

Crystal Structure Prediction via Oblivious Local Search

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Abstract

We study Crystal Structure Prediction, one of the major problems in computational chemistry. This is essentially a continuous optimization problem, where many different, simple and sophisticated, methods have been proposed and applied. The simple searching techniques are easy to understand, usually easy to implement, but they can be slow in practice. On the other hand, the more sophisticated approaches perform well in general, however almost all of them have a large number of parameters that require fine tuning and, in the majority of the cases, chemical expertise is needed in order to properly set them up. In addition, due to the chemical expertise involved in the parameter-tuning, these approaches can be *biased* towards previously-known crystal structures. Our contribution is twofold. Firstly, we formalize the Crystal Structure Prediction problem, alongside several other intermediate problems, from a theoretical computer science perspective. Secondly, we propose an oblivious algorithm for Crystal Structure Prediction that is based on local search. Oblivious means that our algorithm requires minimal knowledge about the composition we are trying to compute a crystal structure for. In addition, our algorithm can be used as an intermediate step by *any* method. Our experiments show that our algorithms outperform the standard basin hopping, a well studied algorithm for the problem.

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1 Introduction

The discovery of new materials has historically been made by experimental investigation guided by chemical understanding. This approach can be both time consuming and challenging because of the large space to be explored. For example, a “traditional” method for discovering inorganic solid structures relies on knowledge of material chemistry coupled with repeating synthesis experiments and systematically varying elemental ratios, each of which can take lots of time [25, 26]. As a result there is a very large unexplored space of chemical systems: only 72% of binary systems, 16% of ternary, and just 0.6% of quaternary systems have been studied experimentally [27].

These inefficiencies forced physical scientists to develop computational approaches in order to tackle the problem of finding new materials. The first approach is based on data mining where *only* pre-existing knowledge is used [7, 11, 13, 15, 23]. Although this approach has proven to be successful, there is the underlying risk of missing best-in-class materials by being biased towards *known* crystal structures. Hence, the second approach tries to fill this gap and aims at finding new materials with *little, or no*, pre-existing knowledge, by *predicting the crystal structure* of the material. This approach has led to the discovery of several new, counterintuitive, materials whose existence could not be deduced by the structures of previously-known materials [6].

Several heuristic methods have been suggested for crystal structure prediction. All these methods are based on the same fundamental principle. Every arrangement of ions in the 3-dimensional Euclidean space corresponds to an energy value and it defines a point on the *potential energy surface*. Then, the crystal structure prediction problem is formulated as a *mathematical optimization* problem where the goal is to compute the structure that corresponds to the global minimum of the potential energy surface, since this is the most likely structure that corresponds to a stable material. The difficulties in solving this optimization problem is that the potential energy surface is *highly non convex*, with *exponentially many*, with respect to the number of ions, local minima [17]. For this reason, several different algorithmic techniques were proposed ranging from simple techniques, like *quasi-random sampling* [9, 21, 20, 22], *basin hopping* [12, 28], and *simulated annealing* techniques [18, 24], to more sophisticated techniques, like *evolutionary and genetic algorithms* [4, 8, 14, 16, 30], and *tiling* approaches [6, 5]. A recent comprehensive review on these techniques can be found in [17].

The simple searching techniques are easy to understand, usually easy to implement, and they are *unbiased*, but they can be slow in practice. On the other hand, the more sophisticated approaches perform well in general, however almost all of them have a large number of parameters that require fine tuning and, in the majority of the cases, chemical expertise is needed in order to properly set them up. In addition, due to the chemical expertise involved in the parameter-tuning, these approaches can be *biased* towards previously-known structures.

The majority of the abovementioned heuristic techniques work, at a very high level, in a similar way. Given a current solution x for the crystal structure prediction problem, i.e., a location for every ion in the 3-dimensional space, they iteratively perform the following three steps.

1. Choose a new potential solution x' . This can be done by taking into account, or modifying, x .
2. Perform gradient descent on the potential energy surface starting from x' , until a local minimum is found. This process is called *relaxation* of x' .
3. Decide whether to keep x as the candidate solution or to update it to the solution found after relaxing x' .

For example, basin hopping algorithms randomly choose x' , they relax x' and if the energy of the relaxed structure is lower than the x , or a Metropolis criterion is satisfied, they accept this as a current solution; else they keep x and they randomly choose x'' . The procedure usually stops when the algorithm fails to find a structure with lower energy within a predefined number of iterations. The more sophisticated algorithms take into account knowledge harvested from chemists and put constraints on the way x' is selected. For example, the MC-EMMA [6] and the FUSE [5] algorithms use a set of building blocks to construct x' . These building blocks are local configurations of ions that are present in, or similar to, known crystal structures. These approaches restrict the search space, which accelerate search, but reduce the number of possible solutions.

This general algorithm is easy to understand, however there are some hidden difficulties that make the problem more challenging. Firstly, it is not trivial even how to *evaluate* the potential energy of a structure. There are several different methods for calculating the energy of a structure, ranging from *quantum mechanical* methods, like *density functional theory*¹, to *force fields* methods², like the *Buckingham-Coulomb* potential function. All of which though, are hard to compute (see Section 2.1) from the point of view of (theoretical) computer science and thus only numerical methods are known and used in practice for them [10]; still there are cases where some methods need considerable time to calculate the energy of a structure. This yields another, more important, difficulty, the relaxation of a structure. Since it is hard to compute the energy of a structure, it is even harder to apply gradient descent on the potential energy surface. For these reasons, the majority of the heuristic algorithms depend on *external*, well established, codes [10] for computing the aforementioned quantities. Put differently, both energy computations and relaxations of structures are treated as *oracles* or *black boxes*.

