

A Big Step from Finite to Infinite Computations

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Abstract

The known is finite, the unknown infinite
– Thomas Henry Huxley

The behaviour of programs can be described by the final *results* of computations, and/or their interactions with the context, also seen as *observations*. For instance, a function call can terminate and return a value, as well as have output effects during its execution.

Here, we deal with semantic definitions covering both results and observations. Often, such definitions are provided for *finite* computations only. Notably, in big-step style, infinite computations are simply not modelled, hence diverging and stuck terms are not distinguished. This becomes even more unsatisfactory if we have observations, since a non-terminating program may have significant infinite behaviour.

Recently, examples of big-step semantics modeling divergence have been provided [3, 4] by means of *generalized inference systems* [2, 5], which allow *corules* to control coinduction. Indeed, modeling infinite behaviour by a purely coinductive interpretation of big-step rules would lead to spurious results [6] and undetermined observation, whereas, by adding appropriate corules, we can correctly get divergence (∞) as the only result, and a uniquely determined observation. This approach has been adopted in [3, 4] to design big-step definitions including infinite behaviour for lambda-calculus and a simple imperative Java-like language. However, in such works the designer of the semantics is in charge of finding the appropriate corules, and this is a non-trivial task.

In this paper, we show a general construction that extends a given big-step semantics, modeling finite computations, to include infinite behaviour as well, notably by generating appropriate corules. The construction consists of two steps:

1. Starting from a monoid O modeling finite observations (e.g., finite traces), we construct an ω -monoid (O, O_∞) also modeling infinite observations (e.g., infinite traces). The latter structure is a variation of the notion of ω -semigroup [7], including a *mixed product* composing a finite with a possibly infinite observation, and an *infinite product* mapping an infinite sequence of finite observations into a single one (possibly infinite).
2. Starting from an inference system defining a big-step judgment $c \Rightarrow \langle r, o \rangle$, with c denoting a configuration, $r \in R$ a result, and $o \in O$ a finite observation, we construct an inference system with corules defining an extended big-step judgment $c \Rightarrow \langle r_\infty, o_\infty \rangle$ with $r_\infty \in R_\infty = R + \{\infty\}$, and $o_\infty \in O_\infty$ a “possibly infinite” observation. The construction generates additional rules for *propagating divergence*, and corules for *introducing divergence* in a controlled way.

The exact corules added in the construction depend on the type of observations that one starts with. To show the effectiveness of our approach, we provide several instances of the framework, with different kinds of (finite) observations.



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Finally, we prove a correctness result for the construction. To this end, we assume the original big-step semantics to be equivalent to (finite sequences of steps in) a reference small-step semantics, and we show that, by applying the construction, we obtain an extended big-step semantics which is still equivalent to the small-step semantics, where we consider possibly infinite sequences of steps. As hypotheses, rather than just equivalence in the finite case (which would be not enough), we assume a set of *equivalence conditions* between individual big-step rules and the small-step relation. This proof of equivalence holds for deterministic semantics; issues arising in the non-deterministic case and a possible solution are sketched in the conclusion of the full paper.

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