

interpolation. 2. it is C^1 everywhere except for the data sites where it is only C^0 . 3. it has local support. 4. it has linear precision.

We have investigated the following: instead of considering a discrete set of data sites, check what happens if all data sites are along a given closed boundary curve, and also let the number of data sites go to infinity. The interpolant now becomes transfinite -- finite sums in Sibson's interpolant become convolution integrals for the transfinite case. The limits for these integrals have to be computed using the continuous Voronoi diagram of the boundary, a structure closely related to its medial axis.

For the special case of a circular boundary, we obtain parametric surfaces that are close to minimal. If we replace the planar circular domain by a spherical cap, we are able to reproduce Enneper's minimal surface exactly, but we only have a numerical "proof" for this.

and relative differential geometry. The latter uses the cutter surface (more precisely, the smooth surface of revolution determined by the cutting tool under rotation) as indicatrix and serves for formulation of local millability criteria. Under certain additional assumptions, local millability implies global millability.

Using the dual B-spline representation of rational surfaces, all surfaces which possess rational general offsets with respect to a given rational cutter can be explicitly constructed. In particular, a remarkable class of rational surfaces is studied, which have rational classical and general offsets for all toroidal cutters of a fixed axis direction. The support function of these surfaces is described with tensor products of trigonometric splines.

Multiresolution analysis with non-nested spaces

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The purpose of this paper is to present two multiresolution analyses (MRA) intended to be used in scientific visualization, and that are both based on a non-nested set of approximating spaces. The need for non nested spaces arises from the fact that the required scaling functions do not fulfill any refinement equation. Therefore we introduce in the first part the concept of approximated refinement equation, that allows to generalize the filter bank and exact reconstruction algorithms. The second part show how this concept enables to define a one parameter family of wavelet bases that realizes a blend between the Haar and the linear wavelet bases. These so-called BLaC (Blending of Linear and Constant) wavelets are used for the analysis of volume data defined on uniform grids. The effect of the blending parameter is shown on examples. In the last part of the paper, the approximated refinement is applied to build a MRA scheme for data defined on an arbitrary planar triangular mesh. The ability to deal with arbitrary triangular meshes, without subdivision connectivity, can be achieved only through the use of non nested approximating spaces, as introduced in the first part.

Transfinite Sibson surfaces

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Sibson's interpolant is a simple scattered data interpolant with the following properties: 1. it is the natural generalization of univariate piecewise linear

Polyhedral Approximation of 3D Objects

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The reconstruction of 3D objects from scattered 3D data is a problem which arises frequently since 3D scanning technology becomes more and more available. In this paper we present a new approach for the construction of a polyhedral approximation Q from an unorganized set of points P on or near the surface S of an unknown 3D object O .

Our algorithm is based on a minimal spanning tree (Voronoi minimal spanning tree VMST) of the Voronoi diagram of P . With the VMST a tessellation of the convex hull of P into polyhedra is formed from which each polyhedra is either inside or outside of Q and consists itself of a set of Delaunay tetrahedra. Applying afterwards an inside/outside classification method on this tessellation the desired approximation Q results.

Variational Design and parameter optimized surface fitting

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Siegfried Heinz (Karlsruhe)

Computer Aided Geometric Design has emerged from the need for freeform surfaces in CAD/CAM technologies; it has become a major research topic in computer science with direct applications for all engineering sciences. A major topic is the generation of smooth curves and surfaces which can be immediately supplied to the NC-process. The fundamental idea of the so called variational design methods is the use of modelling tools which minimize certain functionals which can be interpreted in the sense of physics or geometry. The purpose of this paper is to present a method to include the parametrization as an additional parameter in the variational design process.

General Offset Surfaces

Helmut Pottmann
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During 3-axis NC milling of a free form surface, a reference point on the axis of the cutting tool is moved on a general offset surface.