1.1 Our contribution

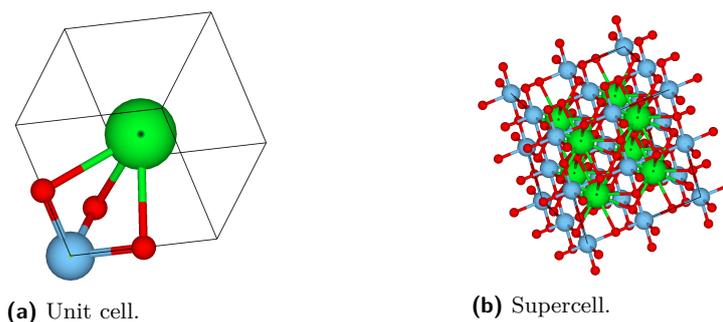
Our contribution is twofold. Firstly, we formalize the Crystal Structure Prediction problem from the theoretical computer science perspective; to the best of our knowledge, this is among the few papers that attempt to connect computational chemistry and computer science. En route to this, we introduce several intermediate open problems from computational chemistry in CS terms. Any (partial) positive solution to these questions can significantly help computational chemists to identify new materials. On the other hand, any negative result can formally explain why the discovery of new materials is a notoriously difficult task.

Our second contribution is the partial answer for some of the questions we cast. In general, our goal is to create *oblivious* algorithms that are easy to implement, they are fast, and they work well in practice. With oblivious we mean that we are seeking for *general procedures* that require *minimal input* and they have zero, or just a few, parameters chosen by the user.

- We propose a purely combinatorial method for estimating the energy of a structure, which we term *depth energy computation*. We choose to compare our method against GULP [10], which is considered to be the state of the art for computing the energy of a structure and for performing relaxations when the Buckingham-Coulomb energy is used. Our method requires only the charges of the atoms and their corresponding Buckingham coefficients to work; see Eq. 2 in Section 2.1. In addition, it needs only one parameter, the

¹ https://en.wikipedia.org/wiki/Density_functional_theory

² [https://en.wikipedia.org/wiki/Force_field_\(chemistry\)](https://en.wikipedia.org/wiki/Force_field_(chemistry))



■ **Figure 1** Most stable configuration of SrTiO_3 .

- depth k . We experimentally demonstrate that our method monotonically approximates with respect to k the energy computed by GULP and that it achieves an error of 0.0032 for $k = 6$. Our experiments show that the structure that achieves the minimum energy in depth 1 is likely to be the structure with the minimum energy overall. In fact we show something much stronger. If the energy of x is lower than the energy of x' when it is computed via the depth energy computation for $k = 1$, then, almost always, the energy of x will be lower than the energy of x' when it is computed via GULP.
- We derive oblivious algorithms for choosing which structure to relax next. All of our algorithms are based on local search. More formally, starting with x and using only local changes we select x' . We define several “combinatorial neighborhoods” and we evaluate their efficiency. Our neighborhoods are oblivious since they only need access to an oracle that calculates the energy of a structure. We show that our method outperforms basin hopping. Moreover, we view our algorithms as an intermediate step before relaxation that can be applied to *any* existing algorithm.

2 Preliminaries

A *crystal* is a solid material whose atoms are arranged in a highly ordered configuration, forming a *crystal structure* that extends in all directions. A crystal structure is characterized by its *unit cell*; a parallelepiped that contains atoms in a specific arrangement. The unit cell is the *period* of the crystal; unit cells are stacked in the three dimensional space to form the crystal. In this paper we focus on *ionically bonded crystals*, which we describe next; what follows is relevant only on crystals of this type. In order to fully define the unit cell of a ionically bonded crystal structure, we have to specify a *composition*, *unit cell parameters*, and an *arrangement* of the ions.

Composition. A composition is the chemical formula that describes the ratio of ions that belong to the unit cell. The chemical formula contains *anions*, negatively charged ions, and *cations*, positively charged ions. The chemical formula is a way of presenting information about the *chemical proportions* of ions that constitute a particular chemical compound, and it does not provide any information about the exact number of atoms in the unit cell. More formally, the composition is defined by a set of distinct chemical elements $\{e_1, e_2, \dots, e_m\}$, their multiplicity n_i , and a non-zero integer charge q_i for each element i . The number m denotes the total number of distinct chemical elements, and $n_i / \sum_{j=1}^m n_j$ is the proportion of the atoms of type e_i in the unit cell. It is required that the sum of the charges adds up to zero, i.e. $\sum_{i=1}^m q_i n_i = 0$, so that the unit cell is charge neutral. For example, the

composition for Strontium Titanate, SrTiO_3 , denotes that the following hold. For every ion of strontium (Sr) in the unit cell, there exists one ion of titanium (Ti) and three ions of oxygen (O). Furthermore, the charge of every ion of Sr is +2, of Ti is +4, and of O is -2. Hence, when the ratios of the ions are according to the composition, the charge of the unit cell is zero.

