Coordinated Motion Planning: The Video

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Abstract

We motivate, visualize and demonstrate recent work for minimizing the total execution time of a coordinated, *parallel* motion plan for a swarm of N robots in the absence of obstacles. Under relatively mild assumptions on the separability of robots, the algorithm achieves *constant stretch*: If all robots want to move at most d units from their respective starting positions, then the total duration of the overall schedule (and hence the distance traveled by each robot) is $\mathcal{O}(d)$ steps; this implies constant-factor approximation for the optimization problem. Also mentioned is an NP-hardness result for finding an optimal schedule, even in the case in which robot positions are restricted to a regular grid. On the other hand, we show that for densely packed disks that cannot be well separated, a stretch factor $\Omega(N^{1/4})$ is required in the worst case; we establish an achievable stretch factor of $O(N^{1/2})$ even in this case. We also sketch geometric difficulties of computing optimal trajectories, even for just two unit disks.

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1 Introduction

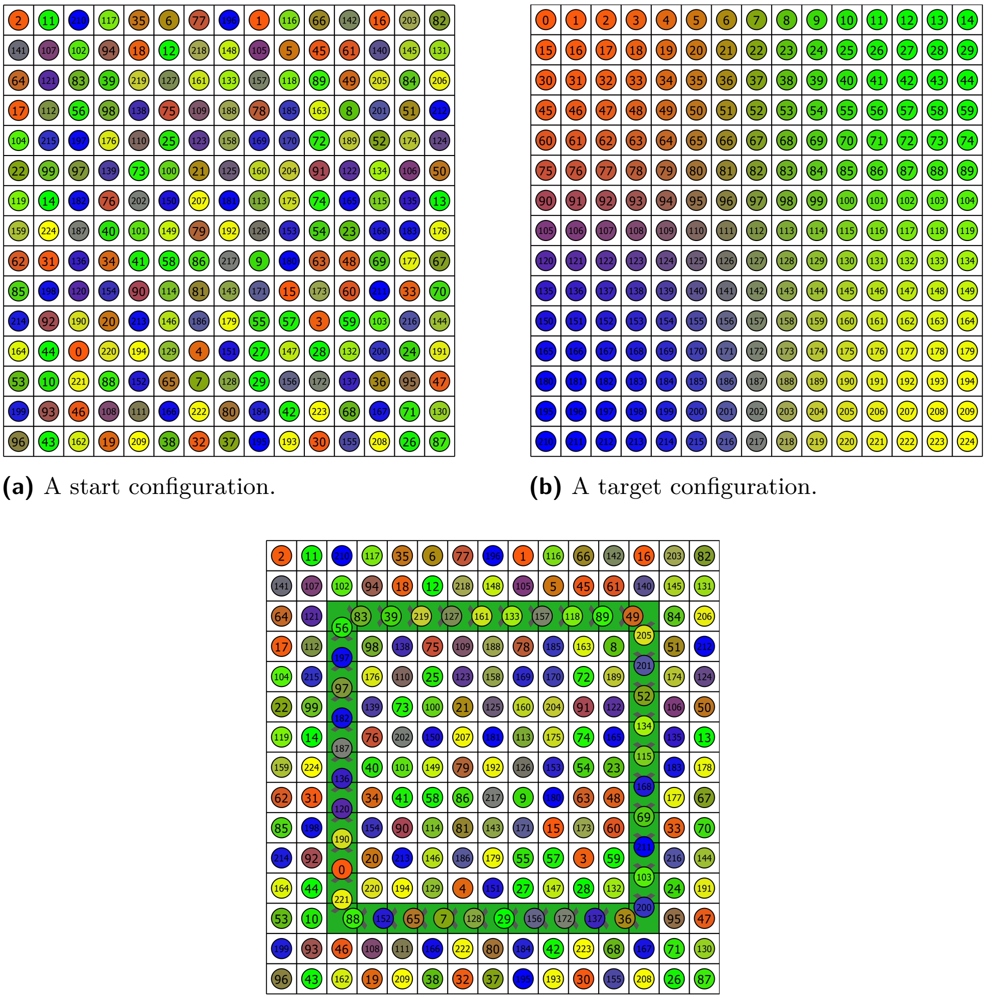
From the early days of computational geometry, robot motion planning has received a large amount of algorithmic attention. In the groundbreaking work by Schwartz and Sharir [2] from the 1980s, one of the challenges was coordinating the motion of *several* disk-shaped objects among obstacles. Their algorithms run in time polynomial in the complexity of the obstacles, but exponential in the number of disks, illustrating the significant challenge of coordinating many individual robots.