We study differential geometric properties of general offsets, both within Euclidean

Local Fairing of bicubic B-Spline Surfaces

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The problem of fairing free form curves and surfaces occurs in many applications of industrial geometric design. B-Spline surfaces are mostly used to represent complex shapes. But the design or reconstruction process of complex surfaces often leads to small imperfections of the surface (for example due to measurement errors). Once a misbehaviour of the surface was located by an appropriate surface interrogation method, a locally working fairing method improves the visual pleasantness of the surface. The purpose of this paper is to give an automatic and local fairing method for bicubic B-spline surfaces. Motivated by the works from Kjellander and Farin et al., we use the following fairing principle: a C^2 piecewise bicubic B-spline surface which is locally C^3 continuous is locally fairer. The sum of all local C^3 discontinuities in the surface area of interest is a scalar value which is minimized by an iterative algorithm. The fairing step consists of locally vanishing the C^3 discontinuity in each iteration. Applying the fairing algorithm only to a part of a surface one can keep unchanged the remaining surface areas. An extension of this algorithm was presented by introducing so called "fairing filters". This concept of filters allows to construct many different fairing methods by combining some basic fairing filters. Therefore it is possible to fair the surface in a more or less local manner.

A Smooth Surface Model from Contours

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We present a method for reconstructing a C^1 -continuous Bezier surface model of an object from a set of contours. The smooth surface is based on the construction of isoparametric curves, both horizontal and vertical, that inherit optimal properties from a triangulation of the contours. Smoothing is performed at the level of metatubes (maximal components of the object without branching) and canyons between metatubes. The point data is approximated at a controllable tolerance level. Contour reconstruction from CT and MR images is widely applicable in anatomical modeling and biomedical visualization, where smooth surface models have the capacity to yield more accurate analysis than triangulations.

Spatial Curves in Industrial Design: Shape-Interrogation and Fairing Methods

by K.G. Pigounakis, N.S. Sapidis, P.D. Kaklis
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Three-dimensional curves are playing an increasing role in computer-aided product design. Current industrial practices to evaluating and improving the fairness of such a curve rely on planar curve-fairing techniques applied to projections of a 3-D curve. This work demonstrates that these practices lack any mathematical foundation on the basis of a necessary and sufficient condition for an inflection on a projection involving the direction of projection and the Frenet frame of the 3-D curve.

Various visualization techniques are proposed, based on differential geometry, which in combination with standard curvature- and torsion-plots allow a complete evaluation of a spatial curve and of any fairing technique. Systematic analysis of industrial data leads to a definition for a fair 3-D curve and various fairness measures and algorithmic techniques for achieving this objective. Implementations are presented for three techniques representing current trends in fairing: (i) global fairing based on “energy” minimization, (ii) local fairing based on “energy” minimization, and (iii) local fairing using knot removal. The three algorithms are applied to industrial test-cases and the results are analyzed producing an evaluation of the three algorithms and of the underlying fairness measures.

Bezier Representations of Cyclide Patches

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Based on a joint work with Prof. Dr. W.L.F. Degen a new approach for generating rational Bezier patches on the symmetric Dupin horn cyclide, a special Dupin cyclide from which the other types may easily be obtained as offset surfaces, is presented. A new formulation of rectangular cyclide patches bounded by lines of curvature is obtained, and a representation of rational triangular cyclide Bezier patches is given for the first time. The approach is based on the concept of inversion. The main idea is to map the symmetric Dupin horn cyclide, given in its implicit form, to a right circular cone through an inversion with respect to one of the real conical points of the cyclide. A rational rectangular/triangular Bezier patch on this cone is then mapped back to the Dupin cyclide, thus obtaining its desired parametric Bezier formulation. Whereas the resulting rectangular Bezier patch on the cyclide is biquadratic, it is impossible to obtain quadratic triangular cyclide patches -- they turn out to be quartic.

Fill-in Regions for Space Curves

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RPI

We consider the problem of shape preserving three dimensional interpolation for a special type of three dimensional curve that we will call coils. A coil is essentially a non-strict version of a curve of geometric order three in three dimensions. This provides a natural generalization of a convex curve in the plane by replacing the planar concept of not crossing any line more than twice, by the spatial concept of not crossing any plane more than three times. The exact same definition can be applied to continuous curves and the shape preserving interpolation problem is then to fit a continuous coil to a discrete sequence of points that are a coil.