Another parameter for every atom is the *atomic radius*. This usually corresponds to the distance from the center of the nucleus to the boundary of the surrounding shells of electrons. Since the boundary is not a well-defined physical entity, there are various non-equivalent definitions of atomic radius. In crystal structures though, the *ionic radius* is used and usually is treated as a hard sphere. Thus, we will use ρ_i to denote the ionic radius of the element e_i .

Unit cell parameters. Unit cell parameters provide a formal description of the parallelepiped that represents the unit cell. These include the lengths y_1, y_2, y_3 of the parallelepiped in every dimension and the angles $\theta_{12}, \theta_{13},$ and θ_{23} between the corresponding facets. For brevity, we denote $y = (y_1, y_2, y_3)$ and $\theta = (\theta_{12}, \theta_{13}, \theta_{23})$, and we use (y, θ) to denote the unit cell parameters.

Arrangement. An arrangement describes the position of each atom of the composition in the unit cell. The position of ion i is specified by a point $x_i = (x_{i1}, x_{i2}, x_{i3})$ in the parallelepiped defined by the unit cell parameters; fractional coordinates x_i denote the location of the nucleus of the ion i in the unit cell. A unit cell parameters-arrangement combination (y, θ, x) in a unit cell with n ions is a point in the $3n + 6$ -dimensional space. For any two points x_i and x_j we will use $d(x_i, x_j)$ to denote their Euclidean distance.

As we have already said, a unit cell parameters-arrangement configuration (y, θ, x) defines the period of an infinite structure that covers the whole 3d space. To get some intuition, assume that we have an orthogonal unit cell, i.e., all the angles are 90 degrees. Then for every ion with position (x_{i1}, x_{i2}, x_{i3}) in the unit cell, there exist “copies” of the ion in the positions $(k_1 \cdot y_1 + x_{i1}, k_2 \cdot y_2 + x_{i2}, k_3 \cdot y_3 + x_{i3})$ for every possible combination of integers $k_1, k_2,$ and k_3 . A unit cell parameters-arrangement configuration is *feasible* if the hard spheres of any two ions of the crystal structure do not overlap; formally, it is feasible if for every two ions i and j it holds that $d(x_i, x_j) \geq \rho_i + \rho_j$.

2.1 Energy

Any unit cell parameters-arrangement configuration of a composition corresponds to a *potential energy*. When the number of ions in the unit cell is fixed, the set of configurations define the *potential energy surface*.

Buckingham-Coulomb potential is among the most well adopted methods for computing energy [3, 29] and it is the sum of the Buckingham potential and the Coulomb potential. The Coulomb potential is *long-range* and depends only on the charges and the distance between the ions; for a pair of ions i and j , the Coulomb energy is defined by

$$CE(i, j) := \frac{q_i q_j}{d(x_i, x_j)}. \quad (1)$$

Note, ions i and j can be in *different* unit cells.

The Buckingham potential is *short-range* and depends on the species of the ions and their distance. More formally, it depends on positive composition-dependent constants

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A_{e_i, e_j} , B_{e_i, e_j} , and C_{e_i, e_j} for every pair of species e_i and e_j ; here i can be equal to j ³. So, for the pair of ions i , of specie e_i , and j , of specie e_j , the Buckingham energy is

$$BE(i, j) := A_{e_i, e_j} \cdot \exp(-B_{e_i, e_j} \cdot d(x_i, x_j)) - \frac{C_{e_i, e_j}}{d(x_i, x_j)^6}. \quad (2)$$

Again, ions i and j can be in different unit cells.

Let $S(x_i, \rho)$ denote the sphere with centre x_i and radius ρ . The total energy of a crystal structure whose unit cell is characterized by n ions with arrangement $x = (x_1, \dots, x_n)$ is then defined

$$E(y, \theta, x) = \lim_{\rho \rightarrow \infty} \sum_{i=1}^n \sum_{j \neq i, j \in S(x_i, \rho)} (BE(i, j) + CE(i, j)).$$

$E(y, \theta, x)$ *conditionally converges* to a certain value [19] and usually numerical approaches are used to compute it. For this reason, and since we aim for an oblivious algorithm, we view the computation of the energy of a structure as a *black box*. More specifically, we assume that we have an oracle that given any structure (y, θ, x) , it returns its corresponding energy.

► **Open Question 1.** *Given a composition and Buckingham parameters for it, find a simple, purely combinatorial way that approximates the energy for every crystal structure.*

► **Open Question 2.** *Given a composition $\{e_1, e_2, \dots, e_m\}$ and an oracle that computes the energy of every structure for this composition, learn efficiently (with respect to the number of oracle calls) the Buckingham parameters A_{e_i, e_j} , B_{e_i, e_j} , and C_{e_i, e_j} for every $i, j \in [m]$.*

Relaxation. The *relaxation* of a crystal structure (y, θ, x) computes a stationary point on the potential energy surface by applying gradient descent starting from (y, θ, x) . The relaxation of a structure can change *both* the arrangement x of the ions in the unit cell *and* the unit cell parameters (y, θ) of the unit cell. We follow a similar approach as we did with the energy and we assume that there is an oracle that given a crystal structure (y, θ, x) it returns the relaxed structure.

