Brief Announcement: What Can(Not) Be Perfectly Rerouted Locally

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Abstract

In order to provide a high resilience and to react quickly to link failures, modern computer networks support fully decentralized flow rerouting, also known as local fast failover. In a nutshell, the task of a local fast failover algorithm is to *pre-define* fast failover rules for each node using locally available information only. Ideally, such a local fast failover algorithm provides a *perfect resilience* deterministically: a packet emitted from any source can reach any target, as long as the underlying network remains connected. Feigenbaum et al. showed [3] that it is not always possible to provide perfect resilience; on the positive side, the authors also presented an efficient algorithm which achieves at least 1-resilience, tolerating a single failure in any network.

Interestingly, not much more is known currently about the feasibility of perfect resilience. This brief announcement revisits perfect resilience with local fast failover, both in a model where the source can and cannot be used for forwarding decisions. By establishing a connection between graph minors and resilience, we prove that it is impossible to achieve perfect resilience on *any non-planar graph*; On the positive side, we can derive perfect resilience for outerplanar and some planar graphs.

2012 ACM Subject Classification Networks → Routing protocols; Computer systems organization → Dependable and fault-tolerant systems and networks; Theory of computation → Distributed algorithms

Keywords and phrases Resilience, Local Failover

Digital Object Identifier 10.4230/LIPIcs.DISC.2020.46

Related Version A full version of the paper is available at `https://arxiv.org/abs/2006.06513`.

Funding Research supported by WWTF project WHATIF, ICT19-045, 2020-2024.

Acknowledgements We would like to thank Jukka Suomela for several fruitful discussions.

1 Introduction

The dependability of distributed systems often critically depends on the underlying network, realized by a set of routers. To provide high availability, modern routers support local fast rerouting of flows: routers can be pre-configured with conditional failover rules which define,

34th International Symposium on Distributed Computing (DISC 2020).
Editor: Hagit Attiya; Article No. 46; pp. 46:1–46:3
Leibniz International Proceedings in Informatics
Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

Table 1 Summary of our results on perfect resilience for specific graph classes.

Graph class	Without source matching	With source matching
Outerplanar	Perfect resilience: Thm 1	Perfect resilience (see left)
K_4	Perfect resilience: Thm 2	Perfect resilience (see left)
Planar graphs	$\|V\| = 7$ counterexample: Thm 5	$\|V\| = 8$ counterexample: Thm 5
Non-planar graphs	Perfect res. impossible: Thm 4	Exact classification open

for each incoming port and desired target, to which port a packet arriving on this incoming port should be forwarded deterministically depending on the status of the *incident* links only: as routers need to react quickly, they do not have time to learn about remote failures.

This paper is motivated by the following fundamental question introduced by local fast rerouting mechanisms: *Is it possible to pre-define deterministic local failover rules which guarantee that packets reach their target, as long as the underlying network is connected?* This desired property is known as *perfect resilience*. The challenge of providing perfect resilience hence lies in the decentralized nature of the problem, where routers only have local information on failed links; achieving perfect resilience is straightforward with global knowledge.

Unfortunately, perfect resilience cannot be achieved in general: Feigenbaum et al. [3] presented an example with 12 nodes for which, after certain failures, no forwarding pattern on the original network allows each surviving node in the target's connected component to reach the target. Chiesa et al. [2] expanded on their result to require only two failures, but required over 30 nodes. On the positive side, Feigenbaum et al. showed that it is at least always possible to tolerate one link failure, i.e., to be 1-resilient. Interestingly, not much more is known today about when perfect resilience can be achieved, and when not. Our results are summarized in Table 1, with the full proofs and further results being available in [4].