For this task we use the fill in regions which are regions in space which must contain all interpolating coils for a given set of points. We discuss how a coil preserving interpolation scheme must restrict these regions in order to localize the problem. These restrictions ultimately form the the osculating simplex for the continuous interpolating coil. The fill-in regions are an intrinsic part of all interpolation schemes that aid in visualizing the shapes of the coil.

NURBS and Face Octrees for Ship Hull Design

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A Surface-Surface Intersection Algorithm with applications to ship hull design is presented. The intersection algorithm is based on a hybrid surface model, involving a primary NURBS representation and a secondary Face Octree representation. Octrees give a fast way of detecting connected components of the intersection curves, together with initial estimates of starting points for the contour iterative tracking algorithm.

Along with their use in surface intersection algorithms, Face octrees are also a very convenient data structure for model interrogation and interference detection between the hull and inside components (pipes, etc). An outline of the involved algorithms is presented.

The result relies on a comparison of the projections into the plane of the characteristics of the nonlinear equation and the characteristics of the equation obtained through linearization around the initial function, i.e. the function determining the initial surface.

A good behavior of the characteristics of the linearized equation is ensured by the Poincaré-Bendixon theory. Benefiting from this and a characterization of the global diffeomorphisms between open, simply connected plane regions, we infer that the projections into the plane of the characteristics of the non-linear equation do not cross. From the latter, smooth solvability is readily seen.

The solvability theory is illustrated by numerical solutions of a few typical cases. These numerical solutions are obtained by finding the characteristics of the non-linear equations numerically. For these, in turn, Mathematica's facilities for numerical solution of systems of ordinary differential equations are used.

Blossoming and Apprimation in extended Chebychev spaces

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The blossoming principle, introduced by L. Ramshaw in 1987, is a powerful tool to define the control points of a polynomial curve and to express the De Casteljau algorithm. Moreover, using its characterizations by osculating flats or by De Boor-Fix formula, it is possible to generalize it to some more general spaces, namely extended Chebychev spaces (denoted EC spaces). Nevertheless, for such spaces, we can not assure the multiaffinity of the blossom, and that implies a dependence on the considered reference interval. Still, we can define Bezier points of a curve for a given interval I in R , and we can apply the De Casteljau algorithm for curves. Then, two kinds of splines can be defined: piecewise functions belonging to a same EC space, and more general ones, generated by functions belonging to different EC spaces. For a given knots vector, it is then possible to define a splines by its control points. By duality, we obtain a 'B-spline like' basis. This allows us to compute in Chebychev spline spaces a mean square approximation of a set of points, and then we can use the tension parameter relative to the non affinity of the blossom. This generalized splines offer a lot of shape parameters (especially in the case of geometric Chebychev splines), nevertheless the algorithms are much more costly as in the polynomial case.

A Tale of Two Resultants: Implicitizing Rational Curves by the Method of Moving Conics even in the Presence of Base Points

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Ron Goldman, Rice Univ.

A collection of new resultants is introduced which are hybrids of the Sylvester and Bezout resultants. For two univariate polynomials f, g of degree n , a resultant $R_r(f, g)$ of order $(n+r) \times (n+r)$, $r=0, \dots, n$, is constructed consisting of $n-r$ rows of the Bezout resultant and $2r$ rows of the Sylvester resultant. We show that R_0 is the Bezout resultant, R_n is the Sylvester resultant, and $\det(R_{r+1}) = \det(R_r)$, $r=0, \dots, n-1$.

These hybrid resultants are then applied to prove that implicitizing a rational curve with base points by the method of moving conics always succeeds in the absence of low degree moving lines that follow the curve. In particular, given a rational curve of degree $2n$ with $2r$ base points and no moving lines of degree $n-r-1$ that follow the curve, we prove that there is always an $n \times n$ matrix consisting of $2r$ linear rows and $n-r$ quadratic rows whose determinant is the implicit equation of the rational curve. We also show how to construct this matrix using only simple techniques from linear algebra.

Smooth solvability and numerical solutions of the shape-from-shading problem

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To recover a smooth surface from a single image of it, one has to solve a non-linear, partial differential equation, known as the image irradiance equation. Improving a computer-based representation of a surface by improving its image intensity or isophotes, one is led to the same equation.