► **Open Question 3.** *Find an alternative, quicker, way to compute an approximate local minimum when:*

- a) *the unit cell parameters (y, θ) of the unit cell are fixed;*
- b) *the arrangement x of the ions is fixed;*
- c) *both unit cell parameters and arrangement are free.*

2.2 Crystal Structure Prediction problems

In crystal structure prediction problems the general goal is to minimize the energy in the unit cell. There are two kinds of problems we are concerned. The first cares only about the value of the energy and the second one cares for the arrangement and the unit cell parameters that achieve the minimum energy. From the computational chemistry point of view, both questions are interesting in their own right. The existence of a unit cell parameters-arrangement that

³ The Buckingham constants are composition-dependent since they can have small discrepancies in different compositions. For example the constants $A_{\text{Ti}, \text{O}}$, $B_{\text{Ti}, \text{O}}$, and $C_{\text{Ti}, \text{O}}$ for SrTiO_3 can be different than those for MgTiO_3 . There is a long line of research in computational chemistry that tries to learn/estimate the Buckingham constants for various compositions. In addition, more than one set of Buckingham constants can be available for a given composition.

achieves lower-than-currently-known energy usually suffices for constructing a new material. On the other hand, identifying the arrangement and the unit cell parameters of a crystal structure that achieves the lowest possible energy can help physical scientists to predict the properties of the material.

MINENERGY

Input: A composition with its corresponding Buckingham constants, a positive integer n , and a rational \hat{E} .

Question: Is there a crystal structure (y, θ, x) for the composition with n ions that is neutrally charged and achieves Buckingham-Coulomb energy $E(y, \theta, x) < \hat{E}$?

MINSTRUCTURE

Input: A composition with its corresponding Buckingham constants, a positive integer n .

Task: Find a crystal structure (y, θ, x) for the composition with n ions that is neutrally charged and the Buckingham-Coulomb energy $E(y, \theta, x)$ is minimized.

The second class of problems, the ones that ultimately computational chemists would like to solve, take as input only the composition and the goal is to construct a unit cell, with any number of atoms, such that the *average energy per ion* is minimized.

AVGENERGY

Input: A composition with its corresponding Buckingham constants and a rational \hat{E} .

Question: Is there a crystal structure for the composition that is neutrally charged and $\frac{E(y, \theta, x)}{n} < \hat{E}$?

AVGSTRUCTURE

Input: A composition with its corresponding Buckingham constants.

Task: Find a crystal structure for the composition that is neutrally charged and the average Buckingham-Coulomb energy per ion in the unit cell, $\frac{E(y, \theta, x)}{n}$, is minimized.

Although the problems are considered to be intractable [17], only recently the first *correct* NP-hardness result was proven for a variant of CSP [1]. However, for the problems presented, there are no correct NP-hardness results in the literature.

► **Open Question 4.** *Provide provable lower bounds and upper bounds for the four problems defined above.*

► **Open Question 5.** *Construct a heuristic algorithm that works well in practice.*

3 Local Search

Local search algorithms start from a feasible solution and iteratively obtain better solutions. The key concept for the success of such algorithms, is given a feasible solution, to be able to *efficiently find* an improved one. Put formally, a local search algorithm is defined by a *neighbourhood function* N and a *local rule* r . In every iteration, the algorithm does the following.

- Has the current best solution x .
- Computes the neighbourhood $N(x)$.
- If there is an improved solution x' in $N(x)$, then it updates x according to the rule r , i.e. $x' = r(N(x))$; else it terminates and outputs x .

The neighborhood $N(x)$ of a solution x consists of all feasible solutions that are “close” in some sense to x . The size of the neighborhood can be constant or a function of the input. In principle, the larger the size of the neighborhood, the better the quality of the locally optimal solutions. However, the downside of choosing large neighborhoods is that, in general, it makes each iteration computationally more expensive. Running time and quality of solutions are competing considerations, and the trade off between them can be determined through experimentation.

We study the following *combinatorial* neighborhoods for Crystal Structure Prediction. All of them keep the unit cell parameters fixed and change only the arrangement x of the n ions. Thus, for notation brevity, we define the neighbourhoods only with respect to the arrangement x .

1. **k -ion swap.** This neighborhood consists of all feasible arrangements that are produced by swapping the locations of k ions. The size of this neighbourhood is $O(n^k k!)$.
2. **k -swap.** This neighborhood is parameterized by a discretization step δ . Using δ we discretize the unit cell and then we perform swaps of k ions with the content of every point of the discretization. So, an ion can swap positions with another ion, or simply move to another vacant position. Again, we take into account only the feasible arrangements of the ions. The size of this neighbourhood is $O(n^k k! / \delta^{3k})$.
3. **Axes.** This neighborhood has a parameter δ and computes the following for every ion i . Firstly, for every dimension it computes a plane parallel to the corresponding facet of the unit cell and contains the ion i . The intersection of any pair of these planes defines an “axis”. Then, this axis is discretized according to δ . The neighborhood locates the ion to every point on the discretization on the three axes and we keep only the arrangements that are feasible. The size of this neighbourhood is $O(n/\delta)$.

In all of our neighborhoods, we are using a *greedy* rule to choose x' ; x' is an arrangement that achieves the minimum energy in $N(x)$.

