

# Modeling Road Traffic Takes Time

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## Abstract

To model dynamic road traffic environment, it is imperative to integrate spatial and temporal knowledge about its evolution into a single model. This paper introduces temporal dimension which provides a method to reason about time-varying spatial information in a spatio-temporal graph-based model. Two types of evolution, topological and attributed, of time-varying graph (TVG) are considered which require the time domain to be discrete and/or continuous, and the TVG proposed includes time-varying node/edge presence and labeling functions. Theoretical concepts presented in this paper will guide us through the process of application development in future.

**2012 ACM Subject Classification** Information systems → Spatial-temporal systems, Computing methodologies → Modeling methodologies, Mathematics of computing → Graph theory

**Keywords and phrases** Qualitative Spatio-temporal Model, Time Varying Graph, Road Traffic, Intelligent Transportation Systems

**Digital Object Identifier** 10.4230/LIPIcs.GIScience.2018.52

**Category** Short Paper

**Acknowledgements** This work takes part in the DAISI project. This project has been funded with the support from the European Union with the European Regional Development Fund (ERDF) and from the Regional Council of Normandy.

## 1 Introduction

Road traffic evolves in space-time. This evolution is made explicit by time-varying spatial relations between different objects like vehicles, pedestrians, buildings etc. which directly affect the flow of traffic in an urban environment. To extract useful information from the movement of traffic, we need to model it in a reasoning system. To this effect, we proposed a spatial model in [7]. It includes different physical objects present in an urban area and, using quantitative information, aims to extract qualitative spatial knowledge which can enhance the robustness of Advanced Driver Assistance Systems (ADAS) currently in use. The model uses



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10th International Conference on Geographic Information Science (GIScience 2018).

Editors: Stephan Winter, Amy Griffin, and Monika Sester; Article No. 52; pp. 52:1–52:7

Leibniz International Proceedings in Informatics



LIPICs Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

graphs as data structure which abstract the real world information. Furthermore, dynamic phenomenon can be described using time-varying graphs.

In this paper, we propose a time formalization which provides a temporal dimension to the spatial model described in [7]. We define a time-varying graph (TVG) which models the change in its structure as well as in its node/edge attributes. The objective of this paper is to propose a theoretical formalization of TVG and link it with spatial graph proposed in our previous work.

The paper is organized as follows. Section 2 presents the related work. In Section 3, we describe, in brief, different spatial graphs. Then, in Section 4, we propose the formalization of time-varying graph and Section 5 concludes the paper along with future work.

## 2 Related Work

In this section, we will first mention some research related to modeling of road traffic and urban environment in Intelligent Transport Systems (ITS) domain. Then we will mention some techniques for including time and modeling time-varying graphs present in the literature.

Although there is a lot of research which exists for modeling road traffic environment for ITS applications, the one we would like to focus on and compare with our work is Local Dynamic Map (LDM) [4], a multi-level database, which has been standardized in Europe. It has been developed for cooperative systems and uses a four layered model to store data. Although we plan to use similar database architecture as in LDM, the main contribution of our model is data abstraction using graphs, which can initiate the use of graph algorithms to comprehend the evolution of road traffic.