In the first case, there is a physical, existing surface one wants to represent. In the second case, there is no guarantee that such a surface, having the modified image intensity, does exist. In the talk, nonexistence of this kind will be illustrated.

In practical surface design, however, the modifications of the image intensity is most often carried out in steps, with each step fairly small and without rapid changes. In particular, this holds for modifications of the image intensity introduced through modifications of isophotes.

For such small changes, we have a perturbation problem for the equation. By using the classical method of characteristics it is shown that for perturbations, whose first- and second order derivatives are small enough, and if a few other natural conditions are satisfied, the non-linear equation has a smooth solution.

rendering performance, transmission bandwidth, and storage capacities. This paper introduces the progressive mesh (PM) representation, a new scheme for storing and transmitting arbitrary triangle meshes. This efficient, lossless, continuous-resolution representation addresses several practical problems in graphics: smooth geomorphing of level-of-detail approximations, progressive transmission, mesh compression, and selective refinement.

In addition, we present a new mesh simplification procedure for constructing a PM representation from an arbitrary mesh. The goal of this optimization procedure is to preserve not just the geometry of the original mesh, but more importantly its overall appearance as defined by its discrete and scalar appearance attributes such as material identifiers, color values, normals, and texture coordinates. In particular, special attention is given to discontinuities such as material boundaries and creases. We demonstrate construction of the PM representation and its applications using several practical models.

A new family of de Casteljau subdivision algorithms

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The standard de Casteljau subdivision algorithm computes new Bezier representations of curves or surfaces on subdomains obtained after inserting a point in the domain. If the point lies outside the domain, the algorithm requires non-convex combinations. We present a variation of the de Casteljau algorithm that always uses convex combinations for any inserted point inside or outside the domain. Moreover we can always choose this combination to be a mid-point insertion. These algorithms require scaling of given control points. We also provide a geometric interpretation and a geometric construction for these algorithms.

We further present a new algorithm that computes Bezier representations on subdomains obtained after inserting a line in the domain. This algorithm is derived using a point-line duality. This algorithm can be used to trim a triangular Bezier surface very conveniently along a Bezier curve that is the image of any line in the domain.

A Discrete Approach for Removing Slight Shape Failures from Tensor-Product B-splines Surfaces

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Let be given a tensor-product B-spline surface, henceforth referred to as the reference surface, which exhibits in a sub-domain of its parametric domain of definition a shape failure, i.e., the sign of its gaussian and/or mean curvature is therein opposite to the designer needs. Furthermore, this shape failure should be slight, in the sense that the maximum of the wrongly-signed gaussian/ mean curvature should be "small". We seek to construct a perturbation of the reference surface, which is free from any shape failure in the whole parametric domain, while staying as close as possible to the reference surface.

For this purpose, we first introduce a series of new notions, which classify the effect of moving a single control vertex of a B-spline surface onto the value of its gaussian and mean curvature at a specific point of the parametric domain. More analytically, we introduce the notion of parabolic/minimal loci of a control vertex wrt a parametric point, which are the loci of this vertex for which the gaussian/mean curvature vanishes at the parametric point under question. It is shown that the parabolic/minimal loci constitute a quadric/cubicoid in E^3 . Furthermore, these notions imply, in their turn, the notions of elliptic/hyperbolic and convex/concave domain of a control vertex wrt a parametric point and can be naturally generalized for a set of vertices.

Then, based on the above notions, we proceed to formulate two discrete versions of the above problem. In both versions, the new locations of the control vertices, which affect the shape-failure domain, have to minimize their Euclidean distance from the initial ones, while satisfying constraints on the sign of the gaussian and mean curvature over a grid covering the shape-failure domain. Next, exploiting the assumption that the final surface should be a perturbation of the reference one, we linearize the shape constraints, which leads to a quadratic-programming discrete version of the initial problem. The numerical performance of this last version is then tested and discussed for a set of industrial examples.