4 Algorithms

We propose two algorithms. The first one is a step towards answering Open Question 1 while the second is a heuristic for MINSTRUCTURE problem.

For Open Question 1, we propose the *depth energy computation* for estimating the energy of a structure. Our algorithm has a single parameter, the depth parameter k , and works as follows. Given a crystal structure, it creates k layers around the unit cell with copies of the structure. So, for the unit cell parameters (y, θ) and the arrangement x of n atoms the energy is $E(y, \theta, x) = \sum_{i=1}^n \sum_{j \neq i, j \in D(k)} (BE(i, j) + CE(i, j))$, where $D(k)$ denotes the set of ions in the k layers of unit cells, and $BE(i, j)$ and $CE(i, j)$ are computed as in Equations 2 and 1 respectively.

For MINSTRUCTURE problem, we slightly modify basin hopping. In a step of basing hopping, a structure is randomly chosen and it is followed by a relaxation. Our algorithm applies a combinatorial local search using the Axes neighborhood, since this turned out to be the best among our heuristics, before the relaxation. So, we will perform a relaxation, *only after* combinatorial local search cannot further improve the solution. Our algorithm can be

used as a standalone one and it can also be integrated into *any other* heuristic algorithm for the Crystal Structure Prediction problem since it is oblivious. In addition, it provides a very fast criterion that when it succeeds it guarantees finding a lower energy crystal structure.

5 Experiments

In this section we evaluate our algorithms via experimental simulations. We first focus on SrTiO₃ which we use as a benchmark. We do this because it is a well studied composition for which the Crystal Structure Prediction problem is solved. We have implemented the algorithms in Python 2.7 and we use the Atomic Simulation Environment (<https://wiki.fysik.dtu.dk/ase/>) package for setting up, manipulating, running, visualizing and analyzing atomistic simulations. All experiments were performed on a 4-core Intel i7-4710MQ with 8GB of RAM.

■ **Table 1** Comparison between depth approach and GULP for SrTiO₃. Energy is in electronvolts (eV). The energy difference shows the average difference in energy between the depth approach and the energy calculated by GULP. Results averaged over 2000 random feasible structures.

k	Energy difference					
	1	2	3	4	5	6
15 atoms	0.0639	0.0226	0.0114	0.0068	0.0045	0.0032
20 atoms	0.0670	0.0238	0.0120	0.0072	0.0047	0.0033

We evaluate the depth energy computation in several different dimensions. For all the experiments we performed for energy computation, we fixed the unit cell to be cubic. Firstly, we evaluate how depth energy computation behaves with respect to k . We see that the method converges very fast and $k = 6$ already achieves accuracy of three decimal points. Then, we compare our depth approach against GULP; see Table 1. Our goal is to provide an intuitively simpler to interpret and work with method for computing the energy. Even though the energy calculated by the depth approach differs from the one calculated by GULP, we observe that the relative energies between two random arrangements remain usually the same even for $k = 1$. To be more precise, let $E_1(x)$ denote the energy of a feasible arrangement x when $k = 1$ and let $E_G(x)$ denote the energy of this arrangement as it is computed by GULP. Our experiments show that if for two random feasible arrangements x_1 and x_2 it holds that $E_1(x_1) < E_1(x_2)$, then $E_G(x_1) < E_G(x_2)$ for 99.8% of 1000 pairs of arrangements. This percentage reaches 100% for $k = 6$. For the “special” arrangement of ions x^* that minimizes the energy computed by GULP, that is $x^* = \operatorname{argmin} E_G(x)$, our experiments show that it is *always true* that $E_k(x^*) < E_k(x)$, for every $k = 1, \dots, 6$, where x is a random feasible structure over 10000 of them. So, this is a good indication that the arrangement that minimizes the energy for $k = 1$, also minimizes the energy overall. We view this as a striking result; it significantly simplifies the problem thus new, analytical, methods can be derived for the problem.

The next set of experiments compares the three neighborhoods described in Section 3 for SrTiO₃⁴. We compare them in several different dimensions: the average CPU time they need in order to find a local optimum with respect to their combinatorial neighborhood and the average drop in energy until they reach such a local optimum (Tables 2 and 4); the average CPU time the relaxation needs starting from such local minimum and the average drop in

⁴ The values of the Buckingham parameters can be found in the full version of the paper [2]

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energy from relaxation (Tables 3 and 5). In addition, for the case of SrTiO_3 , we compare how often we can find the optimal arrangement from a single structure. We observe that the Axes neighborhood has the best tradeoff between energy drop and CPU time. The 2-ion-swap neighbourhood outperforms the other two in terms of running time, however it seems to decrease the probability of finding the best arrangement when performing a relaxation on the resulting structures. This renders the use of 2-ion-swap neighbourhood inappropriate. Axes neighborhood is significantly faster and performs smoother in terms of running time than the 2-swap neighbourhood. However, the latter one performs better with respect to the energy drop, which is expected since axes is a subset of the 2-swap neighbourhood. In addition, the relaxation from the local minimum found by 2-swap significantly improves the probability of finding the best arrangement with only one relaxation. We should highlight that there exist structures where the relaxation cannot improve their energy, but the neighborhoods do; hence using Axes neighborhood we can escape from some local minima of the continuous space.