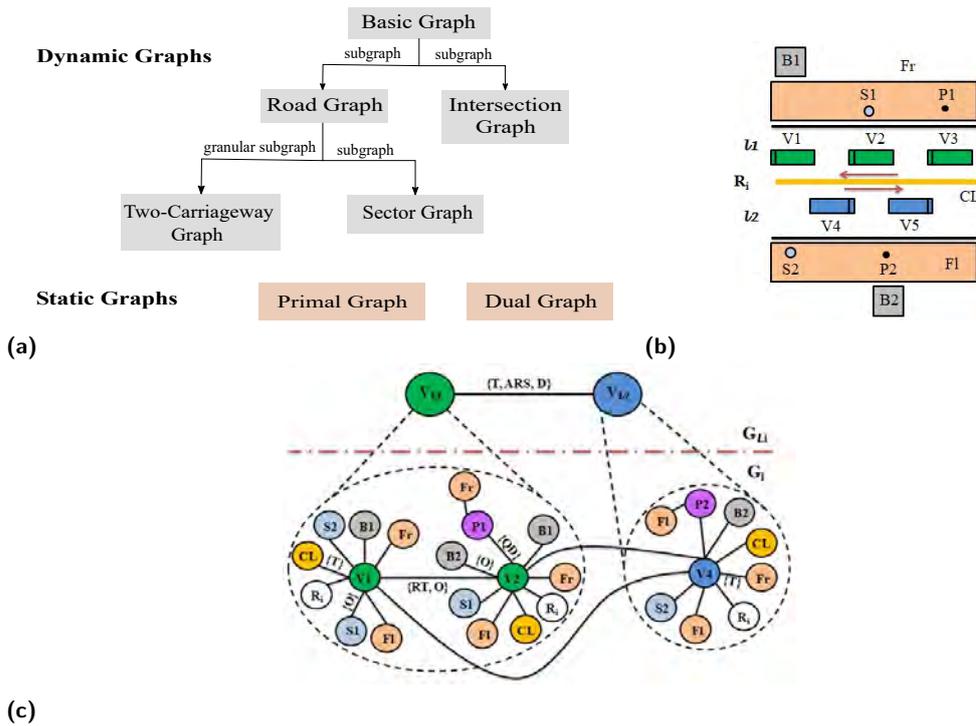
Time modeling has applications in various domains. It can be modeled using intervals or instants [5]. In our model, we consider time to consist of both. A lot of researchers have proposed different methods for time modeling and a survey of such methods is given in [8].

A time-varying phenomenon (like road traffic) can be modeled using time-varying graphs (TVG). Some models for TVGs are described in [3] and [9]. However, the model proposed in this paper is motivated from the one described in [2], as it is suitable for highly dynamic networks and it uses continuous time domain.

## 3 Spatial Graph Model

The qualitative spatial model we proposed in [7] is based on graph theory in which different objects are represented as nodes and qualitative spatial relations between those objects are included as edges. The objective is to have a spatio-temporal model which can help to understand the dynamics of road traffic in an urban environment from the perspective of evolving spatial relations between static and/or non-static objects.

Figure 1a shows the hierarchy of spatial graphs which are derived from Basic Graph  $G = (V, E)$  containing information about the urban environment at finest level of detail.  $V$  is the set of nodes which represent real-world objects and  $E$  is the set of edges which represent spatial relations between different nodes. From  $G$ , a Road Graph for each road segment in that environment is derived, which contains the spatial relations between objects present on that road segment.  $G$  also contains the relations between objects present at each intersection and hence an Intersection Graph (for each intersection) is derived from it. If a road segment is divided into bi-directional carriageways, Two-Carriageway Graph for that road segment is computed which gives information at a coarser level than Road Graph. If, on the other hand, it is divided into sectors, Sector Graph for each sector present on that road segment is



■ **Figure 1** (a) Hierarchy of graphs (b) An  $i$ th road segment divided into two carriageways (c) (Finer) Road graph ( $G_i$ ) and (coarser) Two-carriageway graph ( $G_{Li}$ ).

defined. Graphs which provide static information about the road network, primal and dual graphs, are also included in the model.

Road graph in Figure 1c includes different objects (vehicles, buildings, pedestrians, road markings, vertical structures, roadsides), present on the road segment in Figure 1b, as nodes with its edges representing spatial relations [7]. The nodes of two-carriageway graph represent groups of vehicles moving in opposite directions (for bi-directional road segment).

## 4 Time and Temporal Graph Model

Let us now add a temporal dimension to the proposed spatial model to theorize about dynamic aspects of the road traffic. In this section, we will first describe the structure of the temporal domain along with the temporal primitives considered. Then we will move onto the formalization of the temporal graph model, which is the main contribution of this paper.

