meaningful parametric infinite state systems with decidable decision problems. We achieved here to prove a powerful result for two strict syntactical subclasses of parametric Petri nets: interestingly, the set of all valid valuations of parameters, allowing to cover a given marking, is effectively computable for parametric Petri nets where parameters are restricted to only input arcs or only output arcs.

Indeed, we have shown how the computability of the synthesis set for coverability in preT-PPNs and postT-PPNs can be reduced to a decision problem, respectively, universal coverability and existential coverability, which is then used in Valk and Jantzen's procedure.

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[5] Notice that this is only an inclusion. Indeed, contrarily to postT-PPNs, in ωPNs, the effect of an arc can change along the same execution.

We proved that these two decision problems are both ExpSpace-complete. Putting the two types of parameters together while forbidding any parameter to be used as both an input and output weight preserves the decidability of the emptiness of the solution set. However, we have proved that, even with this restriction, the solution set can in general not be represented using any formalism for which emptiness of the intersection with equality constraints is decidable, which seems a big restriction in practice.

Future work includes studying (simultaneous) unboundedness for classes other than preT-PPNs that is to say postT-PPNs and distinctT-PPNs. Most problems (such as universal simultaneous unboundedness for postT-PPNs and distincT-PPNs) can be settled easily by adapting the proofs of [7], except for existential simultaneous unboundedness for postT-PPNs. The translation to ωPNs proposed here is not sufficient to conclude its decidability either since we have to ensure that the increasing markings are all reached for a common parameter valuation.

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