More recently, a growing spectrum of applications has increased the importance of multi-robot motion planning. However, previous work has largely focused on *sequential* schedules, in which one robot moves at a time, with objectives such as the number of moves; this differs from the practically important task of minimizing the overall makespan (i.e., the total time until completion) of a coordinated *parallel* motion schedule in which many robots are allowed to move simultaneously.

In a separate paper [1], we provide a number of breakthroughs for parallel motion planning.

- We show that it is strongly NP-complete to minimize the makespan for reconfiguring a system of labeled circular robots in a grid environment.
- We give an $\mathcal{O}(1)$-approximation for the long-standing open problems of parallel motion-planning with minimum makespan in a grid setting. This result is based on establishing an *absolute* performance guarantee: We prove that for any labeled arrangement of robots, there is always an overall schedule that gets each robot to its target destination with bounded *stretch*, i.e., within a constant factor of the largest individual distance.
- We extend our results to the scenario with continuous motion and arbitrary coordinates, provided the distance between a robot's start and target positions is at least one diameter. This implies that efficient multi-robot coordination is always possible under relatively mild separability conditions; this includes non-convex robots.
- For the continuous case with more densely packed objects, we establish a lower bound of $\Omega(N^{1/4})$ and an upper bound of $\mathcal{O}(\sqrt{N})$ on the achievable stretch.

See [1] for technical aspects and a more detailed discussion of related work. The purpose of the video submitted with this abstract is to explain and visualize the theoretical approach.



(a) A start configuration.

(b) A target configuration.

(c) A feasible reconfiguration step.

Figure 1 The base scenario of N robots placed in a rectangular grid.

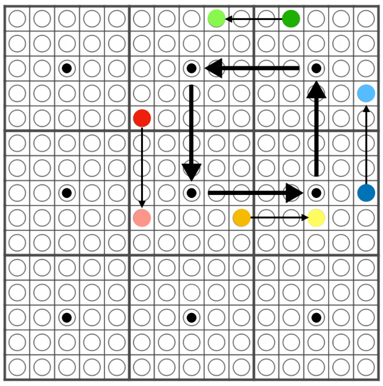
2 A base problem: labeled grid permutation

A fundamental scenario for multi-robot motion planning is the situation in which robots are placed at N grid positions (Fig. 1(a)); the desired target positions lie on the same grid, corresponding to a permutation of size N (Fig. 1(b)). In each step, a robot can move to an adjacent grid position if this is being vacated during the same step (Fig. 1(c)).

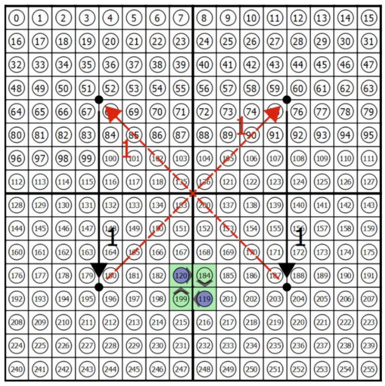
3 Complexity

We can show that even the base problem is NP-hard; see [1] for details.

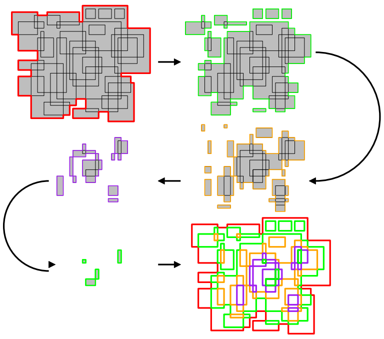
▶ **Theorem 1.** *The minimum makespan parallel motion planning problem on a grid is strongly NP-hard.*



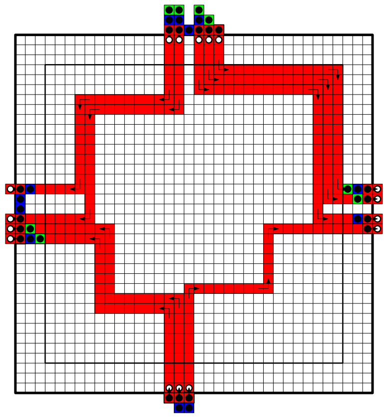
(a) Step 1: Computing the tiling T and the corresponding flow G_T.



(b) Step 2: Preprocessing G_T to remove intersecting and bidirectional edges.



(c) Step 3: Computing a partition into $\mathcal{O}(d)$ d-subflows.



(d) Step 4: Realizing the $\mathcal{O}(d)$ subflows using $\mathcal{O}(d)$ transformation steps.

Figure 2 The steps in Phase I.