Progressive Meshes

Hugues Hoppe

Microsoft Research

Highly detailed geometric models are rapidly becoming commonplace in computer graphics. These models, often represented as complex triangle meshes, challenge

Reverse Engineering of Geometric Models

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Hungarian Academy of Sciences

Reverse Engineering of shapes involves creating geometric models for objects for which no such model is available. Typically a large set of measured data points is taken and "converted" into some continuous model of boundary representation type. The most important algorithmic steps from data acquisition through segmentation and surface fitting to the final model creation are presented. One of the key problems of free-form surface reconstruction is to find a natural segmentation associated with a point cloud. Unlike the case of canonical surface representations, for free-form surfaces we need some underlying patch topology to perform surface fitting. Four different strategies to solve this problem are presented with pros and cons. These strategies differ in the surface representation used, the distance and fairing constraints, the underlying topology and the amount of interactive help required. Finally, it is an important issue whether functional elements or certain free-form features can be identified during the reconstruction process. Finding free-form blending surfaces which join larger primary surfaces is also part of this problem.

Interrogative Visualization of Scalar Fields

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Interrogative visualization refers to the process of interactive display and quantitative querying of data for metric, combinatorial and topological information. To support this paradigm, we build a search structure over the scalar field while adaptively modeling the data with tensor product splines. We approximate dense sets of (multiple) volumetric scattered scalar values with C^1 -continuous trivariate tensor product polynomial spline functions of low degree. A preprocessing step selects a subset S of the domain cells of the scalar field and builds a binary search tree over S . Given a particular isovalue, the domain cells in S which intersect a given isocontour are extracted using a fast range search. each connected component is swept out using a fast isocontour propagation algorithm.

dynamics are employed to model the flow, including uniform, obstacle and vortex simulation, and combinations thereof. Methods develop from the system for efficient computation of the flow. A complex fluid flow is recreated in real time.

Conformally mapped regions satisfy the 'Hamilton principle' in that a minimum energy path in one regions maps to a minimum energy path in the other. This is the basis for many physical simulations that are model in one domain and then mapped to another. Introduction of CAGD techniques in the design of these new domains improves both modeling facility and computational efficiency.

The Sensitivity of a Spline Function to Perturbations of the Knots

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In many applications of splines, like design, intersection problems, offsets, and data compression, the knot vector is the result of long computations or interactive manipulations. It is then possible that knots may be placed very close together without actually being equal. To get a "nicer" knot vector, it may then be tempting to perturb some knots a little and in this way get a more evenly spaced knot vector, or introduce knots that are identical instead of almost identical. Another situation where one would like to change knots is in lofting, where a family of spline curves need to be defined over a common knot vector. The problem with moving knots is of course that the spline function in general will change when a knot is moved. One possibility would be to compute a "good" (or "best") approximation to the given spline on the perturbed knot vector.

In this paper we follow a different approach. We develop simple estimates for how the spline depends on a knot. Then, given a tolerance, it is possible to give an upper limit on how much a knot may be moved without the spline changing more than the tolerance.

The paper will discuss several consequences of the following inequality.

Suppose $f = \sum_{i=1}^n c_i B_{i,k,t}$ and $g = \sum_{i=1}^n c_i B_{i,k,s}$ are univariate splines defined on two different knot vectors, but that they have the same B-spline coefficients $c = (c_i)_{i=1}^n$. Suppose $i_{,j} > 0$ for $j = k + 1, \dots, n$, and $i = j - k + 1, \dots, j$ where

$$i_{,j} = \max\{t_{i+k-1} - s_i, t_{i+k-1} - t_j, s_j - t_j, s_j - s_i\}.$$

Then

$$\|f - g\|_{\infty} \leq \sum_{j=k+1}^n |t_j - s_j| \sum_{i=j-k+1}^j |c_i - c_{i-1}| / i_{,j}.$$

The Design of Conformal Maps

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Conformal maps of the plane are the basis for many physical models including elasticity, electrostatics, heat conduction and fluid flow. Newly emerging applications such as image warping, morphing and data interpolation produce natural looking images and designs. Finite element meshes also benefit from conformal techniques that create right angled elements.