■ **Table 2** Comparison of local neighbourhoods for reaching a combinatorial minimum for SrTiO_3 with 15 atoms per unit cell and $\delta = 1\text{\AA}$ (375 grid points). Time is in seconds and energy in electronvolts (eV). Results averaged over 1000 arrangements.

Neighbourhood	Running time	Time stdev	Energy drop	Energy drop stdev
Axes	5.36	1.54	13.46	10.60
2-ion swap	0.96	0.33	7.75	8.45
2-swap	34.66	14.06	16.21	10.94

■ **Table 3** Evaluation of relaxation procedure after using a combinatorial neighborhood for SrTiO_3 with 15 atoms per unit cell. Time is in seconds and energy in electronvolts (eV). Results averaged over 1000 arrangements.

Neighbourhood	Running time	Time stdev	Energy drop	Energy drop stdev	Global minimum
Random structures-GULP	8.80	6.35	18.60	10.26	6.6%
Axes-GULP	7.92	6.16	5.53	2.28	10.0%
2-ion swap-GULP	8.82	6.25	11.09	5.64	4.7%
2-swap-GULP	5.14	5.08	2.79	1.05	14.8%

■ **Table 4** Comparison of local neighbourhoods for reaching a combinatorial minimum for $\text{Y}_2\text{Ti}_2\text{O}_7$ and $\delta = 1\text{\AA}$ (343 grid points). Time is in seconds and energy in electronvolts (eV). Results averaged over 1000 arrangements.

Neighbourhood	Running time	Time stdev	Energy drop	Energy drop stdev
Axes	7.56	2.08	8.23	3.71
2-ion swap	0.88	0.43	1.16	1.80
2-swap	27.93	9.98	10.26	4.05

Next, we compare our algorithm for MINSTRUCTURE against basin hopping where the next structure to relax is chosen at random. Based on the results of our previous experiments, we have chosen the Axes neighbourhood as an intermediate step before the relaxation. We have run these algorithms 200 times for SrTiO_3 with 15 atoms per unit cell, and 25 times for SrTiO_3 with 20 atoms per unit cell. We report how the energy varies with respect to

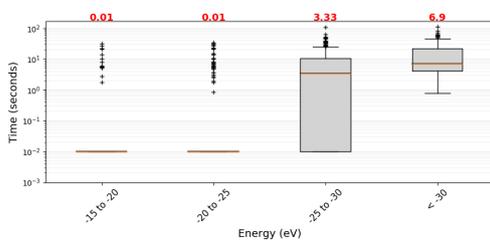
■ **Table 5** Evaluation of relaxation procedure after using a combinatorial neighborhood for $Y_2Ti_2O_7$. Time is in seconds and energy in electronvolts (eV). Results averaged over 1000 arrangements.

Neighbourhood	Running time	Time stdev	Energy drop	Energy drop stdev
Random structures-GULP	2.81	1.45	12.84	4.88
Axes-GULP	2.30	1.20	5.09	1.81
2-ion swap-GULP	2.47	2.25	12.29	4.38
2-swap-GULP	1.99	1.09	3.11	0.93

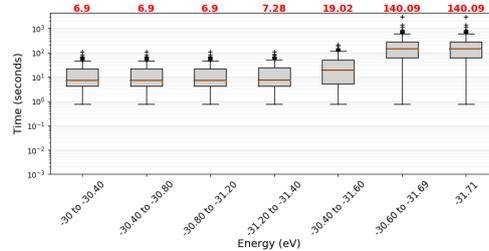
time until the best arrangement is found (Fig. 2) and we report other statistics that further validate our approach (Table 6). As we can see, it is relatively easy to reach low levels of energy and the majority of time is needed to find the absolute minimum. In addition, the overhead posed by the use of the neighbourhood search divided by the time needed by the basin hopping to find the global minimum decreases as the number of the atoms in the unit cell increases.

■ **Table 6** Statistics from the experiments depicted in Figure 2 ($SrTiO_3$ with 15 atoms per unit cell). The corresponding Figures for $SrTiO_3$ with 20 atoms per unit cell can be found in the full version of this paper [2].

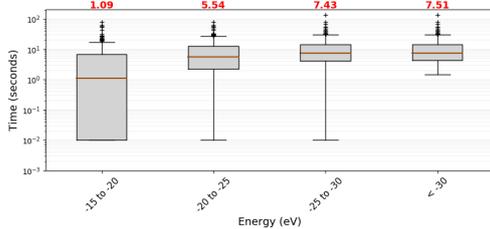
Algorithm	Number of atoms	Total time mean	Total time stdev	Relaxations	Time for relaxations	Time for local search
Axes-GULP	15	227.89	287.21	13.24	126.26	101.63
	20	2280.57	781.66	104.33	1016.72	1049.13
Basin hopping	15	167.89	114.89	18.14	160.79	—
	20	5766.20	4748.33	450.66	4895.60	—



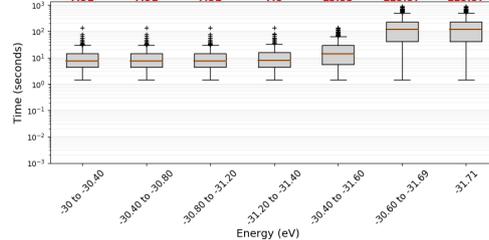
(a) Axes - GULP coarse.



(b) Axes - GULP fine.