### 4.1 Structure of Time

Before diving into the formalization of temporal graph, some characteristics of time [8] need to be clarified. We assume time to be linear, dense and positively unbounded  $[0, \infty[$  (we use  $]$   $]$  notation to represent a closed interval and  $]$   $[$  for open). Consider a time domain  $(\mathbb{T}, \leq)$  which is a set of totally ordered time points with order relation  $\leq$ . Since it is dense, the domain of  $\mathbb{T}$  is  $\mathbb{R}_{>0}$  (set of positive real numbers) and  $\exists t_j \mid t_i < t_j < t_k, \forall t_i, t_j, t_k \in \mathbb{T}, i, j, k \in \mathbb{R}_{>0}$ . That means that there is always a time point between two adjacent time points (digitization of such time points could give different results depending on temporal granularity considered). This time domain has two time primitives: instants and intervals, and we assume that intervals are

bounded by instants [1]. For clarification, term "time instants" is used to represent individual points on a discrete time line whereas term "time points" is used to represent individual points on a continuous time line. We define  $\mathcal{T} \subset \mathbb{T}$  as the time during which our model is functional and we call it the lifetime of the model, which is bounded. We consider that  $\mathcal{T}$  can represent discrete as well as continuous time. In the former case, the domain of  $\mathcal{T}$  is  $\mathbb{Z}_{>0}$  (set of positive integers) and it consists of discrete time instants and intervals. For continuous time,  $\mathcal{T}$  belongs to  $\mathbb{R}_{>0}$  and consists of continuous time intervals. A closed non-zero duration time interval is given as the pair  $[t_{start}, t_{end}] \mid t_{start} < t_{end}, t_{start}, t_{end} \in \mathcal{T}$ , where  $t_{start}$  and  $t_{end}$  are zero duration instants which bind the interval.

## 4.2 Temporal Graph Model

Our model of time varying graph (or TVG) is motivated from [2]. We include node/edge presence functions and define labeling functions. We consider the evolution of the graph in terms of change in its structure (or topology), called "topological evolution", and change in the value of node/edge attributes given as labels, called "attributed evolution". We define a TVG as  $\mathcal{G} = (V, E, \mathcal{T}, \rho_V(\mathcal{T}), \rho_E(\mathcal{T}), A_V(\mathcal{T}), A_E(\mathcal{T}))$  where  $V$  and  $E$  are the sets of nodes and edges, respectively, included in the spatial graph described in Section 3,  $\mathcal{T} \subset \mathbb{T}$  is the lifetime of the model,  $\rho_V : V \times \mathcal{T} \rightarrow \{0, 1\}$  is the time-varying node presence function,  $\rho_E : E \times \mathcal{T} \rightarrow \{0, 1\}$  is the time-varying edge presence function,  $A_V$  is time-varying node labeling function and  $A_E$  is time-varying edge labeling function. For topological evolution,  $\mathcal{T}$  is discrete and for attributed evolution, it is continuous. In both types of evolution, the time at which the change occurs, is called characteristic date [2]. In topological evolution, the characteristic date is when a node or edge is added/removed in the graph. Similarly, in attributed evolution, attribute value changes at a characteristic date. Such dates defined within discrete/continuous lifetime  $\mathcal{T}$  provide an explicit way to model time when different kinds of changes occur.