4 Algorithmic description

The algorithm for achieving constant stretch in grid arrangements hinges on using local permutations for sorting the overall configuration. To achieve this in $\mathcal{O}(d)$ steps, the whole arrangement is subdivided into square tiles of size $\mathcal{O}(d) \times \mathcal{O}(d)$. In Phase I, all robots are moved to the tiles that contain their respective target positions, based on four steps that are based on flow techniques and illustrated in Fig. 2.

In Phase II, the disks within each tile are moved to their respective target positions in $\mathcal{O}(d)$ steps, based on local sorting methods. This results in an overall makespan of $\mathcal{O}(d)$.

5 Continuous motion

The result for the base case can be generalized to arbitrary geometric arrangements, provided that the start and target positions are sufficiently separated.

► **Theorem 2.** *If the distance between the centers of two robots of radius* 1 *in the start and target configurations is at least* $2\sqrt{2}$*, we can achieve a makespan in* $\mathcal{O}(d)$*, i.e., constant stretch, see Figure 3.*

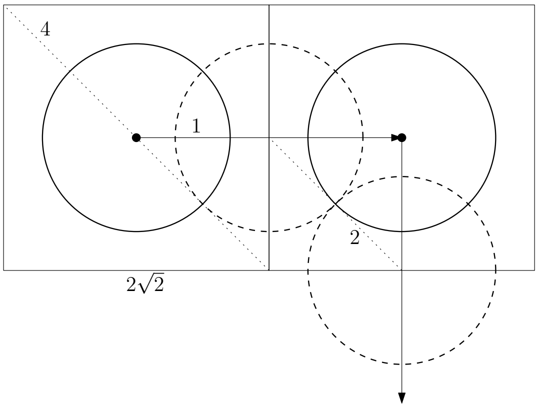
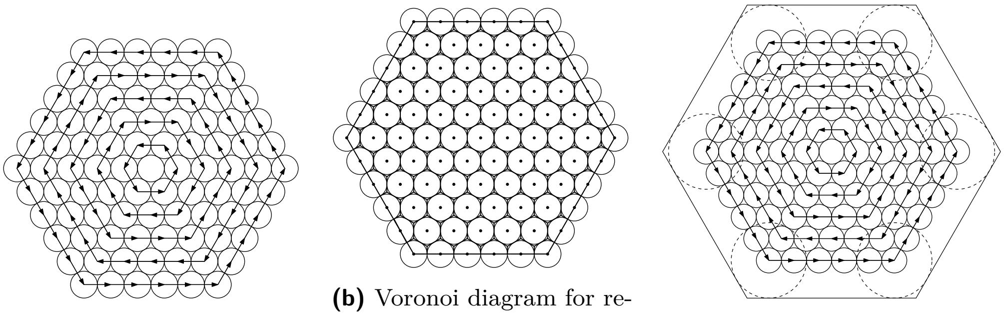


Figure 3 A mesh size of $2\sqrt{2}$ avoids robot collisions, and the cell diagonals have length 4. Note that robots may have arbitrary shape, as the separation argument applies to their circumcircles.



(a) Start and target positions of the robots.

(b) Voronoi diagram for restricted to the convex hull of the midpoints.

(c) Bounding polygon for the moving robots.

Figure 4 Lower-bound construction, with arrows pointing from start to target positions.

For more densely packed arrangements of disks, no constant stretch can be achieved.

▶ **Theorem 3.** *There is an instance with optimal makespan in $\Omega(N^{1/4})$, see Figure 4.*

Conversely, there is a non-trivial upper bound on the stretch, as follows.

▶ **Theorem 4.** *For a set of unit disks at arbitrary start and target positions there is a trajectory set with continuous makespan of $\mathcal{O}(d + \sqrt{N})$, implying a $\mathcal{O}(\sqrt{N})$-approximation algorithm.*

6 The video

The video opens with a number of practical illustrations for parallel motion planning, followed by a discussion of the work by Schwartz and Sharir, the underlying problem description and our complexity result. The main part focuses on the algorithm for achieving constant stretch, visualizing the individual steps of Phase I and Phase II. Finally, the basic aspects for continuous motion are sketched: achieving constant stretch for separable arrangements, as well as the lower and upper bounds for densely packed disks. In the very end, the geometric difficulty of finding optimal trajectories are shown, even for just two disks.

References

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2 Jacob T. Schwartz and Micha Sharir. On the piano movers' problem: III. Coordinating the motion of several independent bodies: the special case of circular bodies moving amidst polygonal barriers. *Int. J. Robotics Research*, 2(3):46–75, 1983.