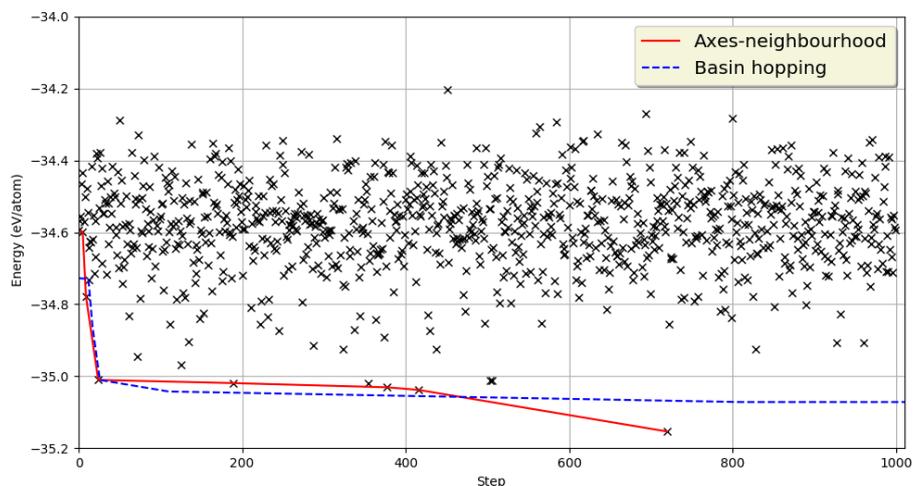


(c) GULP coarse.



(d) GULP fine.

■ **Figure 2** Time to reach specific energy levels for $SrTiO_3$ (15 atoms). Figures (a) and (b) correspond to the algorithm of Section 4. Figures (c) and (d) correspond to basin hopping. The median times needed to reach every energy level are depicted in red on the top of each plot.



■ **Figure 3** Performance of the Axes algorithm for $\text{Y}_2\text{Ti}_2\text{O}_7$. The black points correspond to the energy found after the relaxation of a point computed by the Axes neighborhood at each step. The red line is the lower envelope of the energy found by our algorithm while the blue line corresponds to the lower envelope of basin hopping.

In our last set of experiments, we compare our algorithm against basin hopping algorithm for $\text{Y}_2\text{Ti}_2\text{O}_7$ which contains 22 atoms in its primitive unit cell. In this set of experiments, we produced 3500 random structures. We simulated basin hopping by sequentially relaxing the constructed structures. However, none of the relaxations managed to find the optimal configuration. Our algorithm, using the same order of structures as before, first used the Axes neighborhood as an intermediate step followed by a relaxation; it managed to find the optimal configuration after visiting only 720 structures (Fig. 3).

6 Conclusions

In this paper we have introduced and studied the Crystal Structure Prediction problem through the lens of computer science. This is an important and very exciting problem in computational chemistry, which computer scientists are not actively studying yet. We have identified several open questions whose solution would have significant impact to the discovery of new materials. These problems are challenging and several different techniques and machineries from computer science could be applied for solving them. Our simple-to-understand algorithms are a first step towards their solution. We hope that our algorithms will be used as benchmarks in the future, since more sophisticated techniques for basin hopping can be invented. For the energy computation via the depth approach, we conjecture that the arrangement that minimizes the energy for $k = 1$ or $k = 2$, matches the arrangement that minimizes the energy when it is computed via GULP. Our numerical simulations provide significant evidence towards this. A formal result of this would greatly simplify the objective function of the optimization problem and it would give more hope to faster methods for relaxation. In addition, it could provide the foundations for new techniques for crystal structure prediction. Our algorithm that utilizes the Axes neighborhood as an intermediate step before relaxation, seems to speed up the time the standard basin hopping needs to find

the global minimum. Are there any other neighborhoods that outperform the Axes one? Can local search, or Axes neighborhood in particular, improve existing methods for crystal structure prediction by a simple integration as an intermediate step? We believe that this is indeed the case.