Past methods for determining conformal maps to complex boundaries required a piecewise linear boundary approximation. Three main approaches to computing conformal maps have been: (1) Integrate the Schwarz Christoffel Formulae, (2) Approximate the problem with Jacobi polynomials and solve the resulting system of equations and (3) Solve by finite element methods. Method (1) is very time consuming. Method (2) uses non integral generalizations of Legendre polynomials. Methods (2) and (3) yield systems of equations that can have large dimensions depending on the number of line segments in the boundary. Design interfaces are based on generation of the polygonal boundary. Sometimes circular arcs can be included in the boundary.

To increase design capability and computing efficiency, a conformal or near conformal deformation of the plane is defined which has a Bezier or B-spline curve boundary. The mappings result from minimization of a functional based on the Cauchy Reimann equations. This leads to a sparse system of linear equations, i.e. a $2n$ by $2m$ system where n and m are the parametric degrees of the deformation.

The Bezier derivation is simple and exact based on the Bezier curve written in complex polynomial form which is an analytic function and therefore conformal. Unfortunately the only way to improve an approximation in this form is through degree elevation which makes design more awkward.

In parametric form we are able to minimize a functional of the Cauchy Riemann equations to produce a conformal map if the degrees of both parameters match; or a near conformal match if the transverse parameter degree is less than the boundary degree. The latter case allows extension to a B-spline boundary. This becomes a far more useful design interface and allows for iterated subdivision to improve approximations locally. The error is typically less than a 1 degree maximum deviation on the entire region before any subdivision. The error is even smaller within the interior region of the deformation. Larger error is found at the ends of the B-spline region, a characteristic that can be exploited while designing the model.

Applications are demonstrated for potential flows which use the near conformal map about a B-spline to create and visualize stream flow. Techniques from fluid

Ottrees meet splines

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Maximum subdivision classical octrees are one of the extended discrete models for solids. They are very efficient in some respects, notably interference checking, but at times a smooth representation of the object's boundary may be needed, for example for model exportation.

We present a combination of techniques in other areas to extract a smooth model of an object's boundary given a maximal subdivision classical octree representation, where the vertices of terminal grey nodes carry a tag indicating their classification with respect to the solid.

The method combines the construction of an implicit spline surface which is then smoothed using fairing techniques. An effort is made to handle only the relevant portion of the surface; a B-spline basis is used but although the knots are very dense (associated to the smallest subdivision) the coefficient matrix is sparse, and most of the functions are plainly not used. Information in the octree itself is used to make up for the missing coefficients.

Design of Developable Surfaces Using Riemannian Geometry

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This paper presents parametric and implicit methods for designing developable surfaces that uses Riemannian geometry and generalizes de Casteljau method for constructing Bezier curves. A developable surface is typically represented parametrically, as a one parameter family of rulings defined along a base curve, or implicitly, as the envelope of a one parameter family of planes in space. In our approach, the control points in de Casteljau's algorithm are replaced by control lines. The resulting developable surface can then be viewed as an approximation to the control lines, in the sense that in the parametric method the rulings of the generated surface approximate the control lines, while in the implicit method the control lines are approximated by a set of lines normal to the surface envelope. The unifying thread between the two methods is the use of Riemannian geometry involving the identification of the orientation of lines and planes in R^3 with a two dimensional real projective space. The developable surfaces constructed in this way are coordinate invariant, and independent of the choice of origin and length scale for the physical space.

minimal error. Considering this set of triangles, we determine the one with largest local approximation error and split it into subtriangles using variable split points along the edges. Subdivision terminates when a triangulation is obtained whose associated global approximation error is smaller than some specified tolerance. We have tested the method for scalar- and vector-valued data.

Representations for d-dimensional polyhedra

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University of Bern

A Nef polyhedron is any set in the d-dimensional Euclidean space which can be obtained by applying finitely many set operations "complement" and "intersection" to finitely many linear halfspaces (1978). Linear polyhedra and polyhedral sets are special Nef polyhedra. 4 representations for Nef polyhedra are discussed: Binary construction tree (CSG tree), cell decomposition, binary space partitioning (BSP forest) and a kind of boundary representation (RWS) which seems especially appropriate for most applications. Using RWS ("reduced Wuerzburg structure"), a Nef polyhedron is essentially represented by the locally adjoined pyramids belonging to the minimal faces. These pyramids themselves are represented by cell decomposition. Algorithms are given for converting these 4 representations for Nef polyhedra into each other.