### 4.2.1 Time-varying Node Labels

In our previous work [6], we described nine classes into which real world objects, present in an urban environment, can be classified. These objects, along with groups of vehicles belonging to each carriageway on a bidirectional road segment, are included as nodes in our model. For simplification and homogeneity, we consider that the nodes representing groups of vehicles belong to a separate class called "Group". Each (of now ten) class of nodes, given by the set of classes  $C_V = \{c_1, c_2, \dots, c_{10}\}$ , is assumed to have a unique set of attributes during  $\mathcal{T}$ , given by  $K_{c_i} = \{\kappa_1, \kappa_2, \dots, \kappa_m\}$ , where the number of attributes  $m$  varies for every  $c_i \in C_V$ ,  $1 \leq i \leq 10$ . As in [10], we assign an attribute vector  $[\kappa_1(v_i), \kappa_2(v_i), \dots, \kappa_m(v_i)]$  to  $i$ th node  $v_i \in V_{c_j}$ ,  $1 \leq j \leq 10$  with  $V_{c_j}$  being the set of nodes which belong to a class  $c_j \in C_V$ . An element of the attribute vector  $\kappa_x(v_i)$ ,  $1 \leq x \leq m$  is the value of the attribute  $\kappa_x$  for a node  $v_i$ . This attribute vector of a node  $v_i$  is considered to be the label for that node. Assume that classes for all nodes in  $\mathcal{G}$  and set of attributes  $K_{c_j}$  for all classes  $c_j \in C_V$ ,  $1 \leq j \leq 10$  are given as *a priori*. For a node  $v_i \in V_{c_j}$ , a node labeling function can be given as  $A_{v_i}(v_i, K_{c_j}) = [\kappa_1(v_i) \ \kappa_2(v_i) \ \dots \ \kappa_b(v_i)]_{1 \times b}$  where  $b = |K_{c_j}|$ . Considering

all nodes in a class  $c_j \in C_V$ , a node labeling function for this class is written as

$$A_{V_{c_j}}(V_{c_j}, K_{c_j}) = \begin{bmatrix} \kappa_1(v_1) & \kappa_2(v_1) & \dots & \kappa_b(v_1) \\ \kappa_1(v_2) & \kappa_2(v_2) & \dots & \kappa_b(v_2) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \kappa_1(v_a) & \kappa_2(v_a) & \dots & \kappa_b(v_a) \end{bmatrix}_{a \times b}$$

with  $a = |V_{c_j}|$ . When the time is included with  $A_{V_{c_j}}$ , we get a time-varying node labeling function  $A_{V_{c_j}}(V_{c_j}, K_{c_j}, \mathcal{T})$  for a class  $c_j \in C_V$ . This function will give as output a 3-D vector with dimensions  $|V_{c_j}| \times |K_{c_j}| \times |\mathcal{T}|$ . Since in case of attributed evolution  $\mathcal{T}$  is continuous, the value of  $|\mathcal{T}|$  will change according to the temporal granularity considered. Finally, we define a time-varying node labeling function  $A_V(\mathcal{T})$  which gives as output the value of all attributes for the nodes present in  $\mathcal{G}$  during  $\mathcal{T}$  in the form of 4-D vector, where fourth dimension has ten entries, one for each class.

## 4.2.2 Time-varying Edge Labels

To define the time varying edge labeling function, we classify an edge in TVG on the basis of the classes of nodes which are its end points. For example, an edge between a vehicle and a building is classified as Vehicle-Building edge. In [6], sets of relations for thirteen different classes of edges are proposed. Since we have defined class "Group" of nodes in the previous section, we classify edge between two group nodes into Group-Group class, and hence fourteen classes of edges are possible. The value of relations on these edges, given by a corresponding attribute vector, acts as the edge label. Given the set of node classes  $C_V = \{c_1, c_2, \dots, c_{10}\}$ , set of edge classes can be written as  $C_E = \{c_x c_y \mid c_x, c_y \in C_V, 1 \leq x \leq 10, 1 \leq y \leq 10\}$  and has fourteen elements. It is possible to have a class of type  $c_x c_x \in C_E$  provided  $c_x = Vehicle \vee c_x = Group \vee c_x = Intersection \vee c_x = Roadsegment$  since relations between two vehicles, groups, intersections and road segments are allowed. Each class  $c_p c_q \in C_E, 1 \leq p \leq 10, 1 \leq q \leq 10$  is made up of two classes  $c_p$  and  $c_q$  of nodes, between which an edge exists belonging to  $c_p c_q$ . For every  $c_p c_q \in C_E, R_{pq} = \{r_1, r_2, \dots, r_n\}$  is the set of relations corresponding to that class, where  $n$  depends on the class [6]. Given an edge  $e_{vv'} \in E, v, v' \in V$ , it belongs to class  $c_p c_q \in C_E \iff v \in V_{c_p} \wedge v' \in V_{c_q}, V_{c_p}, V_{c_q} \subset V$ . An edge labeling function for  $e_{vv'}$  given  $R_{pq}$  is written as  $A_{e_{vv'}}(e_{vv'}, R_{pq}) = [r_1(e_{vv'}) \ r_2(e_{vv'}) \ \dots \ r_d(e_{vv'})]_{1 \times d}$  where  $d = |R_{pq}|$ . The attribute vector, given as output, contains the values of different relations which exist on edge  $e_{vv'}$ . Let  $E_{c_p}^{c_q}$  represent the set of edges which belong to class  $c_p c_q \in C_E$ . Then edge labeling function for  $E_{c_p}^{c_q}$  gives a 2D vector as output