References

- 1 Duncan Adamson, Argyrios Deligkas, Vladimir V. Gusev, and Igor Potapov. On the hardness of energy minimisation for crystal structure prediction. In *SOFSEM 2020: Theory and Practice of Computer Science - 46th International Conference on Current Trends in Theory and Practice of Informatics, SOFSEM 2020, Limassol, Cyprus, January 20-24, 2020, Proceedings*, pages 587–596, 2020. doi:10.1007/978-3-030-38919-2_48.
- 2 Dmytro Antypov, Argyrios Deligkas, Vladimir Gusev, Matthew J Rosseinsky, Paul G Spirakis, and Michail Theofilatos. Crystal structure prediction via oblivious local search. *arXiv preprint arXiv:2003.12442*, 2020.
- 3 Richard A Buckingham. The classical equation of state of gaseous helium, neon and argon. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, 168(933):264–283, 1938.
- 4 Seth T Call, Dmitry Yu Zubarev, and Alexander I Boldyrev. Global minimum structure searches via particle swarm optimization. *Journal of computational chemistry*, 28(7):1177–1186, 2007.
- 5 C Collins, GR Darling, and MJ Rosseinsky. The flexible unit structure engine (fuse) for probe structure-based composition prediction. *Faraday discussions*, 211:117–131, 2018.
- 6 C Collins, MS Dyer, MJ Pitcher, GFS Whitehead, M Zanella, P Mandal, JB Claridge, GR Darling, and MJ Rosseinsky. Accelerated discovery of two crystal structure types in a complex inorganic phase field. *Nature*, 546(7657):280, 2017.
- 7 Stefano Curtarolo, Gus LW Hart, Marco Buongiorno Nardelli, Natalio Mingo, Stefano Sanvito, and Ohad Levy. The high-throughput highway to computational materials design. *Nature materials*, 12(3):191, 2013.
- 8 David M Deaven and Kai-Ming Ho. Molecular geometry optimization with a genetic algorithm. *Physical review letters*, 75(2):288, 1995.
- 9 CM Freeman, JM Newsam, SM Levine, and CRA Catlow. Inorganic crystal structure prediction using simplified potentials and experimental unit cells: application to the polymorphs of titanium dioxide. *Journal of Materials Chemistry*, 3(5):531–535, 1993.
- 10 Julian D Gale and Andrew L Rohl. The general utility lattice program (gulp). *Molecular Simulation*, 29(5):291–341, 2003.
- 11 Romain Gautier, Xiuwen Zhang, Linhua Hu, Liping Yu, Yuyuan Lin, Tor OL Sunde, Danbee Chon, Kenneth R Poeppelmeier, and Alex Zunger. Prediction and accelerated laboratory discovery of previously unknown 18-electron abx compounds. *Nature chemistry*, 7(4):308, 2015.
- 12 Stefan Goedecker. Minima hopping: An efficient search method for the global minimum of the potential energy surface of complex molecular systems. *The Journal of chemical physics*, 120(21):9911–9917, 2004.
- 13 Geoffroy Hautier, Chris Fischer, Virginie Ehrlacher, Anubhav Jain, and Gerbrand Ceder. Data mined ionic substitutions for the discovery of new compounds. *Inorganic chemistry*, 50(2):656–663, 2010.
- 14 David C Lonie and Eva Zurek. Xtalopt: An open-source evolutionary algorithm for crystal structure prediction. *Computer Physics Communications*, 182(2):372–387, 2011.
- 15 Nicola Nosengo. Can artificial intelligence create the next wonder material? *Nature News*, 533(7601):22, 2016.

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- 16 Artem R Oganov and Colin W Glass. Crystal structure prediction using ab initio evolutionary techniques: Principles and applications. *The Journal of chemical physics*, 124(24):244704, 2006.
- 17 Artem R Oganov, Chris J Pickard, Qiang Zhu, and Richard J Needs. Structure prediction drives materials discovery. *Nature Reviews Materials*, page 1, 2019.
- 18 J Pannetier, J Bassas-Alsina, J Rodriguez-Carvajal, and V Caignaert. Prediction of crystal structures from crystal chemistry rules by simulated annealing. *Nature*, 346(6282):343, 1990.
- 19 Chris J Pickard. Real-space pairwise electrostatic summation in a uniform neutralizing background. *Physical Review Materials*, 2(1):013806, 2018.
- 20 Chris J Pickard and RJ Needs. High-pressure phases of silane. *Physical Review Letters*, 97(4):045504, 2006.
- 21 Chris J Pickard and RJ Needs. Ab initio random structure searching. *Journal of Physics: Condensed Matter*, 23(5):053201, 2011.
- 22 Martin U Schmidt and Ulli Englert. Prediction of crystal structures. *Journal of the Chemical Society, Dalton Transactions*, 1996(10):2077–2082, 1996. doi:10.1039/DT9960002077.
- 23 J Christian Schön. How can databases assist with the prediction of chemical compounds? *Zeitschrift für anorganische und allgemeine Chemie*, 640(14):2717–2726, 2014.
- 24 J Christian Schön and Martin Jansen. First step towards planning of syntheses in solid-state chemistry: determination of promising structure candidates by global optimization. *Angewandte Chemie International Edition in English*, 35(12):1286–1304, 1996.
- 25 Theo Siegrist and Terrell A Vanderah. Combining magnets and dielectrics: Crystal chemistry in the ba₀- fe₂o₃- tio₂ system. *European Journal of Inorganic Chemistry*, 2003(8):1483–1501, 2003.
- 26 TA Vanderah, JM Loezos, and RS Roth. Magnetic dielectric oxides: subsolidus phase relations in the ba₀: Fe₂o₃: Tio₂ system. *Journal of Solid State Chemistry*, 121(1):38–50, 1996.
- 27 P Villars, K Brandenburg, M Berndt, S LeClair, A Jackson, Y-H Pao, B Igel'nik, M Oxley, B Bakshi, P Chen, et al. Binary, ternary and quaternary compound former/nonformer prediction via mendeleev number. *Journal of alloys and compounds*, 317:26–38, 2001.
- 28 David J Wales and Jonathan PK Doye. Global optimization by basin-hopping and the lowest energy structures of lennard-jones clusters containing up to 110 atoms. *The Journal of Physical Chemistry A*, 101(28):5111–5116, 1997.
- 29 Jearl D Walker. *Fundamentals of physics extended*. Wiley, 2010.
- 30 Yanchao Wang, Jian Lv, Li Zhu, and Yanming Ma. Crystal structure prediction via particle-swarm optimization. *Physical Review B*, 82(9):094116, 2010.