Smooth Subdividable Spline Surfaces of arbitrary Shape

Hartmut Prautzsch
Univ. Karlsruhe

Constructing C^k -spline surfaces of arbitrary shape by existing methods is quite cumbersome. Moreover, the degree of such surfaces is relatively high, namely $O(k^2)$, and it is not clear how to present such surfaces by a single spline control net as one can do it with tensor product or box spline surfaces.

Here a new method is introduced to construct G^k -surfaces with regular parametrizations of low degree from one single spline control net. The piecewise degree is $2k+2$, $2k+2$ at extraordinary points and $k+1$, $k+1$ elsewhere. For certain limiting choices of some free parameters in the construction one can also obtain singularly parametrized C^k splines.

Furthermore, subdivision algorithms are derived for the control nets of the regular spline surfaces above. Thus the existence of such algorithms is proved.

original data.

We use topological methods for visualizing the flow over a spherical domain. A new explicit method based upon linear variation over triangular domains will be described.

Some new Haar wavelets for triangular subdivisions will be described. These new wavelets are only biorthogonal, but have the property that when the areas of the subtriangles of a subdivision have uniform area, the wavelets become fully orthogonal.

n-sided patches

Hans Wolters
SDRC

In solid modeling applications it is often necessary to fill in holes which have been created by application such as edge fillets. Furthermore it is required that the patch used to fill in the hole is tangent continuous with respect to neighbouring surfaces. We present a new construction technique for a 4-sided patch in NURBS representation. The basic building blocks are C^0 and C^1 Coons blending schemes. Starting with a C^0 Coons blending surface we iteratively produce a patch which is tangent continuous to the neighbouring patches. This is achieved by a constrained modification of the first inner layer of control points, followed by a subsequent C^1 Coons blend. Each iteration step requires the solution of a least squares problem. The main idea is that we modify the control points such that the normals along the boundary coincide with the corresponding of the neighboring patches. Subsequently we show how the 4-sided patch can be used for the construction of general n-sided patches. The n-sided domain is subdivided into 4 sided regions and the technique described above is applied to each of the regions.

Yet another data-dependent triangulation scheme for scattered data in the plane

James C. Barnes, Bernd HAMANN and Shiming Xie
UC Davis

We describe a multiresolution representation for multi-valued, scattered data given in the plane. In an initial step we determine a data-dependent triangulation of those points defining the convex hull of the point set in the plane. We choose the triangulation whose associated error (piecewise linear approximation of the data) has

THIRD DAGSTUHL WORKSHOP ON GEOMETRIC MODELING JUNE 1996

The third Dagstuhl workshop on Geometric Modeling was organized by Hanspeter Bieri (Univ. Bern), Guido Brunnett (Univ. Kaiserslautern), Tony DeRose (Pixar), and Gerald Farin (Arizona State Univ.). The Dagstuhl workshops are now clearly established as one of the essential meetings in this discipline.

Underlining the importance of the workshop, several awards were given to eminent researchers in the field of Geometric modeling. The awardees were

P. de Casteljau,
G. Nielson,
P. Bezier,
W. Boehm,
R. Barnhill,
W. Gordon.

Credit for the initiation of these awards goes to Hans Hagen, funding was provided by TechMat, Kaiserslautern.

Modeling and Multiresolution Analysis and Visualization of Flow Data over a Spherical Domain

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This report will discuss some work on the development of multiresolution methods for the analysis and visualization of scattered, vector-valued data over a spherical domain. An example data set would consist of wind direction and magnitude measured at various scattered locations over the surface of the earth. We wish to model and display this data at several different levels of resolution. We will describe in varying detail three aspects of this problem: Modeling scattered vector-valued data over the sphere, visualization techniques for these models and multiresolution methods for spherical vector data.

The methods for modeling the vector-valued data over a spherical domain are based upon a lifting scheme for scalar interpolants defined over a sphere. This lifting scheme guarantees that the vector valued interpolant has the property that the range vector is perpendicular to the domain evaluation point which is a constraint on the