$$A_{E_{c_p}^{c_q}}(E_{c_p}^{c_q}, R_{pq}) = \begin{bmatrix} r_1(e_1) & r_2(e_1) & \dots & r_d(e_1) \\ r_1(e_2) & r_2(e_2) & \dots & r_d(e_2) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ r_1(e_c) & r_2(e_c) & \dots & r_d(e_c) \end{bmatrix}_{c \times d}$$

where  $c = |E_{c_p}^{c_q}|$ . Over  $\mathcal{T}$ , time varying edge labeling function  $A_{E_{c_p}^{c_q}}(E_{c_p}^{c_q}, R_{pq}, \mathcal{T})$  for a class  $c_p c_q \in C_E$  gives as output the 3-D vector with dimensions  $|E_{c_p}^{c_q}| \times |R_{pq}| \times |\mathcal{T}|$ . Considering all classes of edges in  $E$ , time varying labeling function  $A_E(\mathcal{T})$  has 4-D vector as output with fourth dimension related to the total number of edge classes (fourteen).

### 4.2.3 Graph Evolution

As mentioned before, the evolution of a TVG can be topological or attributed. The change in the graph topology is considered to be discrete in our model. That means, the addition/removal of a node/edge is instantaneous. In addition, we ignore the change in attribute values of nodes/edges and only focus on their presence/absence. Hence, the definition of TVG can be modified to  $\mathcal{G}_T = (V, E, \mathcal{T}, \rho_V(\mathcal{T}), \rho_E(\mathcal{T}))$ . However, in case of attributed evolution, the TVG is formalized as  $\mathcal{G}_A = (V, E, \mathcal{T}, A_V(\mathcal{T}), A_E(\mathcal{T}))$  with  $\mathcal{T}$  representing continuous time, which can be discretized if variation in an attribute value is instantaneous. In this case, change in graph topology is ignored. It is noteworthy, that both types of evolution happen simultaneously and hence, the lifetime of the system is the same in both cases.

## 5 Conclusion and Future Work

In this paper, we proposed the formalization of a time-varying graph (TVG) which provides the temporal dimension to the spatio-temporal graph-based model we are developing, to understand the dynamics of road traffic in a given urban environment. Two types of graph evolution are considered and node/edge presence and labeling functions are defined. Due to limited number of pages, we skip the description of related concept of underlying graph, a static graph which relates spatial and time-varying graphs, and the notion of defining different point of views for visualizing the evolution of TVG. The next step for our work is to define the conceptual framework of the system and implement the ideas proposed. To do this, we first need to compare the existing spatio-temporal data models (conceptual and physical) and adapt one to our needs. The required real-world traffic data is collected by CEREMA, Rouen (France). Our long-term goal is to develop graph algorithms to compute and reason about patterns in evolving road traffic.

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