Coalgebraic Theory of Büchi and Parity Automata: Fixed-Point Specifications, Categorically

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Abstract

Coalgebra is a categorical modeling of state-based dynamics. Final coalgebras – as categorical greatest fixed points – play a central role in the theory; somewhat analogously, most coalgebraic proof techniques have been devoted to *greatest* fixed-point properties such as safety and bisimilarity. In this tutorial, I introduce our recent coalgebraic framework that accommodates those fixed-point specifications which are not necessarily the greatest. It does so specifically by characterizing the accepted languages of *Büchi* and *parity* automata in categorical terms. We present two characterizations of accepted languages. The proof for their coincidence offers a unique categorical perspective of the correspondence between (logical) fixed-point specifications and the (combinatorial) parity acceptance condition.

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Studies of automata, and state-based transition systems in general, have been shed a fresh categorical light in the 1990s by the theory of *coalgebra* [7, 5]. In the theory, a state-based dynamics is modeled by a coalgebra, that is, an arrow $c\colon X \to FX$ in a category $\mathbb{C}$; and this simple modeling has produced numerous results that capture mathematical essences and provide general techniques.

Final coalgebras as "categorical greatest fixed points" play a central role in the theory of coalgebra. Somewhat analogously, most coalgebraic proof methods have focused on greatest fixed-point properties – a notable example being a span-based categorical characterization of *bisimilarity*.

In this tutorial, I will outline our recent results [10, 8] about how we can accommodate, in the theory of coalgebra, those fixed-point properties which are not necessarily the greatest. This takes the concrete form of characterizing the accepted languages of *Büchi* and *parity* automata in the language of category theory. Our framework, based on the so-called *Kleisli* approach to coalgebraic trace semantics [6, 4, 2, 1], is generic and covers both automata with nondeterministic and probabilistic branching. It covers both word and tree automata, too.

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We present two characterizations of the accepted languages of Büchi and parity automata. The first one is called *logical* fixed points; it is formulated in terms of the order-enriched structure of the underlying Kleisli category (where the monad in question models branching type) [10]. The second one, called *categorical* fixed points, utilizes nested datatypes specified by a functor. The latter resembles repeated application of (co)free (co)monads. We exhibit a proof for the coincidence of the two characterizations. What arises through it is a categorical perspective of one of the key observations that underpin the recent developments in computer science – namely the fact that the *combinatorial* notion of parity acceptance condition represents *logical* specifications given by nested and alternating fixed points.

The tutorial is based on the speaker's joint works with Corina Cîrstea, Bart Jacobs, Shunsuke Shimizu, Ana Sokolova, and Natsuki Urabe [2, 3, 8, 10]. A detailed account of the technical material of the tutorial will be given in a forthcoming paper [9].

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