2 Model

Let $G = (V, E)$ be a network represented by an undirected graph of nodes ("routers") V connected through undirected links E along which packets are exchanged. Initially, an arbitrary set $F \subset E$ of links fail (rendering them unusable in both directions). We study the class of local routing (forwarding) algorithms, in which every node $v \in V$ takes deterministic routing decisions based solely on the target t of the packet to route, the set of incident failed links $F \cap E(v)$, and the receiving or incoming port (*in-port*) of the packet at node i. Note that without knowledge of the in-port, already very simple failure scenarios prevent resilience. We also study the model where the forwarding may depend on a source node s. In particular, this implies that neither the state of the packet nor the state of the node can be changed, e.g., by header rewriting or using dynamic routing tables. We say that a forwarding pattern is *perfectly resilient* if it always succeeds in the connected component of the target.

3 Possibility of Perfect Resilience

For outerplanar graphs there exists a planar embedding such that all nodes are part of the outer face and this property holds also after arbitrary failures as long as the graph remains connected. Thus we can route along the links of the outer face of a planar graph using the well known right-hand rule [1] despite failures. Note that the face-routing pattern is even target-oblivious: starting on any node, it will visit every node. Moreover, this approach can also be applied for planar graphs if source and target are on the same face.

▶ **Theorem 1.** *Let $G = (V, E)$ be i) an outerplanar graph or ii) a planar graph where the packet starts on the same face as the destination. Then, there is a perfectly resilient forwarding pattern which does not require source matching.*

So far, we established that perfect resiliency is possible on outerplanar graphs as well as on the same face of planar graphs. This raises the question if perfect resilience is possible on some non-outerplanar planar graphs, which we answer in the affirmative for the clique K_4: we employ forwarding along a cyclic permutation, unless the destination is a neighbor.

▶ **Theorem 2.** *K_4 allows for perfectly resilient forwarding patterns without source matching.*

4 Impossibility of Perfect Resilience

We observe that a perfectly resilient algorithm is also perfectly resilient on subgraphs and contractions of its original graph, by a simulation argument. As subsetting and contracting are the two fundamental operations in the minor relationship, we deduce that the existence of a perfectly resilient algorithm on a graph G implies its existence on any minor of G.

▶ **Theorem 3.** *If G permits a perfectly resilient forwarding pattern, so do its minors, for both with and without source matching.*

We can hence prove that if a graph is not planar, it does not allow for perfect resilience without source matching, as both K_5 and $K_{3,3}$ do not allow perfect resilience: the latter can be shown by case distinction, where the routing cycles along nodes the three or four nodes no longer neighboring the destination. In contrast, for the K_4 after failures that leave the graph connected, at most two nodes are not direct neighbors of the destination.

▶ **Theorem 4.** *If G is not planar, then it is not perfectly resilient without source matching.*

We now show that there are planar graphs that do not permit perfect resiliency, using case distinction and further arguments in Figure 1. We can extend the result to include a source s, with slightly different argumentation, connected to the nodes 3 and 5:

▶ **Theorem 5.** *There exists a planar graph G on 7 nodes such that no forwarding pattern without source matching will succeed on G (8 nodes with source matching).*

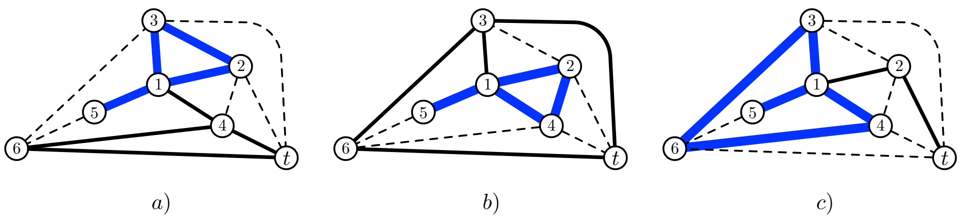

Figure 1 Planar graph with no perfectly resilient forwarding pattern. If the dashed links fail, a pattern that attempts to be perfectly resilient will be stuck in one of the loops shown in bold in the figures, depending on the routing at node 1, even though there is at least one remaining path.

References

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